THE UNIVERSAL MEASURER.

PART I.

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THE UNIVERSAL MEASURER.

IN TWO PARTS:

The FIRST PART.

Contains the Theory of Measuring in Projecting and Dividing the Forms of Superficial and Solid Figures.

The SECOND PART.

Teacheth the Practice of Measuring in all it's various uses whether Artificers Works, Gauging, Surveying, or Mining.

By A. FLETCHER. *Philomath.*

WHITEHAVEN.

Printed by W. MASHEDER 1752.

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THE

PREFACE.



AS *I hope the enſueing Work is able to anſwer for its Truth and Uſefulneſs, ſo I ſhall here take up no more of my* READER's *Time than to give them a brief Account of ſome of the Principal Articles herein contain'd, in Order as they are Printed.*

Firſt, Practical Geometry 72 Definitions *ſhewing the Forms and Names of Figures, how they*

they are conſtituted, and the Nature of Superficial *and* Solid Meaſure.

Secondly, 172 Problems. *The firſt* 31 *of which are taken up with the* Conſtruction *of* Figures *and* Scales. *The next* 12 *following are employ'd in the Rule of* Proportion, Extraction *of the* Square *and* Cube *Roots, whereby thoſe Rules in common Arithmetic, are clearly demonſtrated; and the* Variation *of* Proportion *ſhewn. Here is alſo demonſtrated the* 47 *Propoſition of* E. 1. *with the* 4, 5, 6, *&* 7, *Propoſitions of* E. 6.

Thirdly, Next come in the chief and Practical Problems *relating to the* Circle, *following is a Demonſtration of the* 19 *Propoſition of* E. 6.

Fourthly, Prob. 71 *begins with* Circumſcription, *and* Inſcription *of Figures, continued to* Prob. 84 *then follows a Demonſtration of the* 36, 37, *&* 38, *Propoſitions of* E. 1.

Fifthly, Prob. 86, *begins with* Reduction *and* Diviſion *of* Figures, *as well* Irregular *as* Regular. *Here is ſhewn the Myſtery of Superficial Meaſure Geometrically; and how any* Irregular *Plot of Ground*, &c. *may be divided into any poſſible Proportion, by Lines drawn thro' Points*

any

any how given ; as alſo how to cut from a Plot, any aſſign'd Quantity by a Line drawn thro' any Point any where given.

Sixthly, Prob. 121 *begins with the Caſes of* Plane Trigonometry, *wherein every particular Caſe is Geometrically laid down and ſolv'd, here is alſo* 20 Trigonometrical Definitions.

Seventhly, Prob. 120, *begins with* Menſuration *of* Heights *and* Diſtances, *Acceſſible and inacceſſible perform'd ſeveral ways ; where* NOTE, *that after* Prob. 135, *you have deſcrib'd the principle* Inſtruments *uſ'd in* Surveying *of* Land, Mines, *and in* Levelling, Prob. 140 *begins with* Levelling, *and* Prob. 142, *with* Surveying *of* Mines *both of which are perform'd ſeveral Ways, then are ſeveral difficult* Prob. *in* Navigation, *Geometrically ſolv'd.*

Eightly, Prob. 151, *begins with* Solid Geometry, *how to Project the ſeveral* Regular Solids, *and the five* Regular Bodies, *with the Method of their* Circumſcription & Inſcription *Geometrically.*

Ninethly, Prob. 165, *begins with the Verbal, and Geometrical Demonſtration of Theorems applicable to* Menſuration *and* Gauging.

Tenthly,

Tenthly, Prob. 167, *From the Property of any Curve-Lin'd Figure, to find its Area and Length of its Curve; from whence are deduc'd ſeveral uſeful Theorems relating to the Circle, Ellipſis, Parabola*, &c. *and their Parts.*

Eleventhly, Prob. 168, *From the Property of any Curve Lin'd Figure, turn'd about any of its Parts, forming a Solid; to find the Solidity and Convex Surface thereof, from which are deduc'd a great many Theorems relating to the* Sphere, Spheroid, Parabolic Conoid, Parabolic Spindle, &c. *with their ſeveral Fruſtums and Segments: with a Demonſtration of the limits of the mean Diameters ſhewing how far they may be depended on in Gauging the ſeveral Forms of Casks; with Theorems for finding any Circles Diſtance from the Bung, whereby any ſtanding Cask is eaſily Ullag'd, and any unequal Headed Cask Gauged. Here are alſo Theorems for finding the Forms of Casks*, &c.

Twelfthly, Prob. 169 *To Meaſure any Plane, or Solid, by Aproximation, from this* Prob *alone, I preſum'd to call this Book* UNIVERSAL, *and was once minded to Demonſtrate no other way for Meaſuring, but this only; But conſidering that*

that it is in ſome Shape New, however not found in any Book of this kind that ever I've met with. Therefore, altho' the Demonſtration of this Problem *is evident, yet I have Demonſtrated the forgoing ones, which are compared with this, to give a further Proof of its Truth. This Method of Meaſuring, has no regard to the Figures Form, but Meaſures all Univerſally as well* Solids *as* Planes ; *Its Inventor was* Sir Iſaac Newton, *and ſince Proſecuted by ſeveral able Mathematicians, particularly* Mr. Jones, Mr. Coats. Mr. De Moivre, Mr, Sterling, *in a whole Treatice, and by* Mr. Shertcliffe, *in his Book of Gauging, whoſe method I eſteem as moſt plain and ſo have holden neareſt to it. From this* Prob. *are deduc'd ſeveral Theorems, Particularly for Meaſuring* Priſmoids, Cylindroids, Hoofs, *&c. With a general Method for Gauging and Ullaging without any Notice taken of the Casks Form and yet gives the Content more exact than the common way of gueſſing at the Form with the mean Diameter, both theſe Methods are well illuſtrated in the* II. PART, (viz) *in Gauging, where the uſe of the* Double Sliding Rule *is ſhewn, with two New Lines for Ullaging, contriv'd by* Mr. Shertcliffe.

There are several things here remarkable, such as Definition 62, *of the* 5 Regular Bodies, *that there can be but* 5 *such Bodies, with their several Methods of Representaion* &c. Prob. 31 *how to Project a* Parabole *in Plano*, Prob. 69 *to Inscribe any Number of* Circles *in a given* Circle; Prob. 138, *To Survey a Field and not come in it*, Prob. 168, *Theorems*, 23 *and* 24, *To find the Solidity of any Conical Hoof, from the Properties of the* Cone *or* Ellipsis. Prob. 172, *any two Factors given to express their Ratio's of the* Circle *to its Diameter and how near Truth* &c. Prob. 171, *To Measure any* Spherical Triangle *or* Spherical Polygon. Prob. 170, *from the three sides of any* Plane Triangle *to find its Area* &c. *with many more too tedious to mention. I might here point out several New Articles, but tho' I have seen all Books of this kind that ever I could hear of, yet there may be some which I have not seen. I presume the Reader will find this Book better stor'd both with* Theory *and* Practice, *and that in more general Terms, than any yet extant: for most Authors who write upon* Theory *and* Practice, *have them mixt, by which means many good Books are hard to be understood. But here the Rules in the* II. PART *are all Demonstrated in*

in the I. PART, *where ſeveral Errors in Practice are pointed out, ſo that this* II. PART *is always upon* Practical *Methods, the Rules being Illuſtrated with proper Cutts and Examples, wrought at large both by the* Pen *and* Sliding Rule.

And as no Author hitherto, has thought fit to lay down this Method in one and the ſame Book, I hope that may be a ſufficient Apology for my Undertaking.

PART II.

SECT. 1. *Deſcription, Conſtruction, and uſe of* Coggleſhal's Sliding Rule, *with Multiplication, Diviſion, and the Rules of Proportion perform'd by one general Rule, ſo order'd, as that the Numbers never go off the Lines, howſoever unequal, Extraction of the Square Root* &c.

SECT,

SECT. 2. *Multiplication of Feet and Inches, commonly call'd Croſs Multiplication, in a plain and Demonſtrative manner.*

SECT. 5. Decimal Arithmetick *made eaſy, with ſeveral Contractions fully explain'd.*

SECT. 4. *Extractions of Roots explain'd, the common Method of Extracting the Square Root. With an Uuiverſal Theorem for Extracting any Root whatever, out of a Simple Power. Illuſtrated with near twenty Examples as eaſy as Decimal Diviſion.*

SECT. 5 *Menſuration of Surfaces, How to take Dimenſions, and work with, or without, Inches, every Caſe ſpoken particularly to.*

SECT. 6. *The ſeveral Artificer's works with an eaſy method for thoſe that deal much to keep a regular Book &c.*

SECT.

SECT. 7. *Solid Meaſure, every particular Solid handled apart.*

SECT. 8. *Surveying, Plotting, and Dividing of Lands, as alſo how to make Maps of the ſame.*

SECT. 9. *Gauging, as before hinted at, in all its parts.*

SECT. 10. *Queſtions with Sollutions at large, wkich exerciſes the whole Work.*

Thus READER's, *I have given you a ſhort account of the Work, and if you are acquainted with the Golden Rule,* (in whole Numbers,) *this Book is ſo eaſy, that with a little pains, you may become Maſters of it.*

I hope the READER, *will favourably paſs over any Inaccuracy of Style he may meet with, for I expect to be charg'd with defects from*

from this Quarter, yet I hope the whole will be as correct as any work of this Nature.

Little Broughton,
near Cockermouth
in Cumberland.
October 27*th*. 1752.

Abraham Fletcher.

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THE

THE UNIVERSAL MEASURER.

PART I.

GEOMETRICAL DEFINITIONS.

GEOMETRY signifies the Art of MEASURING in General.

A POINT is that which hath no Parts as A.

A LINE is made by the Motion of a Point; so has neither Breadth or Thickness. as, B——C

4. LINES are either Right, B——C
as BC; Curv'd or Crooked, D⌒E
as DE: or Mix'd as F G. F⌒G

5. OBJECTS

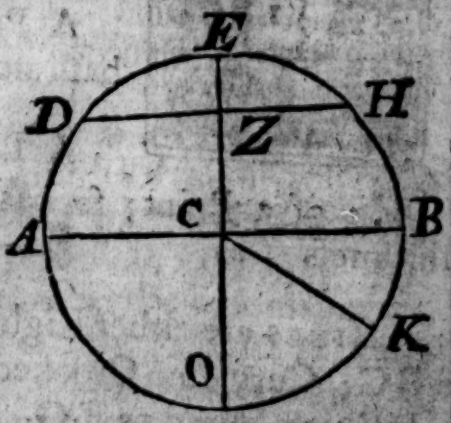
5. OBJECTS to be Meaſur'd are either Lineal, Superficial or Solid.

6. A SUPERFICIES is that which hath Length and Breadth only as Paper, Painting, Land, Glaſs, &c.

7. A SOLID, hath Length, Breadth and Thickneſs, ſuch as Timber, Stone, Walls, &c.

Of theſe two principal Kinds, (*viz.*) Solid and Superficial, there are many Forms or Shapes, both Regular and Irregular. The chief Superficial regular Forms are Three, (viz.) The CIRCLE, SQUARE and TRIANGLE; by which all Irregular Figures are Meaſur'd, they being Reduc'd into that regular Figure which they moſt Reſemble, which commonly falls under the Triangle.

8. A CIRCLE is a Round Superficies, the Line or Ring ſurrounding the ſame, is call'd the CIRCUMFERENCE, or PERIPHERY; as AEBO.



That Point equally diſtant therefrom and in the Middle of the Circle is call'd the CENTER of the Circle, as C is the Center of the Circle AEBO.

9. The

9. The DIAMETER is a Right-line drawn thro' the Center of any Circle, and Divides the ſame into two equal Parts, half Circles or Semicircles. So AB, and EO, are Diameters. And AEB, AOB, are Semicircles.

10. CORD or CHORD, is a Right-line drawn within the Circle, but not thro' the Center; and Divides the Circle into two unequal Parts call'd SEGMENTS, ſo DH is a Chord or Subtenſe; and DEHZD is a Segment leſs than a Semicircle, as DAOKBHZD is a Segment greater than Semicircle.

11. An ARCH is any Part of a Circle's Periphery cut off by a Chord, ſo DEH and DAOKBH are Arches.

12. RADIUS or Semidiameter is that Extent of the Compaſſes, with which the Circle's Periphery is deſcrib'd, ſo CB, CA, CE, CO and CK, are all Semidiameters.

13. A SECTOR is a Part or Portion of a Circle, form'd by two Right-lines or Semi-diameters drawn from the Center to the Periphery, ſo BCK, BCE, and ACK are Sectors.

14. A QUADRANT or Quarter of a Circle, is a Sector equal to half a Semicircle, ſo ECB, BCO, OCA and ACE are all Quadrants.

15. A DEGREE is the 360 Part of a Circle's Periphery,

Periphery, that is; every Circle is ſuppos'd to be Divided into 360 equal Parts call'd Degrees, and every Degree into 60 other equal Parts call'd Minutes, every Minute into 60 more equal Parts call'd Seconds, &c. So that a Semicircle contains 180 Degrees, and a Quadrant 90 Degrees. Thus much of the Circle is ſufficient for Superficial Meaſure.

16. A SQUARE is a Figure contain'd under 4 Right-lines, being all of one Length, and ſtanding Perpendicular, or at Right-angles one to another; as the Line AB, CD, DB, CA, being all one Length and ſtanding at Right-angles one to another do form the Geometrical Square ABCD.



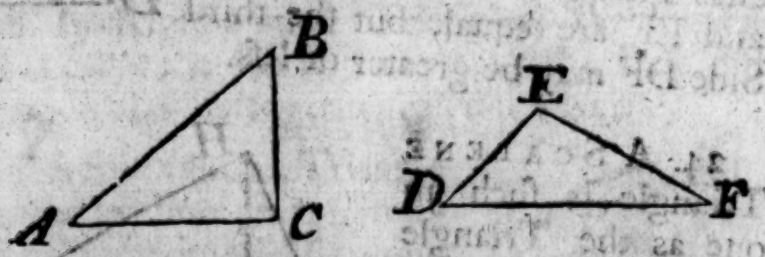
17. If any two Lines cut or croſs each other exactly either in the Middle, Ends, or any other Place (as a MASONS Squre,) thoſe Lines are ſaid to be Perpendicular, or at Right-angles one to the other.

18. A PARALLELOGRAM [or Long-Square,] is a four-ſided Figure contain'd under 4 Right-lines, (ſuch as Doors, Tables, &c.) whoſe Ends are equal, and Sides equal, ſtanding at Right-angles one to another, as the four Right lines, A B, C D, A C, D B, ſtand at Right-angles one to another, and form the Parallelogram ABCD.



19 Be-

19. Becauſe the Sides of Squares or Parallelograms ſtand at at Right-angles to one another, they are call'd Rectangles.



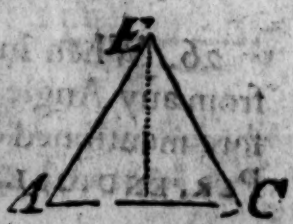
20. All TRIANGLES are made by the meeting or interſecting of three Lines, and are of various Forms, thus ABC, DEF, GHI, are all Triangles.



If in any Triangle there be a Right-angle, it's call'd a Right-angled Triangle, as in the Triangle ACB Right-angled at C. So the Triangle ABC is a Right-angled Triangle. All Angles that are not Right-angles, are call'd OBLIQUE-Angles. So the Triangle DEF and GHI, are Oblique-angled Triangles their Angles being all Oblique.

21. If any Angle be more than 90 Degrees, or a Right-angle, it's call'd an Obtuſe-angle; but if it be leſs, it's call'd an Acute-angle: And both Kinds are call'd Oblique-angles.

22. An EQUILATERAL Triangle is that which hath all its three Sides equal, ſo the Triangle AEC is an Equilateral Triangle having all its three Sides AE, EC, and CA equal

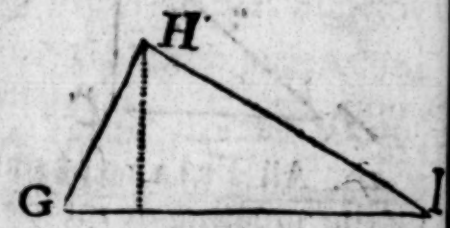


23. An

23. An ISOSCELES Triangle is that which hath only two Sides equal, ſo the Figure DEF is an Iſoſceles Triangle whoſe two Sides ED and EF are equal, but the third Side DF may be greater or leſs.



24. A SCALENE Triangle is ſuch an one as the Triangle GHI, having all its three Sides unequal: By theſe three laſt Definitions Triangles are Named in reſpect to their Sides.



25. Triangles Denominated from their Angles are 1. Right-angled Triangles (as by DEF. 20.) 2. Obtuſe-angled Triangles are ſuch as have one Angle Obtuſe. 3. Acute-angled Triangles are ſuch as have all the Angles Acute: And ſo in the above Figure GHI is an Obtuſe-angled Triangle, having the Angle at H Obtuſe; but the other two Triangles are both Acute-angled Triangles, theſe together are call'd Oblique, (as per DEF. 21.)

26. When in any Triangle a Line is let fall from any Angle upon the oppoſite Side, as in the fore-mentioned Figures, that Line is call'd a PERPENDICULAR.

27. BASE

27. BASE, is always that Side in any Figure upon which the Perpendicular falls, and the Perendicular must always fall from such an Angle, s that it may cut the Side upon which it falls at ight-angles.

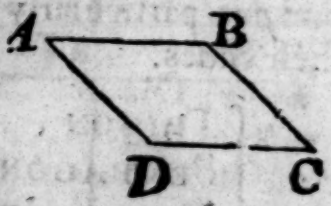
28. In Right-angled Triangles, the longeſt ide is call'd the HYPOTHENUSE; the Baſe and erpendicular are together call'd the LEGS.

29. A SPHERICAL Triangle s formed by three great Circles f the Sphere interſecting one nother as the Fig. ABC.



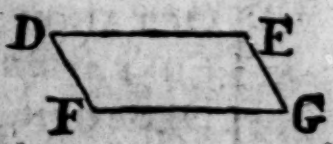
30. If ſome of the Sides of a Triangle be Right-ines and ſome be Arches or Circular-lines, thoſe Triangles are call'd MIXT Triangles.

31. RHOMBUS or Oblique-angled Square is a igure contain'd under 4 equal ides ſtanding at Oblique-an- les one with another, (as a ane of Glaſs) &c



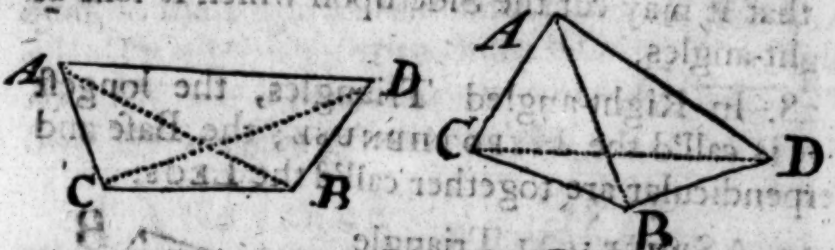
Thus the four Right-lines AB, CD, AD, BC, cut one nother at Oblique-angles, and form the Rhom-us ABCD.

32. A RHOMBOIDES or Oblique-angled Parallelogram s a Figure of 4 Sides, whoſe Ends or Baſes are equal, and ts Sides equal as the Figure DEFG, whoſe Ends re DF and EG, and its Sides FG and DE.



33. A

33. A TRAPEZIUM, Double Triangle, or Quadrangle; is any Figure of unequal Sides and Angles, ſo the Figures ABCD, are both Trapeziums.



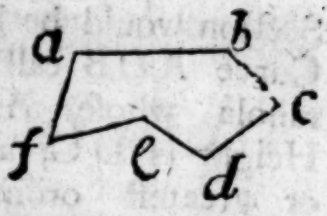
34. A DIAGONAL Line, is a Right-line drawn between any two oppoſite Angles, as in the above Trapezies, the pricked Lines are all Diagonals. A Diagonal drawn within a Square, Parallelogram, Rhombus, or Rhomboides, divides the Figure into two equal Parts, but moſt other Figures into two unequal Parts.

35. Regular POLYGONS, in general, mean all ſuch Figures as have equal Sides and Angles and are particularly Named from the Number of their Sides.

The	TRIGON	having	3	Equal Sides and as many Angles.
	PENTAGON		4	
	TETRAGON		5	
	HEXAGON		6	
	HEPTAGON		7	
	OCTAGON		8	
	ENNEAGON		9	
	DECAGON		10	
	ENDECAGON		11	
	DODECAGON		12	

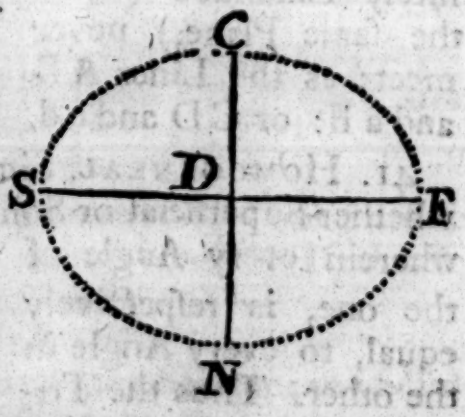
36. Irre-

36. Irregular POLYGONS, are ſuch Figures as have above four unequal Sides and Angles, ſuch as the Figure ABCDEF, of ſix unequal Sides, and as many unequal Angles; and it is called an irregular Hexagon.



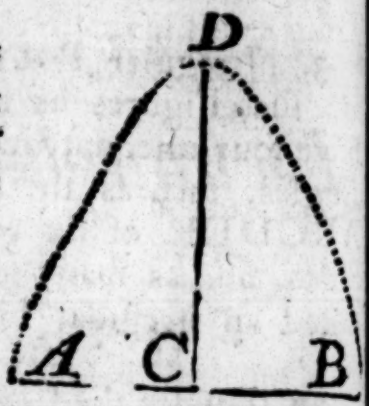
37. Beſides the Circle and its Parts: there are ſeveral other Curve-lin'd Figures, but thoſe of uſe in common Menſurations, are the ELLIPSIS, and PARABOLA, of the common Kinds.

38. The OVAL or ELLIPSIS, is an imperfect Circle longer than 'tis broad, ſuch as the Figure FSCN, whoſe Curve or Periphery is the prick'd outſide, and the two Lines croſſing each other in the Center or the Middle thereof at D, are called Diameters: the longer of which FS is called the Tranſverſe Diameter, and the ſhorter CN is call'd the Conjugate Diameter.

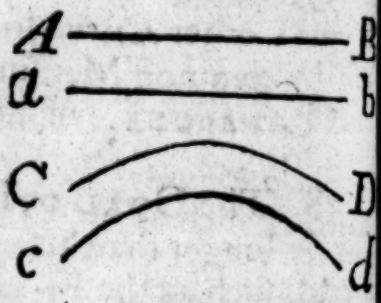


39. The PARABOLA is not much different from a Semi-Ellipſis; and is form'd by a Cone being cut parallel to its Sides, ſo if a Cone were cut parallel to one of its Sides the plane of that

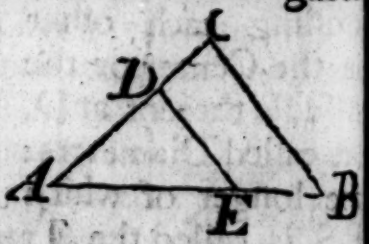
Section would be like the Curve A D B call'd a Parabola whoſe Axis or Height is D C, and Baſe or greateſt ordinate is A B, cutting D C at Right-angles.



40. PARALLEL LINES, are thoſe that lie equally diſtant from one another in all their Parts, viz. ſuch Lines as being infinitely Extended (upon the ſame Plane,) never meet: as the Lines A B and a b: or C D and c d.



41. HOMOGENEAL, Similar, or alike Figures whether Superficial or Solid are all ſuch Figures wherein every Angle of the one, is reſpectively equal, to every Angle in the other. Thus the Triangles A B C, and A D E are alike, having the Angle at A common to both Triangles, and the Angles B and E are equal, alſo the Angles D and C are equal. The Sides of ſuch Figures are Proportional.



42. Equal Figures, are all ſuch whether Superficial or Solid as have their Contents equal.

DE-

Definitions of *Solids.*

43. 

A CYLINDER is a round Solid whoſe Baſes or Ends are equal; that is, 'tis a Long Round Figure like a Rolling Stone in a Garden.

44. A PARALLELOPIPEDON or Solid Parallelogram, is a Figure bounded under 4 equal Sides and equal Ends; as a Cheſt or a piece of Square Timber.

45. A PRISM is a Solid contain'd under ſeveral Planes, and having its Baſes alike, equal, and parallel.

46. A CUBE or Solid Square, is a Solid whoſe Sides and two Ends are all equal and parallel; as a Dye, a Square Box, &c.

47. A CONE, or round ſpire Steeple; is a Solid which hath a round and circular Baſe, and grows equally leſs and leſs untill it end in a Point, call'd the VERTEX, or Top of the Cone.

48. A PYRAMID, is a Body whoſe Baſe is a Polygon, and whoſe Sides are plane Triangles, the ſeveral Tops meeting together in one Point.

49. A GLOBE or Solid Circle, is a round Solid ſuch as, Balls, Bullets, or the like.

59. A SPHERE, or hollow Globe; form'd by moving or turning a Semicircle about its Diameter, and is of the ſame form with a Bullet: Only a Bullet is Solid, and a Sphere is Concave or Hollow.

51. A SPHEROID or Egg like Figure, is a Solid formed by turning a Semi-Ellipſis about its longeſt or tranſverſe Diameter, and is call'd a Right Spheroid.

52. If a Semi-Ellipſis be turned about its Conjugate Diameter it will form a flat Solid call'd an Oblate Spheroid.

53. A PARABOLIC CONOID, is a Solid form'd by a Semi-Parabola turning about its Axis.

54. Alſo if a Parabola be turn'd about its Baſe it will form a Solid call'd a Pyramidoid, but is moſt commonly call'd a Parabolic Spindle.

55, AXIS, is the height of a Solid or a Right-line drawn through its Middle as the Diameter of a Globe or Sphere is its Axis. A Line drawn from the Top of a Cone or Pyramid to the Center or Middle of its Baſe is call'd its Axis, Altitude. or Perpendicular Height.

56. FRUSTUM, is the lower Part of a Cone or Pyra-

Pyramid, there being a part cut off the Top by a Plane parallel to its Baſe.

57. Middle Zone, is the Middle Part of a Globe, Sphere, or Spheroid; there being cut off two pieces equal and oppoſite, and thoſe pieces ſo cut off are call'd Segments, being in the Form of a Bowl, Bottom of a Copper, &c.

☞ Note, Middle Zone is by ſome called Fruſtrum of a Globe.

58. Hoof, if you ſuppoſe the Fruſtrum of a Cone or Pyramid, to be cut by a plane Diagonally from Baſe to Baſe; the two parts ſo cut are call'd Hoofs.

59. A Prismoid is the Fruſtrum of a Pyramid whoſe Baſes are Parallel to one another, but Diſproportional.

60. A Cylindroid is the Fruſtrum of a Cone, having its Baſes parallel to each other, but Diſproportional.

61. Solid Angle, is that which is contain'd under ſome Number of plane Angles more than two, which are not in the ſame Plane, meeting together in a Point. See the Prob. for making Solid Angles.

Of the Five *Regular* Bodies.

AS any Number of equal Sides and equal Angles, make up a Superficies called a regular Polygon; ſo a like Number of regular Polygons (of the ſame kind) make up a Solid, of which there can only be Five, which go under the title of the Five regular Bodies; and that there can be but Five ſuch Bodies, may be thus proved: For, 3 plane Angles (at leaſt) are required to the conſtituting of a Solid Angle, all of which, muſt be leſs than 4 Right-angles; for the Angular Points of 4 Right-angles, being join'd together will lie all in one plane; but 6 Angles of an Equilateral Triangle, 4 Angles of a Square, and 6 of an Hexagon do ſeverally equal 4 Right-angles. 4 of a Pentagon, 3 of a Heptagon, 3 of an Octagon, &c. do exceed 4 Right-angles. Therefore, of 3, 4, or 5 Equilateral Triangles, 3 Squares, or 3 Pentagons, 'tis poſſible to make a Solid Angle; wherefore, beſides the 5 above-mentioned Bodies, there cannot be another Regular Body form'd.

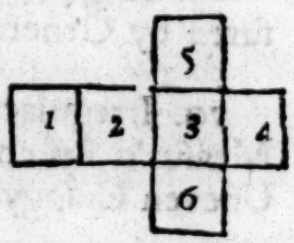
63. The

63. The Repreſentation of theſe Bodies may be expreſſed Three ſeveral Ways. 1. As they are deſcrib'd in Plano upon a plane Superficies. 2. As they repreſent themſelves in a Body-like Form. And 3. As they may be drawn upon pliable Paſte-board, and then in the Paſte-board cut the Lines half way thro' and Bow or Fold in the ſeveral Planes of which the Body conſiſts, and they will cloſe together in ſuch Sort, that they will make the perfect Body deſigned.

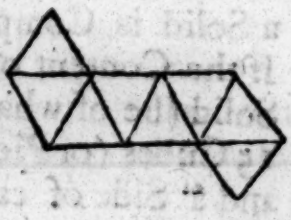
64. A TETRAEDON, is a Solid contain'd under four equal and Equilateral Triangles, and is no more but a Triangular Pyramid whoſe Baſe and 3 Sides are all Equal and Equilateral Triangles, A, B, and E upon the Sides of the Equilateral Triangle D &c.



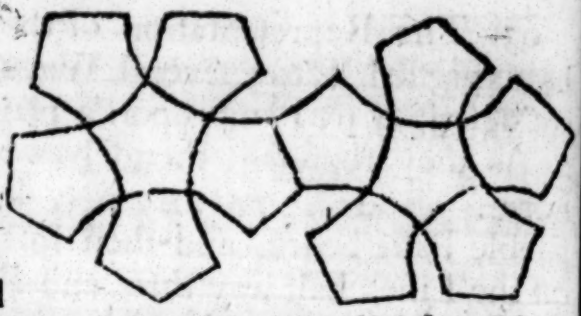
65. HEXAEDRON, is the ſame with a Cube. This Solid is form'd by turning up the equal Squares 2, 4, 5, 6, all upon the Sides of the Square mark'd 3, over which bend the Square marked 1.



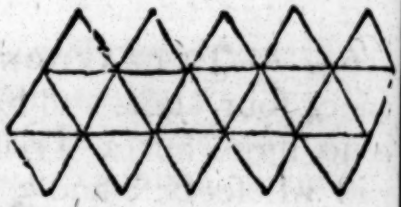
66. An OCTAEDRON is a Solid contained under Eight equal and Equilateral Triangles, for the Repreſentation of this Body turn up the Triangles of this Figure,



67. A



67. A DODECAEDRON is a Solid Body or Figure contained under 12 equal equilateral and Equiangular Pentagons and may be form'd by folding up the 12 Regular Pentagons in this Figure.



68. An ICOSAEDRON is a Solid which conſiſts of 20 Triangular Pyramids, whoſe Vertices meet in the Center of a Sphere, which is imagined to circumſcribe it.

69. Regular Solids, are ſuch as can be Meaſured by General Rules fitted to their Forms.

70. Irregular Solids, are ſuch as cannot be Meaſur'd by any one General Rule; ſuch as, Uneven Craggy Stone, Timber, &c.

71. Solidity, or Solid Content, is to find how many Cubes (whoſe Sides are of a given Length) a Solid is Compoſed of: As for EXAMPLE, If the Content of any Solid be 15 Inches, ſuch a Solid (be of what Form it will) may be cut into 15 Cubes (or Pieces that will make up 15 Cubes,) and a Side of each Cube will be one Inch.

72. Area,

72. Area, or Superficial Content, shews how many Superficial Squares (or Pieces that will make up Squares) any Plane or Surface may be cut into, or is compos'd of. ——— Convex Surface shews how many such Squares will cover the Out-side of any Solid.

GEOMETRICAL PROBLEMS.

PROBLEM I.

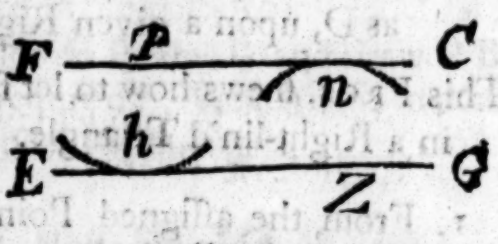
How to draw a Right-line PARALLEL to a given Right-line, at any given Diſtance therefrom.

Let it be requir'd to draw a Line Parallel to the Line AB, and at the Diſtance of the Line *e f* from it.

1. TAKE the given Parallels diſtance *e f* in your Compaſſes and with that Extent and one Foot in any Part of the given Line AB, as at L, Strike the Arch *r*, with the ſame Radius and one Foot in any other Part of the ſaid Line as in R, ſtrike another Arch *p*; draw the Line CD juſt to touch the Tops of both theſe Arches, and it will be the Parallel requir'd.



If it were requir'd to draw a Right-line thro' the Point P and Parallel to the Right-line EG: It may be done thus, Take the neareſt Diſtance between the given Point P and the given Line EG; which is done by ſetting the Compaſſes in P and ſweeping ſuch an Arch *h* as will juſt touch the ſaid Line EG, with which Diſtance P*h* and one Foot in any Part EG; as at Z ſweep an Arch *n*, touching the Top of which and thro' P draw the Right-line FC, which is the Parallel requir'd.

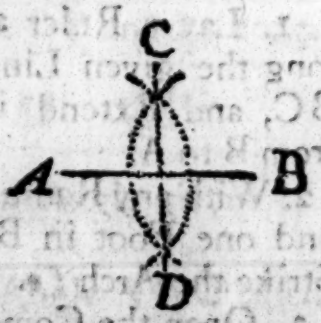


PROBLEM II.

To biſect or divide a given Right-line as AB into two equal Parts.

This PROB. alſo ſhews how to raiſe a Perpendicular in the Middle of any given Right-line.

1. Take any Radius greater than half the given Line AB and ſet one Foot of your Compaſſes in one End of the ſaid Line AB, ſuppoſe at B, and ſweep an Arch; alſo with the ſame Extent and one Foot in the other End A, ſtrike another Arch croſſing the former in C and D.



2. Lay a Ruler to the croſſing or interſecting of thoſe Arches and draw the Line CD and its done, for CD cuts AB at Right-angles in the Middle.

PRO-

PROBLEM III.

To let fall a Perpendicular from any given Point as D, upon a given Right-line AB.

This Prob. shews how to let fall a Perpendicular in a Right-lin'd Triangle.

1. From the assigned Point D to the given Line AB, draw a Right-line DA any how, which divide into two equal Parts in E, (which may easily be done *per* last Prob.)



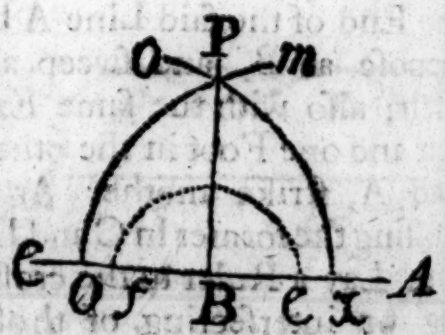
2. Upon E as a Center with the Radius EA equal ED, describe the Semicircle cutting AB in P. Join DP and it will be the Perpendicular reqd.

PROBLEM IV.

To raise a Perpendicular upon the End of any given Right-line, as at B in the given Line BC.

1. Lay a Ruler along the given Line BC, and Extend it from B to A.

2. With any Radius and one Foot in B, Strike the Arch *fe*.



3. Open the Compasses a piece further and with one Foot in *f*, Strike

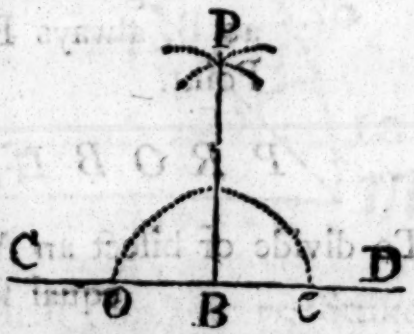
Strike the Arch *o m*; alſo with the ſame Extent and one Foot in *f* ſtrike the Arch *x o* cutting the former Arch in P.

Laſtly lay a Ruler to P and B and draw P B the Perpendicular requir'd: Which alſo makes the Angle CBP to be a Right-angle.

PROBLEM V.

To raiſe a Perpendicular upon any Point aſſigned in a given Right-line as upon B in the Line C D.

1. With any Radius and one Foot in the given Point B, Strike a Semi-circle *o e* cutting C D in *o* and *e*.



2. With any Radius more than B *e* equal B *o* and one foot in *o* ſtrike an Arch P, alſo with that Extent and one Foot in *e* Strike another Arch interſecting the former in P: Laſtly draw the Right-line B P and its done.

PROBLEM VI.

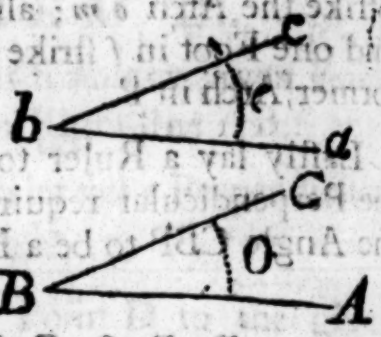
To make an Angle equal to an Angle given.

Let it be requir'd to make the Angle A B C equal to the Angle a b c.

1. Draw

1. Draw a Right-line as B A.

2. With any convenient Radius and one Foot in the angular Point *b*, Strike an Arch as *e*; with the ſame Extent and one Foot in B, Strike the Arch O.



3. Make the Arch *o* equal to the Arch *e*. Draw the Right-line B C, ſo is the Angle C B A equal to the Angle *a b c* as was requir'd.

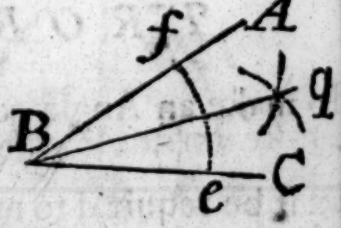
☞ If any three Letters be given concerning an Angle as A B C, the Middle Letter as B, always Repreſents the Angular Point.

PROBLEM VII.

To divide or biſect an Angle as A B C into two equal Parts.

1. With any Radius and one Foot in the Angular Point (B) Strike an Arch as *e f* cutting the Sides in *e* and *f*.

2. With any Radius and one Foot in *e*, Strike an Arch as *q*, with the ſame Extent and one Foot in *f* ſtrike an Arch interſecting the former in *q*. Laſtly lay a Ruler to *q* and B and draw the Right-line *q* B which



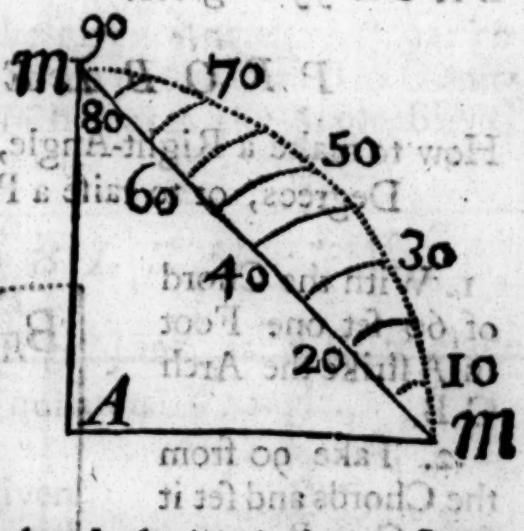
which will divide the Angle as was requir'd. (viz.) Make the Angle *q* B C equal to the Angle *q* B A.

PROBLEM VIII.

How to make a Line of CHORDS.

1. Draw a Right-line as A *m*, upon the End A raise a Perpendicular A *m*.

2 Make A *m* Radius and describe the Arch *m m*, which divide into 90 equal Parts, then draw a Line from the extream points of the Limb or Arch as *m m*, then with one Foot of your compasses in the intersection of the said Line with the Arch (viz.) in *m*, Strike Arches from the Degrees in the Limb to the Line *m m*, as 10, 20, 30 *&c.* So is *m m* a Line of Chords.



PROBLEM IX.

To make an Angle of any given Number of Degrees, (suppose 35.)

1. Draw

1. Draw the Line A B.

2. With the Chord of 60 Degrees and one Foot in A ſtrike the Arch *f e*.

From *f* ſet off 35 from the Chords to *e*.

4. Draw A *e* C the thing is done, the Angle B A C is 35 Degrees.



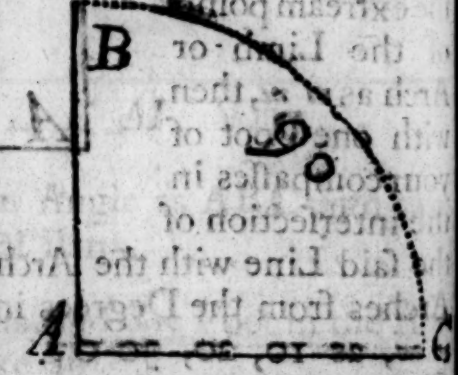
PROBLEM X.

How to make a Right-Angle, or an Angle of 90 Degrees, or to raiſe a Perpendicular.

1. With the Chord of 60 ſet one Foot in A ſtrike the Arch C B.

2. Take 90 from the Chords and ſet it from C to B.

3. Draw A B, then is C A B the Angle requir'd.



PROBLEM XI.

How to make an Obtuſe Angle ſuppoſe of 115 Degrees.

When an Angle is Obtuſe (viz.) more than 90 Degrees then ſet it off at twice.

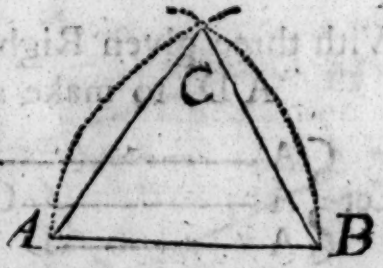
1. Take



1. Take half the given Angle (viz.) 57 : 30 et it twice on the ſweep of 60: from B to C draw he Line A C, which makes the Angle B A C 15 as requir'd.

PROBLEM XII.

Upon any given Right-Line, as A B, to make an Equilateral Triangle.



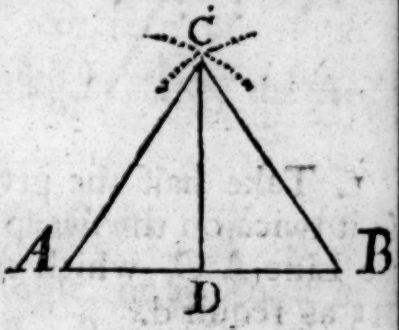
1. Make the given Line, A B Radius, and ſetting one Foot of the Compaſſes in B, ſtrike he Arch A C, alſo with he ſame Radius, and one Foot in A, ſtrike he Arch B C croſſing the former Arch in C.

2. Join C A and C B with Right-lines, and tis done, for the Triangle A B C is an Equi-ſided Triangle.

PROBLEM XIII.

To make an Iſoſceles Triangle of a given Baſe, A B, and Perpendicular C D.

Given { A——————B } Baſe
{ C——————D } Perpendicular



1. By Problem II. Biſect, the given Baſe A B, which falls in D.

2. Make C D equal the given Perpendicular C D.

3. From C draw C A and C B, ſo will the Triangle A B C, be an Iſoſceles Triangle, whoſe Baſe A B is equal to the given Line A B, and its Perpendicular C D equal to the given Line C D; as required.

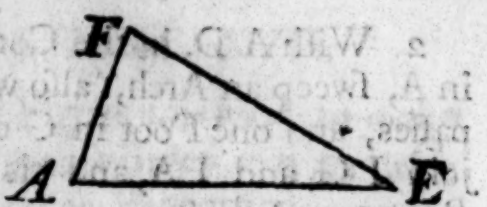
PROBLEM XIV.

With three given Right-lines, A B, A C, and A D, to make a Plane Triangle.

Let { A——————B
{ A——————C } be the given Lines
{ A——————D

NOTE, That ſome two of the Lines, taken together, muſt be longer than the third Line, otherwiſe it cannot poſſibly be a Triangle.

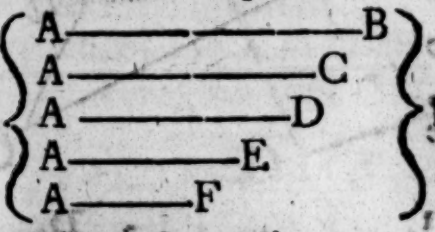
1 Take



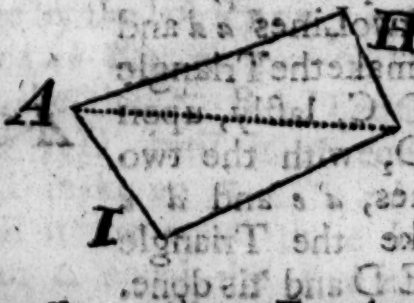
1. Take any of the ven Lines, ſuppoſe B, and lay it from to E in a Right-line. 2. Take either of the ther Lines in your ompaſſes as A G and with one Foot in A, ſtrike n Arch, alſo make the third Line A D, Radius, nd with one Foot in E ſtrike another Arch, croſſng the former Arch in F, join E F and F A nd the Triangle is made.

PROBLEM XV.

Having the Diagonal A B, and the 4 Sides, A C, A D, A E and A F of a Trapezia given, to make the Trapezia.



Let ... be the Lines given.



1. Make A G = A B, nd with any of the given Lines, as A C n the Compaſſes and ne Foot in either A r G, ſuppoſe in A, trike an Arch, alſo with A E in the Compaſſes, and one Foot in G weep another Arch, cutting the former Arch n H, join H A and H G, and one of the Trianles is made.

2. With

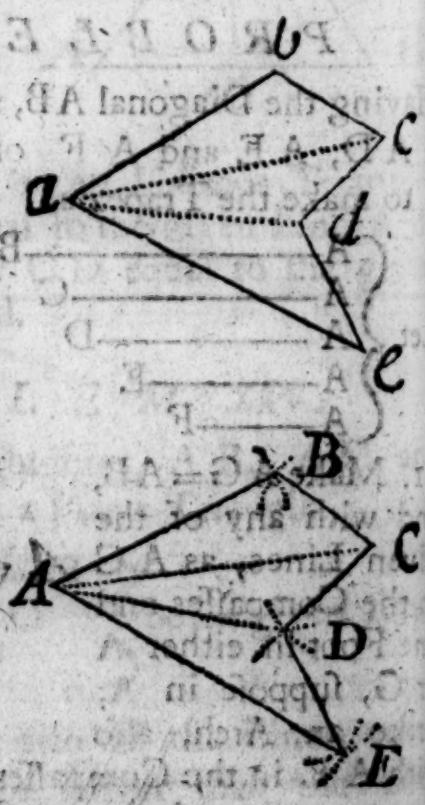
2. With A D, in the Compasses and one Foot in A, sweep an Arch, also with A F in the Compasses, and one Foot in G cut the last Arch in I, join I G and I A, and 'tis done. (See the last PROBLEM.)

PROBLEM XVI.

Given any Right-lin'd Figure, as *a b c d e*; to make another equal and alike thereunto.

1. Divide the given Figure *a b c d e* into Triangles, by drawing the Diagonals *a c* and *a d*.

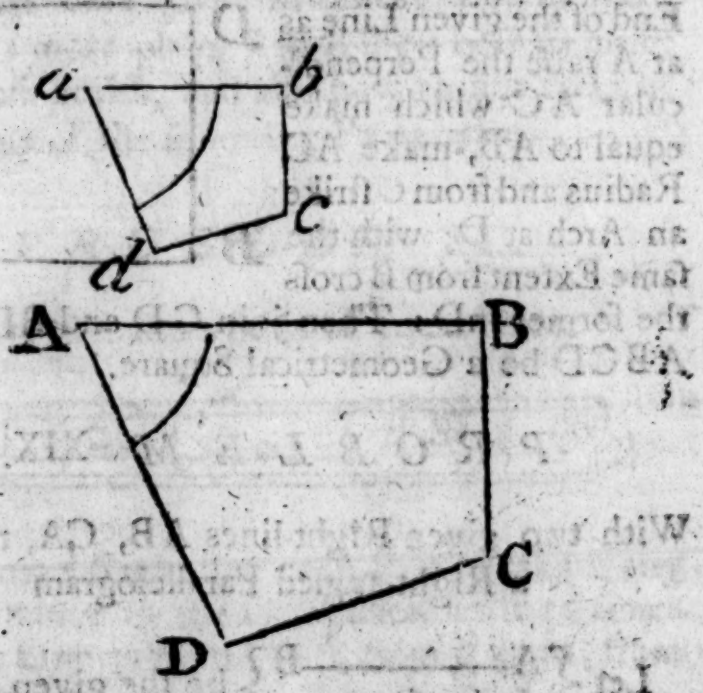
2. By Prob. XIV and XV; of the three Lines *a c*, *a b* and *b c*, make the Triangle A B C also upon A C, with the two Lines *a d* and *d c*, make the Triangle A D C, lastly, upon A D, with the two Lines, *a e* and *d e* make the Triangle A E D and 'tis done.



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PROBLEM XVII.

Given any Right-lin'd Figure as *a b c d*, to make another like Figure whose Sides may be twice as big; or in any other Proportion.



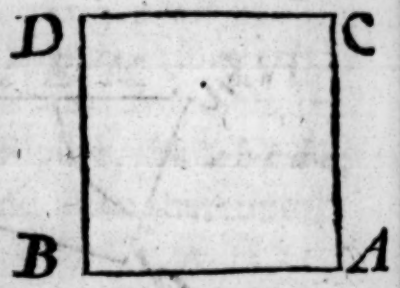
1. This may be done by the last PROB., only making A B equal twice *a b* &c. or more easily thus, by PROB. VI. make the ∠ B A D equal the ∠ *b a d* continuing A D until it be equal twice *a d*, and upon B make an ∠ = ∠ *a b c*, continuing B C, until it be equal twice *b c*, then upon C, make an ∠ equal the ∠ *b c d*, making C D equal twice *c d*, which in this Case will meet A D in D if the Figure be true drawn, and will make the ∠ A D C = ∠ *a d c*.

PRO-

PROBLEM XVIII.

Upon a given Right-line AB to make a Geometrical Square.

By PROB. IV. on one End of the given Line as at A raiſe the Perpendicular AC which make equal to AB, make AC Radius and from C ſtrike an Arch at D; with the ſame Extent from B croſs the former in D: Then join CD and BD ſo will ABCD be a Geometrical Square.

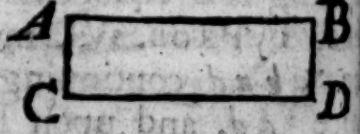


PROBLEM XIX.

With two given Right-lines AB, CA, to make a Right-angled Parallelogram

Let { A——————B / A——C } be the given Lines.

1. Make DC = AB, and at C raiſe the Perpendicular CA which make equal to AC given.



2. Make AB Radius and from A ſtrike an Arch at B.

3. With the Extent AC from D croſs the former Arch in B.

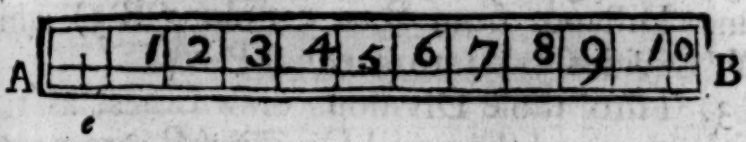
4. Join AB and DB, which gives the Rectangle requir'd.

N.

N. B. That if the Dimenſions of Figures be given in Numbers; the Figures may be made by a Scale of equal Parts, or a Diagonal Scale the ſame way as if the Dimenſions were given in Lines: But to make this more plain, I ſhall ſhew how to make thoſe Scales, and alſo how to uſe them in ſome of the following PROBLEMS.

PROBLEM XX.

How to make a Scale of equal Parts.



1. Draw a Right-line as A B, and taking any ſmall Diſtance in the Compaſſes, ſet it 11 times upon the Line as from A to 0, from 0 to 10, from 0 to 20, *&c.* to 100.

2. Divide the Diſtance A 0 into 10 equal Parts, which ſerves the whole Line for Units, ſo that 25 is the Diſtance *e* 20; alſo 75 is the Diſtance 70, *&c.*

*** If you begin to take, or Meaſure any Figure with a Scale of equal Parts, *&c.* you muſt uſe that ſame Scale, to all the Sides or the Operations will be falſe.

PRO-

PROBLEM XXI.

To Divide a given Line, AB into any Number of equal Parts, (ſuppoſe into five equal Parts.)

1. At each End of the given Line, by PROB. VI make equal Angles viz. ∠ BA 5 = ∠ AB 5.



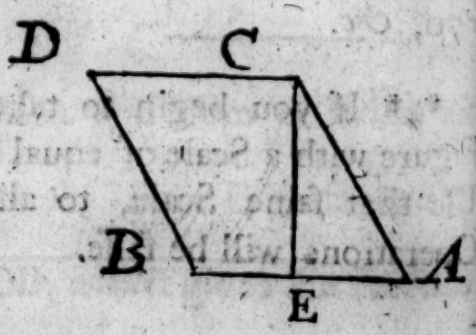
2. Take any ſmall Diſtance in your Compaſſes and run it along your Line A 5, dividing it into 5 equal Parts, alſo run the ſame Diſtance from B to 5 (along B 5) which alſo Divides 5 B into 5 equal Parts.

3. Thro' thoſe Diviſions draw Lines, as 1...4 2...3, *&c.* and they will Divide AB into 5 equal Parts as was requir'd.

PROBLEM XXII.

To make a Rhombus, whoſe Height may be 15 and each Side thereof 20.

1. Draw a Right-line, and take 20 from the Scale of equal Parts, and ſet thereon as from A to B.



2. Make A B Radius, and ſett-

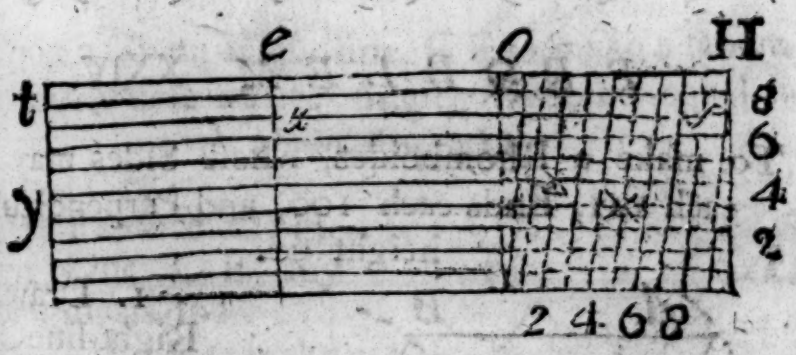
ing

ing one Foot in B ſweep the Arch D, likewiſe with one Foot in A ſweep the Arch C; from the ſame Scale of equal Parts, take 15 and with that Diſtance, by Prob. I. draw CD Parallel to AB, cutting the Arch in C and D.

3. Join CA and DB, ſo is ABCD the Rhombus requir'd.

PROBLEM XXIII.

How to make a Diagonal Scale.



A Diagonal Scale is divided into 10 large equal Diviſions, as the Diſtance between *e* and *o* is one of thoſe large Diviſions, and theſe ſignify Hundreds, then at the Side of the Scale there is another large Diviſion, as *o* H which is divided into 10 equal Parts, and theſe ſignify Tens.

The Figures 1, 2, 3, 4 *&c.* on the End ſignify Unites.

To find any Number on this Scale.

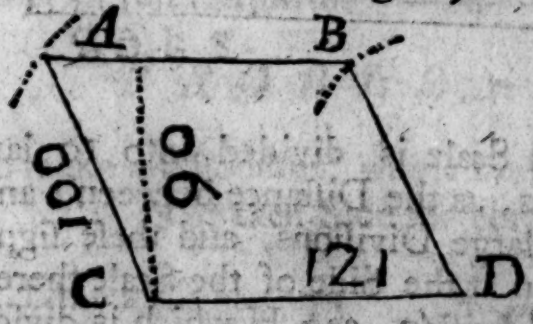
Suppoſe you would find 2 Chains, 25 Links; firſt for the 2 Chains, take 2 of the large Divi-

ſions. Then for the Links, go up to the Di-agonal Line, (*viz.*) on the Side run along that Line until you come under 5 (on the top) and that Place or meeting is 2 Chains 25 Links, which is at Z; ſo the Diſtance y Z is 2 C. 25 L. or 225 of equal Parts:

In like Manner the Diſtance						
	t H		C. L.		EP	
	y x	is	3 : 00	or	300	*of equal Parts.*
	t s		2 : 54		254	
	s u		2 : 88		288	
			1 : 88		188	

PROBLEM XXIV.

To make a Rhomboides, whoſe Sides may be each 121, Ends each 100, and Perpendicular height 90.



1. Draw a Right-line CD making it = 121 from the Diagonal Scale, then (by Prob. 1.) take 90 from your ſame one Foot, in-different Points of CD, ſweep the 2 Arches touching whoſe tops draw your Right-line AB from the ſame Diagonal Scale take 100, and with one Foot in C ſweep the Arch at A cutting A B in it, alſo ſetting one Foot in D ſweep the Arch at B cutting AB in B ſo is ABCD the Rhom-boid

boides required; the Perpendicular C *o* is let fall from the Point C upon A B by Prob. III.

NOTE, All Parallel Right-lines, falling upon or cutting the ſame Right-line make equal Angles as in the following Figure, the Lines B C and A E being Parallel make the Angles A E F and B C E, *viz.* the Arches *b e* and *e F* equal to each other therefore if you make the Angles A E F and B C E, equal the Lines A E and B C will be Parallel, 29 *e.* 1.

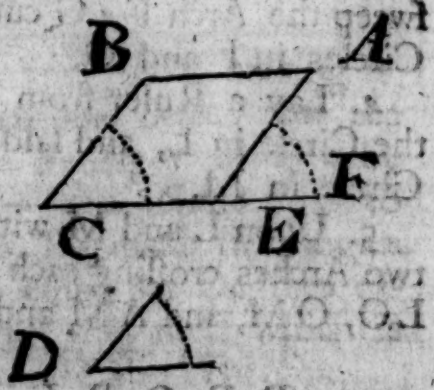
PROBLEM XXV.

Upon a Given Right-line, C E to make a Rhombus whoſe accute Angles ſhall be equal a given Angle D.

1. Produce the given Line C E any Length, as to F.

2. By Prob. 6. make the ∠ E C B and A E F each equal the given Angle D making C B and E A each equal E C.

3. Join A B and 'tis done.

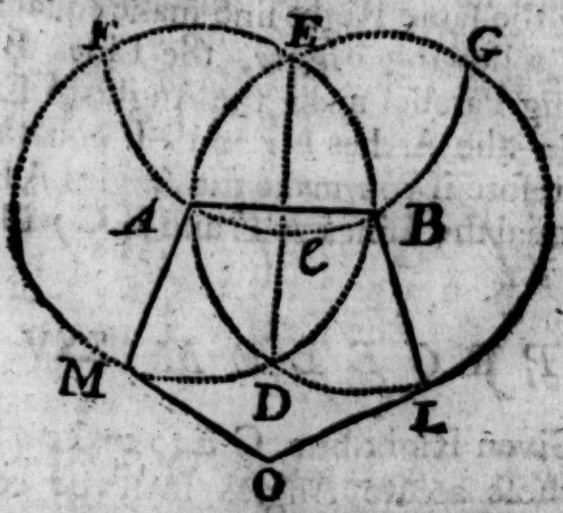


PROBLEM XXVI.

Upon any given Right-line A B to make a Regular Pentagon.

1. Make

1. Make the given Line AB Radius and on each End thereof describe a Circle crossing each other in D and E.



2. Draw the Right-line DE.

3. Make AB Radius and with one Foot in E, sweep the Arch F *e* G cutting ED in *e* and the Circles in F and G.

4. Lay a Ruler from F to *e* and it will cut the Circle in L, and laid from G to *e* cuts the Circle in M.

5. Upon L and M with the Radius AB strike two Arches crossing each other in O: Join BL, LO, OM, and AM and its done.

PROBLEM XXVII.

Upon a given Right-line to make any Regular *Polygon.*

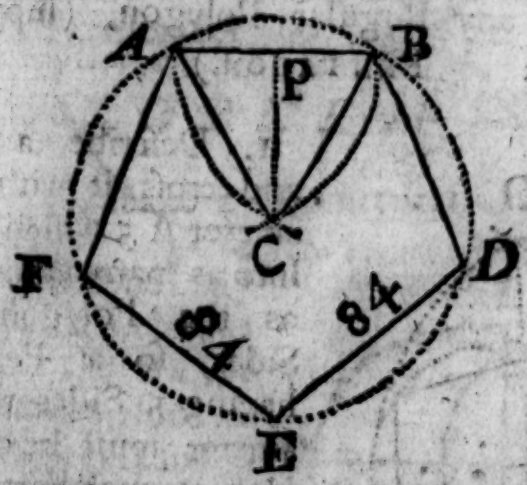
RULE.

Always Divide 180 Degrees by the Number of Sides that the given Polygon is to contain (See DEF. 15) Subtract the Quotient from 90 Deg.

Degrees, and of the remainder make an Angle at each End of the given Line.

EXAMPLE.

How to make a Regular Pentagon, each Side of which may be 84 Inches, Feet, Yards, &c.



1. Draw a Right-line, AB, making it equal 84 from the Diagonal Scale now the Number of Sides in a Pentagon being 5 by the above Rule, $180 \div 5 = 36$ and $90 - 36 = 54$ therefore by Prob. IX. make the Angles ABC and CAB each equal 54 Degrees extending their Sides until they meet which is in C, so is C the Center of the Pentagon.

2. Make CB or CA Radius, and upon C describe the Circle.

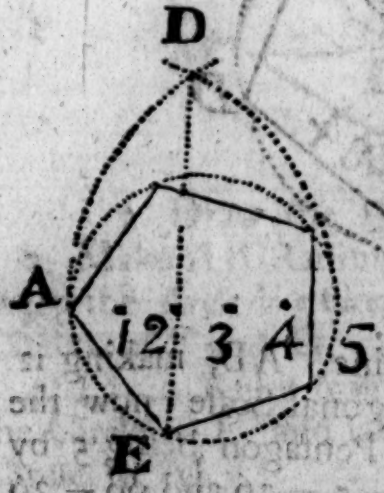
3. Take AB in your Compasses and it will Divide the Circle into 5 equal Parts which join with the Right-Lines BD, DE, EF and FA so is ABDEF the Regular Pentagon requir'd.

NOTE

NOTE. If you Measure CP upon the same Diagonal that you took AB from, it will shew the Perpendicular height of the Pentagon.

PROBLEM XXVIII.

To make any Regular Polygon, (suppose a PENTAGON.)



1. Describe a Circle and cross it with a Diameter A 5 which divide into as many equal Parts as the Polygon hath Sides, so a Pentagon having 5 Sides the Diameter must be divided into 5 equal Parts.)

2. With the whole Diameter in the Compasses, and one Foot in A sweep an Arch also with the same Extent and one Foot in 5 Strike another Arch crossing the former Arch in D.

3. Always lay a Ruler to D and two of those Divisions and draw the Line D E cutting the Circle in E join E A with a Right-line and it will be a Side of the Pentagon.

OTHERWISE.

Divide 360 Degrees (see DEF. 15) by the Number of Sides in the given Polygon, and the Quotient

Quotient will (being taken from the Line of Chords) Divide the Periphery of the Circle into as many equal Parts as the Polygon hath Sides the Circle being drawn with the Chord of 60 Deg.

So if the above Circle be deſcrib'd with the Chord of 60 Deg. and you take 72 Deg. from the ſame Scale of Chords, this 72 will be a Side of the Pentagon for $360 \div 5 = 72$.

PROBLEM XXIX.

To make a Geometrical OVAL.



1. Deſcribe any two equal Circles as T *a* B *e* and S *a* C *e* paſſing each thro' o-thers Center B and C and croſſing each o. ther in *a* and *e*.

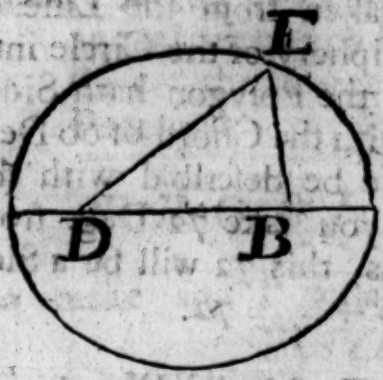
2. Take the Dia-meter of either Circle (*viz*) S C or T B in the Compaſſes and ſetting one Foot in *e* ſweep the Arch I with the ſame Extent and one Foot in *a* ſweep the Arch G, ſo is T I S G an OVAL.

PROBLEM XXX.

Another way to make an OVAL or ELLIPSIS.

1. Upon any Plane that you would have the Oval made, put in 2 Pins or Nails as at B and D.

2. To



2. To these 2 Pin fasten the 2 Ends of Thread or Line, a DEB.

3. In the Double the said Line as at E pu a Pin and move the Pi about with your Han (keeping the Line a ways streight,) so wi the Pin mark out an ELLIPSIS.

PROBLEM XXXI.

To describe a PARABOLA on a given Axis.

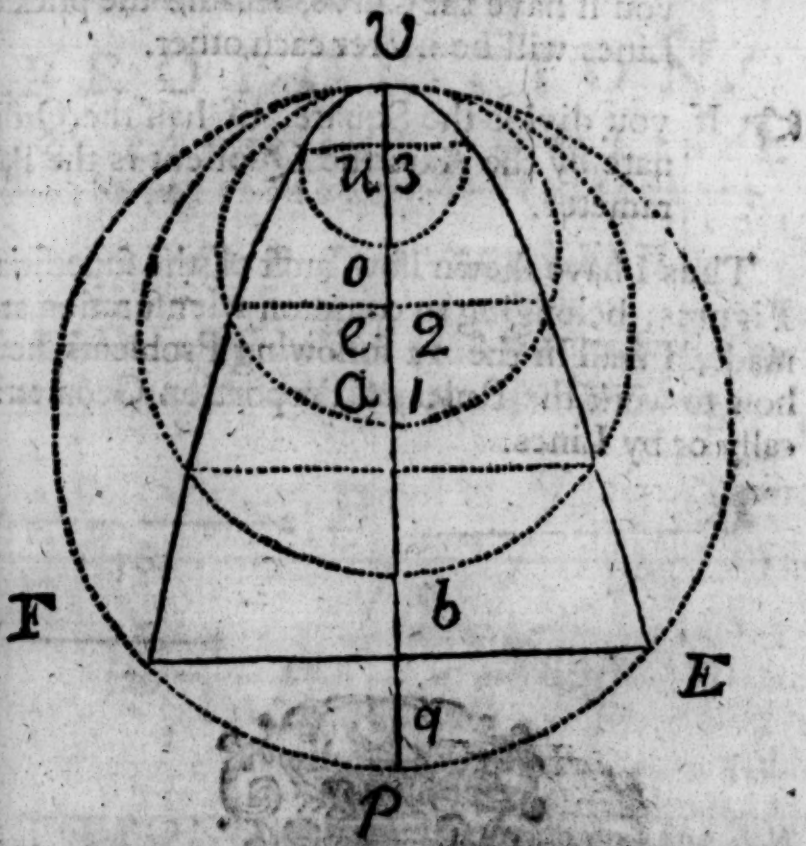
THE Parabola is the section of a Cone and Plane parallel to one of the Sides of th Cone, that is to say, to a Right-line draw from the Vertex of the Cone thro' some point the Circumference of its Base, which is a Circle This Section or Parabola is bounded by a Curv Line called a Parabolical Line, and generally Conic Line, because a Conic Line is the Sectio of a Plane and a Conic Superficies, that is to say the Surface of a Cone. It is evident that this P rabolical Line is a Curve Line, and spreads in it progress not unlike a Rope slack pull'd, or a heav body, which being thrown obliquely into the Air descends with much the same obliquity, describin a Parabolical Line.

EXAMPL

EXAMPLE,

Given A B the Axis, and C D the Parameter of a Common Parabola, to describe the Figure.

Given { A ——————————— B
C —— D



1. Make V q = A B, and upon the said V q sweep as many Circles as you please all of which must pass thro' V as upon the Points *a*, *e*, *o*, and *u* Strike the 4 Circles all meeting in V.

2. Where each of those Circles cut the Axis as in *p b a* and *o* set the Parameter (*viz.*) from *p* to *q* from *b* to 1, from *a* to 2, from *o* to 3 &c.

3. Thro' thoſe marks *q* 1, 2, 3, draw right Lines at Right-angles to the Axis, and where thoſe Lines cuts their reſpective Circles draw the Curve E *v* F and it will be the Parabola.

NOTE, More Circles you draw the exacter you'll have the Curve, becauſe the prick'd Lines will be nearer each other.

☞ If you divide the Square, of half the Ordinate by the Axis the Quotient is the Parameter.

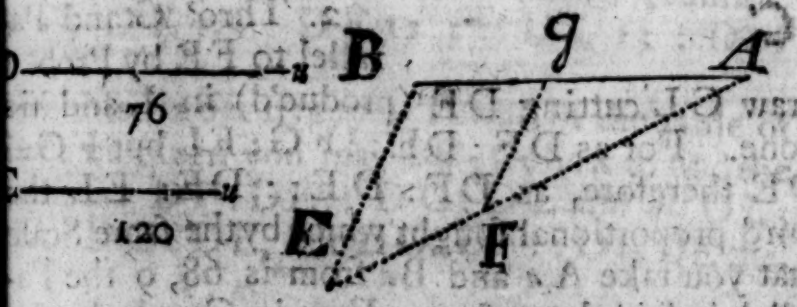
Thus I have ſhewn how moſt of the ſuperficial Figures, belonging to common Menſuration are made, I ſhall in the 12 following Problems ſhew how to work the Rules of Proportion Geometrically or by Lines.

THE RULE OF PROPORTION.

GEOMETRICALLY *or by* LINES.

PROBLEM XXXII.

To Divide a Right-line as AB into such Proportion as the Line D*u* is to the Line C*u*.



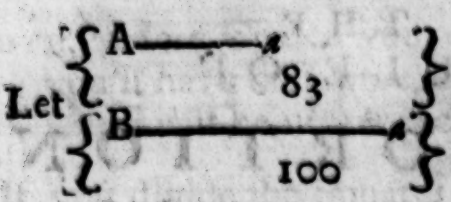
1. UPON either End of the given Line AB, as at A, make any Angle drawing the Line AE.

2. Make AF = D*u*, and FE = C*u* join EB and Parallel to EB thro' F (by PROB. 1) draw q and 'tis done: For as EA (196) is to AB (167) so is AF (76) to A*q* (64,7) and as, AE : AB :: EF : *q*B.

PRO-

PROBLEM XXXIII.

Given two Right-lines A *a* and B *a* to find a third in Proportion to them.





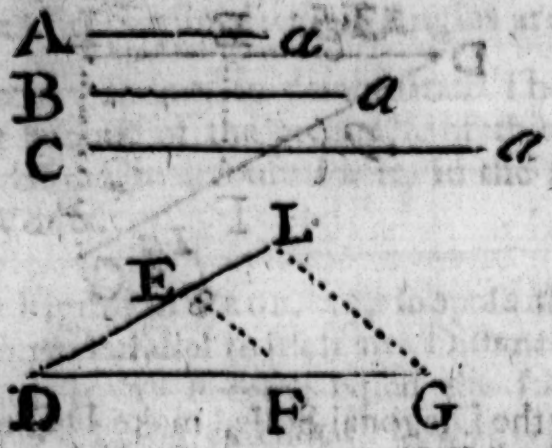
1. With the two given Lines make any Angle (*viz.*) make D E = A *a* and D F = B *a*, Produce D F any length; also make F G = A *a* and join F E.

2. Thro' G and Parallel to F E by Prob. 1 draw G L cutting D E (produc'd) in L and 'tis done. For as D F : D E : : F G : E L but F G = D E therefore, as D F : D E : : D E : E L the third proportional sought which by the same Scale that you take A *a* and B *a* from is 68, 9 the Parallel is had thus, set one Foot in G and take the neareſt Diſtance to E F (produc'd if need be) with that Diſtance and one Foot in E ſweep an Arch by the top of that Arch and thro' G draw G L *&c.*

PROBLEM XXXIV.

Given three Right-lines A *a*, B *a* and C *a* to find a fourth in Proportion to them.

1. Make



1. Make D E equal to A *a*, D F = B *a* and L = C *a* join E F and thro' L parallel to E F aw L G: to cut D F produc'd to G.

So is D G the fourth proportional requir'd, as, D E 7,5 : D F 12 :: D L 13 : D G 9,8.

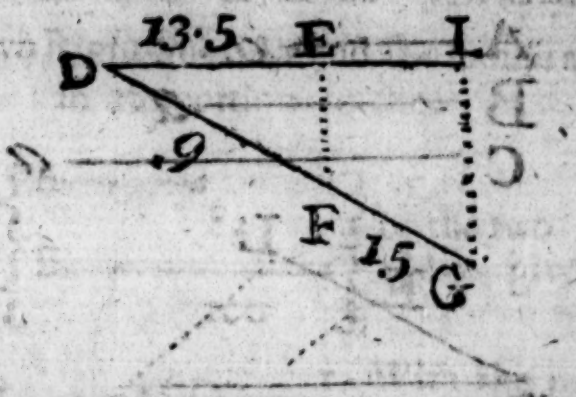
By this Prob if any 3 Terms in the Rule of Three be given you may eafily ſee how to find the Fourth for D F : D E :: D G : D L and as D L : D E : : D G : D F, alſo D G : D F : : D L : D E *&c.*

PROBLEM XXXV.

9 Labourers *do a Piece of Work in* 13½ *Days, in what Time will* 15 *Labourers do the ſame Work.*

1. As in the laſt Prob. make any Angle as D G.

2. By



2. By the Diagonal Scale, make D G equal 15 or 150, and D F = 9 or 90 alſo D L = 13,5 or 135.

3. Join E F, and make the Angle D G L = ∠ D F E drawing E F till it cut D L in E: then take the Diſtance E D in your Compaſſes and lay it upon the Diagonal Scale, and you'll find it 8,1 or 81 the Days required, for as D G : D L :: D F : D E.

NOTE, that if for 13,5 you take from the Diagonal Scale 135 then for 9 you take 90 and for 15, 150 and then in ſtead of calling D E 81 you may call it 8,1

The Reaſon of theſe PROBLEMS about Proportion, is from Euclid, 4, 5, 6, and 7. Propoſitions of the 6*th* Book: And is from this,

THEOREM I.

If two Triangles are alike, their like Sides are Proportional: That is, thoſe Sides which Subtend the equal Angles; as alſo thoſe Sides which are about the equal Angles, will be Proportional

and

d consequently if any two Triangles have eir Sides Proportional; their Angles are equal.

It is not my Intent, to demonstrate Theorems re; yet because of the great Use of this Theom, I presume to account for it, in the few folwing Words.

In the Figure to PROB. 33, suppose the Line F to move Parallel to itself any Distance, as to and there it will become equal the Line L N D E and D F being extended) now it is plain by e Figure, That if E F become L G, D E will come D L and D F will become D G, That is DE : EF :: DL : LG, and DF : EF :: G : LG &c. EF and L G being Parallel, the iangles D E F and D L G are alike *per* DEF. 41.

Also by the same Reason, if you draw F N rallel to D L the 3 Triangles D E F, D L G d F N G are all alike.

THEOREM II.

all Right-angled Plane Triangles the Square the Hypothenuse is equal to the Sum of the uares of the other two Sides (47 EUCLID 1.)

DEMONSTRATION.

Let A B C be a Right-angled Plane △ in which fall the Perpendicular B D from the Right-angle upon the Hypothenuse A C then are the three iangles ABC, BCD and BAD alike, as may be us prov'd by the two following Notes.

1. *Note*

1. *Note*, The Sum of the three Angles of every plane Triangle, is equal to 180° (*per* 32 EUCLID 1. Book.) Therefore if from 180° you Subtract any two Angles of a plane △ the Remainder is the third Angle.

2. In all right-angled plane Triangles, the right-angle being 90°: If you Subtract either of the acute Angles from 90°, you will have the other.

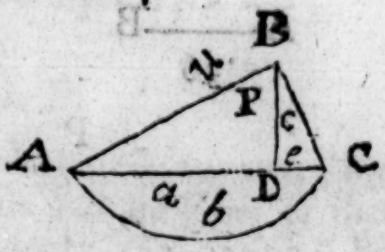
Now ∠ABC (or 90°) —∠C=∠A, and ∠BDC (or 90°) —∠CBD=∠ C, which taken from ∠ABC or 90° leaves ∠A, *&c.* that is the ∢A =∢DBC, and ∢C=∢ABD, *&c.*

Let B=AC, A=AD, E=DC, P=BD, C=BC U=AB.

Then by THEOREM

Fir

First we have	1	A : P :: P : E
	2	B : C :: C : E
	3	B : U :: U : A
1 ∴	4	PP = AE hence $P = \sqrt{AE}$
2 ∴	5	CC = BE or $E \times \overline{A+E}$
3 ∴	6	UU = AB or $A \times \overline{A+E}$
÷ B	7	$\frac{CC}{B} = E$
÷ B	8	$\frac{UU}{B} = A$
÷ 8	9	$\frac{CC+UU}{B} = A+E = B$ by the Figure
× B	10	CC + UU = BB, as was to be proved from the 5th. and 6th Steps we have



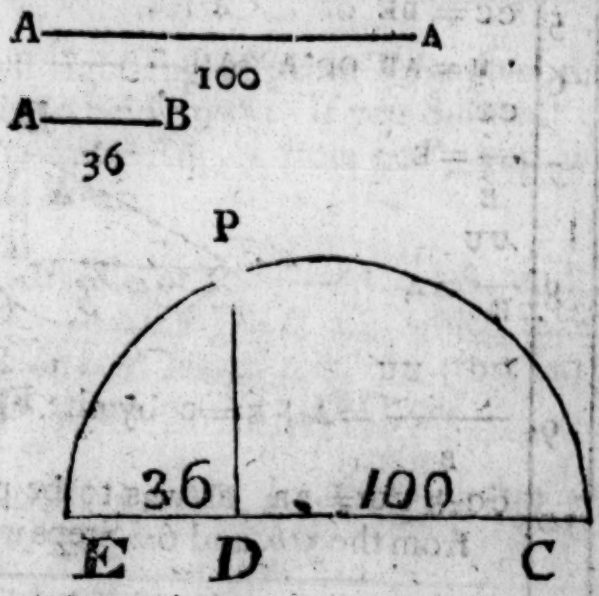
E : E×A+E : : A : A×A+E
that is E : CC : : A : UU and also
A+E : A×A+E : : A : AA
that is B : UU : : A : AA
hence arises *Prob.* 40, 41, 42.

These two PROBLEMS being of so great Use, have thus prov'd their Truth that a Learner may better retain such PROBLEMS in mind, as depend thereon, of which there are a great many.

NOTE, 3. An Angle in a Semi-circle is a Right Angle (31 EUCLID 3) by this and the other Step above, we find mean Proportionals.

PROBLEM XXXVI.

Between two given Lines or Numbers, A B (36) and A A (100) to find a mean Proportional.



1. Draw a Right-line C E on which make C D = A A and D E = A B, and upon D (by PROB. 5) raise the Perpendicular D P.

2. Upon C D E as a Diameter describe the the Semi-circle C P E, cutting D P in P so is D P the mean Proportional required which laid on the Diagonal Scale is found equal 60 : For as C D : P D : : P D : D E (STEP 1. THEOREM 2.) for if Lines be drawn from P to C and E the Angle C P E will be a Right-angle *per* NOTE 3 THEOREM 2.

PRO

PROBLEM XXXVII.

To Extract the SQUARE ROOT *of any Number.*

Suppoſe of 3600 by LINES.

RULE.

A Mean Proportional between any two Lines or Numbers is the Square Root of their Product. Thus 100 × 36 = 3600: But mean Proportional between 100 and 36 is by he laſt PROB. found = 60.

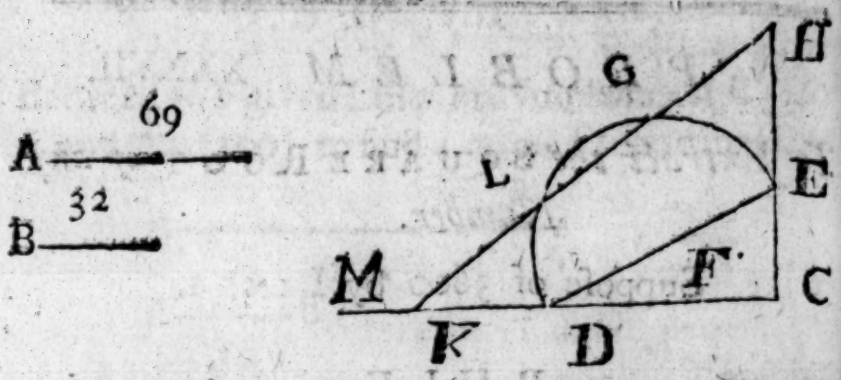
Alſo 360 × 10 = 3600, and a mean Proportional between 10 and 360 will alſo be found = 60 n like manner 90 × 40 = 360 and a mean Proportional between 90 and 40 is alſo found = o *&c.* for any other Number: So the Square oot of 3600 is 60, and thus by the Diagonal cale you may find the Square Root of large umbers, ſee STEP 4 foregoing for proof of is.

PROBLEM XXXVIII.

etween two given Right-lines (A and B) to find two mean Proportional Lines.

1. Draw a Right-line as C M any length, and pon the End C, raiſe the Perpendicular C H, aking it any length.

2. Make



2. Make CE equal the Line B, and make CD equal the Line A, draw the Right-line ED which divide into 2 equal Parts at F, then upon F, as a Center describe the Semi-circle E G D, then take the less Line B and with one Foot in D Strike an Arch crossing the Semi-circle in G.

Lastly, upon the Point G move a Ruler, 'till it cut the Lines C H and C M, in H and K at an equal Distance from F, so shall E H and D K be the two mean Proportional (Lines sought.) For as E C (B) : D K : : D K : E H, and E C : K D : : E H : C D: (A) 13 : e : 6.

PROBLEM XXXIX.

To Extract the Cube Root *Geometrically.*

RULE

THE Cube Root of any Number is the first of two mean Proportionals between one and that Number: But because in great Numbers this method will be tedious, take the following one, *viz.* if any Number be multiply'd

multiply'd by the Square of any other Number the Cube Root of that Product is equal the first of two mean Proportionals between the said two Numbers.

EXAMPLE.

What's the Cube Root of 67584.

Take any Number which you suppose will be less than the Cube Root of the given Number 67584, so here I take 32 which squared is 1024, but the given Number 67584 ÷ 1024 = 66 so the first of 2 mean Proportionals between 32 and 66 will be the Cube Root of 67584 now if you make the Line A = 66 and the Line B = 32 from your Diagonal Scale the Work will be the very same as in the last PROB. wherein D K is the first of two mean Proportionals between 32 and 66 which D K laid on the Diagonal Scale is found = $40\frac{1}{4}$ nearly, so $40\frac{1}{4}$ is nearly the Cube Root of 67584 as was required, and with an exact Scale you may readily find the Cube Root of great Numbers.

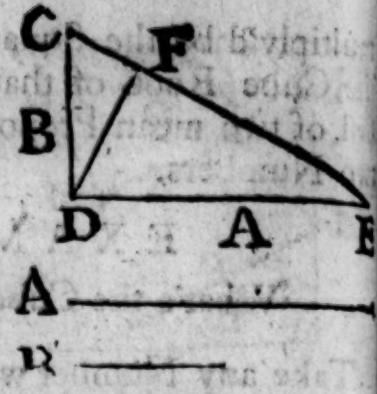
PROBLEM XL.

To find out two Lines which shall have such Proportion to each other as the Square of the Line A hath to the Square of the Line B.

1. Make D E = the Line A and upon D raise the Perpendicular D C making it = the Line B, join E C,

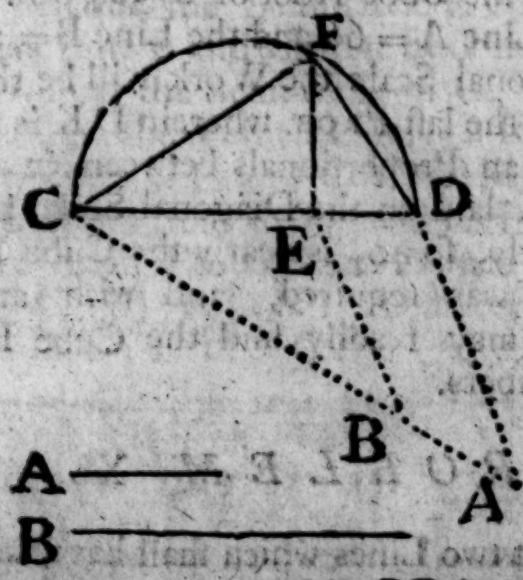
2. Upon

2. Upon the Hypothenuse CE from the ∠D let fall the Perpendicular DF and 'tis done; for: □A : □B : : EF : FC, STEP 5 and 6 THEOREM 2.



PROBLEM XLI.

To divide a Line CD in power as the Line A is to the Line B.



1. By PROB. 32, divide CD into such Proportion as A to B that is make CB = B and BA = A join, AD and make the ∠EBC = ∠DAC then as B : A : : CE : ED.

2. Make half CD Radius, and describe the Semi-circle CFD, then upon E raise the Perpendicular EF cutting the Circle in F.

3. Join

3. Join CF and FD, and 'tis done; for B : A : : □ CF : □ FD Step 5 and 6 Theorem 2. also A + B : □ CD : : B : □ CF, and : : A : □ FD.

PROBLEM XLII.

To enlarge any Line CE in Power according to any Proportion, Suppose as the Line A to the Line B last Fig.

1. By Prob. 32, make it as A : B : : CE : CD then upon E D as a Diameter describe the Semi-circle C F D.

2 Upon E erect the Perpendicular E F cutting the ½ Circle in F, lastly, join CF and 'tis done, for E A : B : : □ C E : □ C F : Step, Theorem 2.

PROBLEM XLIII.

To cut a Right-Line AD in extreme and mean Proportion.



1. Upon A erect the Perpendicular A F making it = AD.

2. Bisect AF in G and join G D and Produce F A making GI = GD.

3. Make A C = A I and 'tis done, for as AD : AC : : AC : CD (30 : E : 6). The

☞ The 26 following PROBLEMS are the chief PROBLEMS belonging to the Circle, where Note, that to inſcribe one Figure in or within another Figure is the ſame thing as to cut the one out of the other, as for *Example* to inſcribe a Square in a Circle, is the ſame as to cut a Circle into the greateſt Square that poſſibly can be *&c.* Any Right-lin'd Figure is inſcrib'd in a Circle, when all its Angles are in the Circles Periphery,

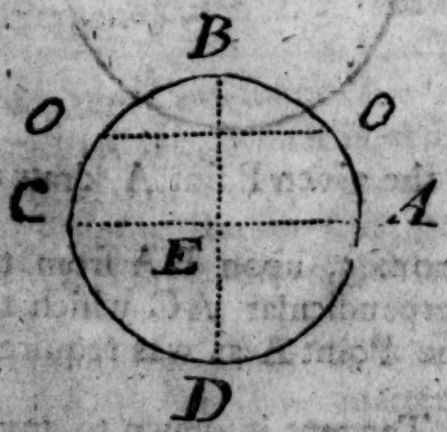


THE

THE CHIEF PRACTICAL Problems RELATING TO THE CIRCLE.

PROBLEM XLIV.

Given a Circle ABCD to find its Center.



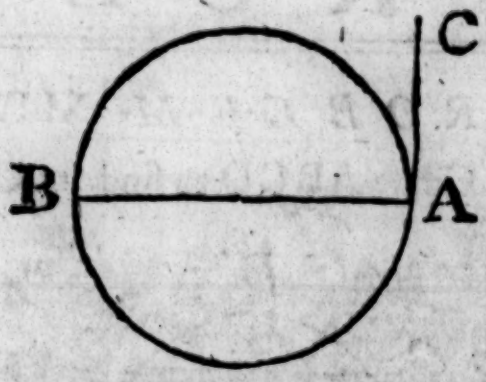
ANY where within the Circle draw a Right-line, as OO which biſect with the Line BD, in like Manner biſect e Line BD with the Line AC, ſo is BD a iameter, as alſo AC, and where they croſs each

other as at E, is the Center of the Circle required, thoſe Lines are biſected by PROB. 2.

NOTE, The Diameter AC and BE cut each other at Right-angles and ſo divide the Circle into four Quadrants.

PROBLEM XLV.

To draw a Right-line that ſhall touch a Circle in any Part of its Periphery, ſuppoſe at the Point A.



1. Thro' the given Point A, draw the Diameter BA.

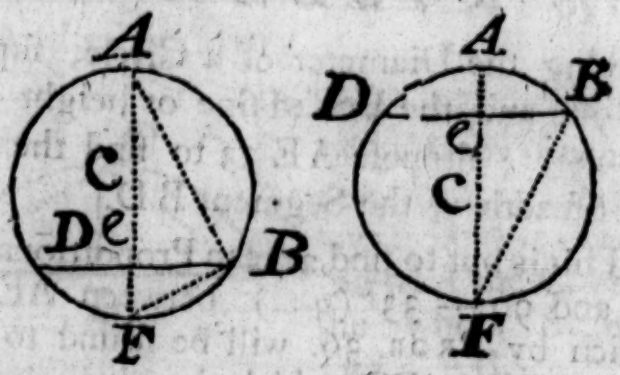
2. By PROB. 5, upon BA from the Point A raiſe the Perpendicular AC which touches the Circle in the Point A as was required.

By this a Tangent is drawn to any Point in a Circle's Periphery.

PRO-

PROBLEM XLVI.

Given the Verſed-ſine A *e* and Chord B D of any Segment of a Circle, to find the Diameter of the whole Circle.



1. Draw the Right-line AB, and upon B erect the Perpendicular B F drawing it any length.

2. Produce the Verſed-ſine A *e* until it meet the Perpendicular B F, which is at F, ſo is F A the Diameter of the whole Circle whoſe middle is at C ſo with the Radius CA you may compleat the whole Circle this is plain *per* NOTE 3.

THEOREM 2.

If the length of the Chord B D be 91 and the Verſed-ſine A E 33 (it is no matter whether or no the Arch B A D be given) you may by the Diagonal Scale make B D = 91 and in *e* the middle thereof raiſe the Perpendicular *e* A making it = 33 then join A B and go on as before and Meaſure the Diameter A F upon the Diagonal Scale and you'll find it 96 nearly.

To

To do this ARITHMETICALLY,
As AE 33 : BE 45,5 : : 45,5 : FE 62,73
which added to EA 33, gives the Diameter AF 95,73 (see PROB. 37.)

PROBLEM XLVII.

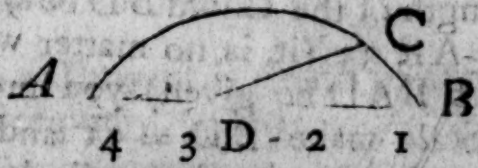
Having the Diameter of a Circle, suppose AF 96, and the Versed-sine or height of a Segment; suppose AE 33 to find the Chord or breadth of the Segment BD.

This is but to find a mean Proportional between 33 and 96 — 33 (*viz.*) between AE and FE which by PROB. 36. will be found to be about 45,5 = EB or ED which doubled gives BD = 91 for the Chord required.

These 2 PROBLEMS are of great use in measuring Segments of Circles, Globes or Spheres.

PROBLEM XLVIII.

To find the length of any Arch-Line as ACB nearly.



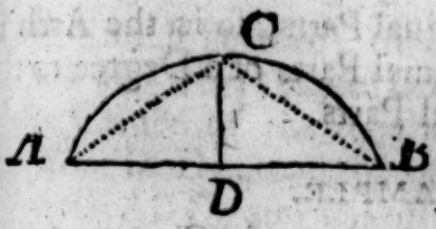
1. Divide the Chord AB into 4 equal Parts, and set one of those equal Parts from B to C upon the Arch.

2. From

2. From C to the third of thoſe Parts, draw the Right line C D which will be nearly equal to half the Arch ACB.

PROBLEM XLIX.

To find the length of any Arch as ACB by common Arithmetic nearly.



1. Multiply the Chord of half the Segment, (*viz.*) A C or C B by 8 and from that Product Subtract the Chord of the whole Segment (*viz.*) AB.

2. Divide the remainder by 3 and the Quotient is the length of the Arch ſought nearly.

CB or AC = 60 Chord of $\frac{1}{2}$ Segment,
8

480
Subtract AB = 100 Chord of the whole Segt.

3) 380

126,66

So the length of the Arch ACB will be nearly equal to 126,66.

But there can be no Rule given for finding the length of an Arch, exactly (untill ſome find a method for exact Squaring the Circle) for this Rule

Rule gives the Quantity ſomewhat too little and the greater the Arch is, the greater is the Error yet it will do for common Practice.

If you know what Degrees are contained in the Arch of any Segment, you may find the Arch Line truly by this following,

RULE.

As the Periphery of any Circle in Degrees is to its Periphery in equal Parts, ſo is the Arch in Degrees and Decimal Parts of a Degree to the ſame Arch in equal Parts.

EXAMPLE.

Suppoſe the circumferrence of a Circle to be 71 and the Arch to contain 52° 15' (the Decimal of 15 is 25°) what is the Arch in length?

Say as 360° is to 71 ſo is 52°25' to 10,304, ſo the 52° 15 will contain 10, 304 of ſuch Parts as the whole Periphery is 71.

PROBLEM L.

To deſcribe a Circle thro' any 3 Points as ABC not lying in a Right-line.

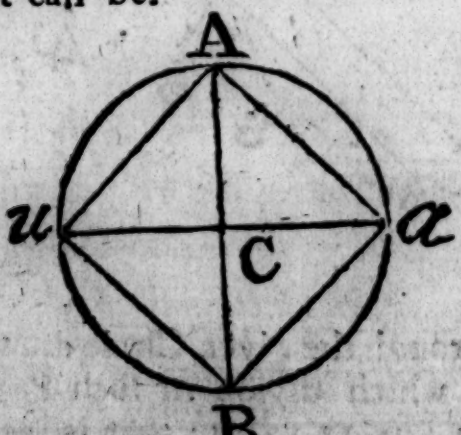
1. By PROB. 2 biſect the diſtance between any two of the given Points thus with one Foot in C Strike the Arch *o* with the ſame extent, in B Strike the Arch *q* thro' the interſection of thoſe 2 Arches draw the Right-line ED, alſo with one Foot in A ſweep the Arch *b* with the ſame extent and one Foot in B ſweep the Arch *d*, draw the Right-



Right-line FD thro' the croſſing of thoſe 2 Arches and where thoſe two Lines meet as in D is the Center of the required Circle, ſo ſetting one Foot in D extend the other to any one of the Points ABC, and it will paſs thro the other Points and ſo deſcribe the Circle.

PROBLEM LI.

To inſcribe a Square in a given Circle, or which is the ſame to cut in a Circle the greateſt Square that can be.



1. Thro' the Circle draw a Diameter, as AB and croſs it at Right-angles with another Diameter

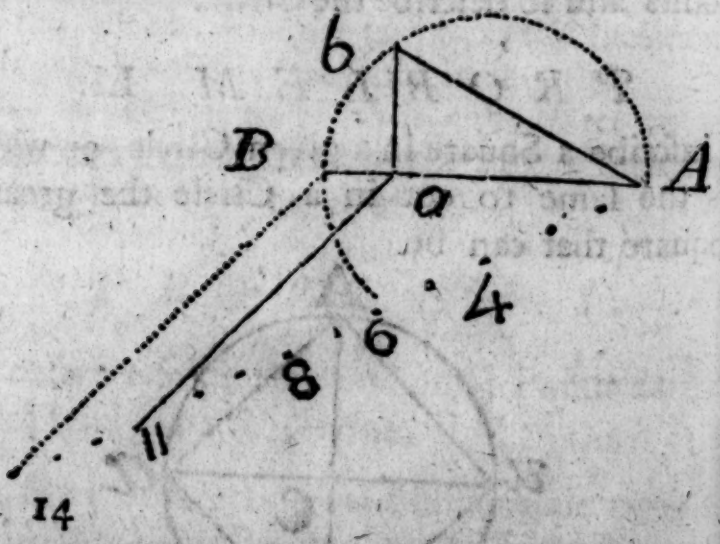
meter *u a*, which is eaſily done by biſecting the Diameter AB.

2. Join the Points *a* B, B *u*, *u* A, and A *a* with Right-lines and 'tis done, for the Square A *a* B *u* is the greateſt that can be cut out of the Circle 6, 7, E. 4.

You may by this method, make a Geometrical Square.

PROBLEM LII.

To find the Side of a Square equal to the Area of any given Circle.



1. Through the given Circle draw a Diameter as A B, which divide in ſuch Proportion (by Prob. 32.) as 11 to 14; that is, as 11 : 14 :: A *a* : A B.

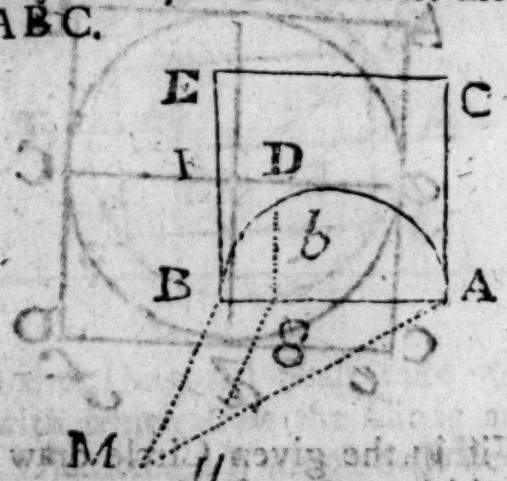
2 Upon

2. Upon *a* raiſe the Perpendicular *a b* cutting he Circle in *b*.

3. Draw the Right-line A *b* which is one Side f the required Square, for the Square of A *b* is qual the Area of the Circle.

PROBLEM LIII.

To find the Diameter of a Circle equal to the Side of a Square, *viz.* to find the Diameter of ſuch a Circle, as that the Area of the ſaid Circle may be equal the Area of the given Square A B C.



1. Upon any Side of the given Square as AB ſcribe a Semi-circle as ADB.

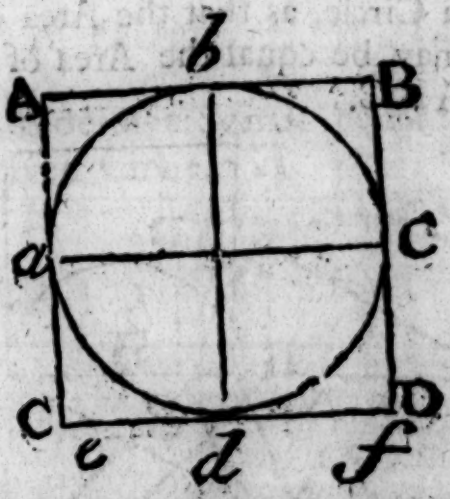
2. By the laſt or 32 Prob. make it as 11 is 8 ſo is AB to A 8, thus from a Diagonal Scale Scale of equal Parts, lay 11 from A to M and in BM, alſo lay 8 from A to 11 and draw 11 8 rallel to BM by making ∠ 8 11 A = ∠ BMA, en 8 A : B A : : 8 : 11.

3. Upon the Point 8 erect the Perpendicular D cutting the half Circle in D.

3. Thro' A and *b* draw the Right-line AF which is the Diameter required.

PROBLEM LIV.

About a Circle to describe a Square whose Sides may all touch the Circles Periphery.



1. Within the given Circle draw a Diameter as *a c*, which cross at Right-angles with the Diameter *b d* (as in PROB. 51.)

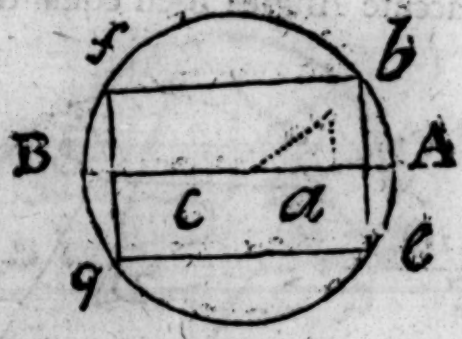
2. With the Radius of the given Circle, and one Foot in *d* strike the Arches *e* and *f*; also with the same Extent and one Foot in *a* and *c*, cross the former in C and D; also do the like at the other Ends of the Diameter,

3. Conect the Intersection of those Arches with the Right-Lines DC, CA BA and BD; is ABCD the Square required; 6, 7, e. 4.

PRO

PROBLEM LV.

A Joiner having a round Table, whose Diameter is 100 Inches, is minded to cut it into the greatest Right-angled Parallelogram that can be, whose length may be to its breadth as 3 to 2, or which is the same as 30 to 20 required the length and breadth of the said Parallelogram.



1. From the Diagonal Scale take 50 (*viz.* of 100) with which draw the Circle and cross with the Diameter AB which will be = 100 the Tables breadth.

2. From the same Scale, take 30 and set from the Center C to *a*.

3. Upon *a* raise the Perpendicular and make 20, and draw the Right-line CB cutting the Circle in *b*, draw *be* Parallel to the Perpendicular.

4. Make $bf = eq$ and $be = fq$ and join *bf*, *fq* and *qe* and 'tis done.

5. Measure *eq* or *bf* upon the Diagonal Scale and you'll find it equal to 83 Inches the length

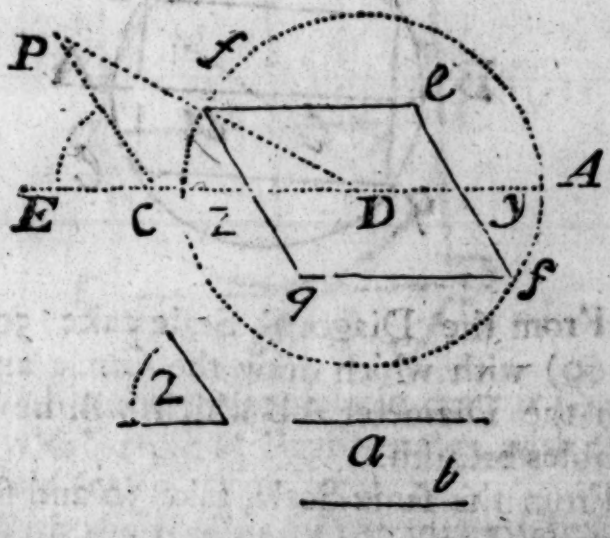
and

and *e b* or *f q* 55$\frac{1}{4}$ Inches the breadth of the long Square requir'd.

This is plain from ſimilar Triangles.

PROBLEM LVI.

Out of a given Circle ABCD, to cut the greateſt Rhomboides that can be whoſe length may be to its breadth as the Line *a* is to the Line *b* and its acute Angles each equal the Angle *z*



1. Thro' the given Circle draw a Diameter, AC (produc'd if need be) take the Line *a* and ſet from the Center D to C and make the ∠ ECP = given ∠ *z* making CP = *b*.

2. Draw the line D P cutting the Circle in *f*.

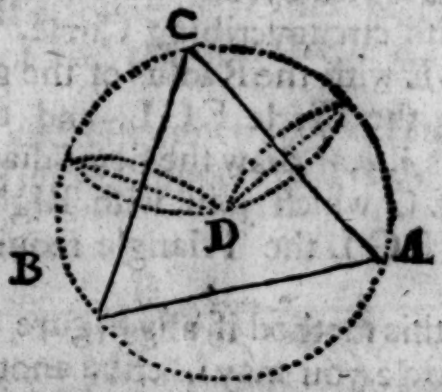
3. Draw *f q* parallel to C P and make *z q* = *f z* alſo make D *y* = D *z* and thro' *y* draw *f e* parallel

allel to $q f$ making $ey = fv$ join fq aud fe and ou'll have fq, fe for the Rhomboides required.

The truth of this is easy for as DC : CP : : Dz ($= \frac{1}{2} fq$ or $\frac{1}{2} fe$) : zf or which is the same hing as DC ($= a$) : CP ($= b$) : : ef : fq : or as the wholes are, so are the halves; that is s $a : b :: \frac{1}{2} a : \frac{1}{2} b$, &c.

PROBLEM LVII.

bout any Triangle as ABC to describe a Circle.

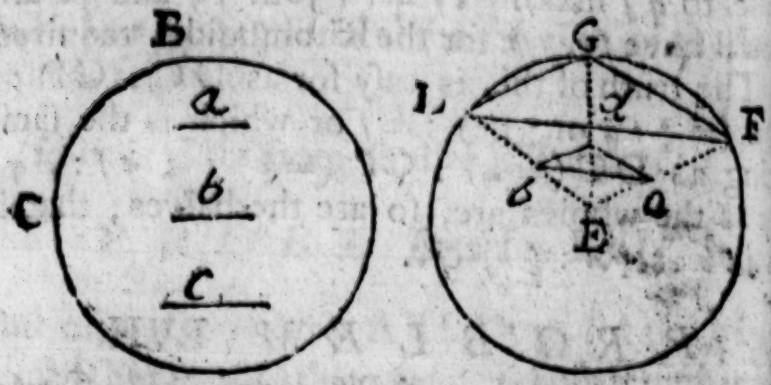


1. This is the same with PROB. 50 therefore sect any 2 Sides of the Triangle suppose the des AC and CB and where the bisection Lines eet as at D is the Center of the requir'd Circle E4.

PROBLEM LVIII.

iven the Circle ABCD to be cut into a Triangle whose Sides may be in a given Ratio as the Lines a, b and c.

1. By PROB. 14 of the 3 given Lines a b c, r any other 3 Lines in the same Ratio) make a Triangle



Triangle *a b d* and by the last PROB. find E the Center of its circumscribing Circle.

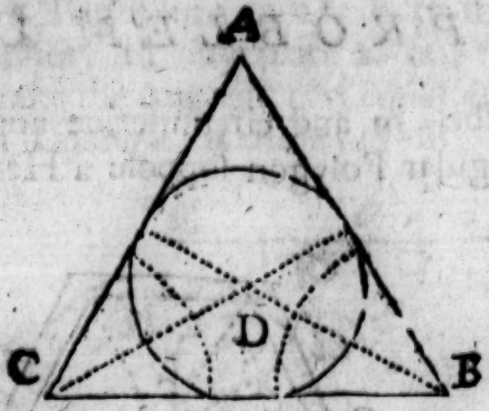
2. Upon E with the Radius of the given Circle ABC draw the Circle F G L, and thro the Angular Points *a*, *b*, *d* draw the Semi-diameters EF, EL, and E G which join with Right-lines and you'll have F G L the Triangle required.

NOTE, By this method if any Figure be inscrib'd in a Circle you may inscribe another like Figure in any other Circle, whether greater or less by describing the one Circle upon the Center of the other and drawing ÷ Diameters thro' every Angle *&c.*

PROBLEM LIX.

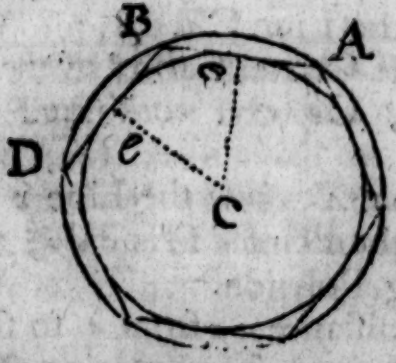
Within any Right-lin'd Triangle to inscribe a Circle.

1. By PROB. 7. Bisect any two Angles of the propos'd Triangle, suppose the Angles B and C and



nd where the biſection Lines meet as at D is e Center of the requir'd Circle 4, E. 4.

o inſcribe and circumſcribe a Circle about any regular Figure, ſuppoſe a Hexagon about a Circle.

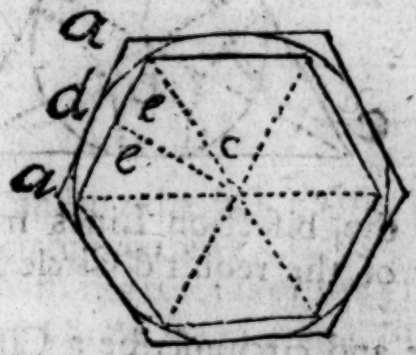


1 By Prob. 2 biſect any two ſides of the gi- en Polygon, Suppoſe the Sides AB and BD, nd where the Lines of biſection meet as at C, is e Center of the Polygon then extend your ompaſſes from C to *e* and that extent will ſweep e inſcribed Circle alſo the Radius CA, CB, D *&c.* will ſweep the circumſcrib'd Circle, 1 E. 4.

PRO-

PROBLEM LX.

To inſcribe in and circumſcribe about a Circle any regular Polygon ſuppoſe a Hexagon.



1. By PROB. 28 in the Circle you may inſcribe the Figure and biſect any Side thereof as the Side *e e* with the Line C*d*.

2. From the Center C thro' every Angle draw the Lines C*e*, C*e* &c. continued without the Circle.

3. By PROB. 45 draw the Line *a d a* to touch the Circle in the Point *d* ſo is *a d a*, a ſide of the circumſcribing Polygon.

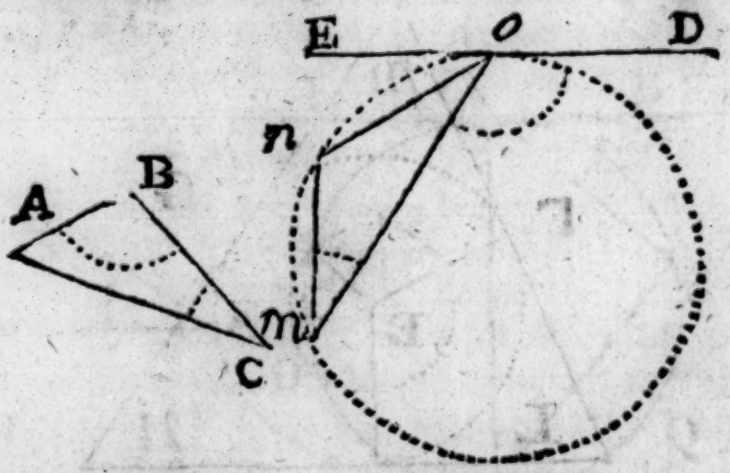
4. Lay the diſtance *a a* from *a* to the next produc'd Diameter and from that to the next produc'd Diameter &c. and you will have Points to draw all the other Sides by.

PROBLEM LXI.

To inſcribe in a Circle a Triangle having equal Angles with a given one.

1. Draw

1. Draw a Right-line as D o E to touch the Circles Periphery in any place as in *o* then (by PROB. 6) make the Angle D *o* M equal any Angle of the given Triangle, as the Angle B.



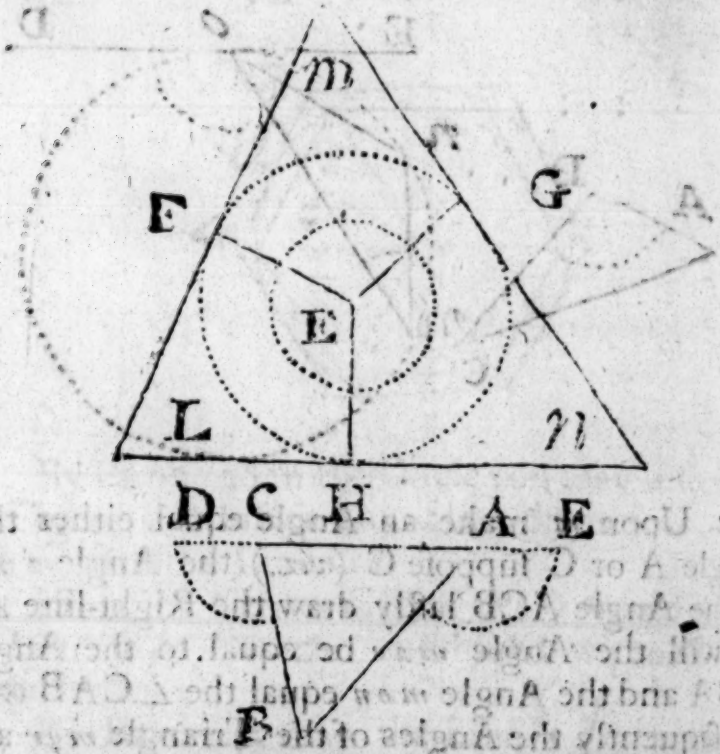
2. Upon *m* make an Angle equal either the Angle A or C ſuppoſe C (*viz.*) the Angle *o m n* = the Angle ACB laſtly draw the Right-line *n o* ſo will the Angle *m n o* be equal to the Angle CBA and the Angle *m o n* equal the ∠ CAB *&c.* conſequently the Angles of the Triangle *m o n* are all equal the Angles of the Triangle ABC as was required; 2, E. 4.

PROBLEM LXII.

To circumſcribe about a Circle a Triangle having equal Angles to a given Triangle as ABC. Both this and the laſt PROB. may be done otherwiſe by PROB. 58, but theſe are EUCLID's method.

L 1. Extend

1. Extend the Side AC making the 2 external Angles DCB and EAB then at the Center of the Circle make the Angle FEH = BCE and the Angle HEG = Angle EAB.

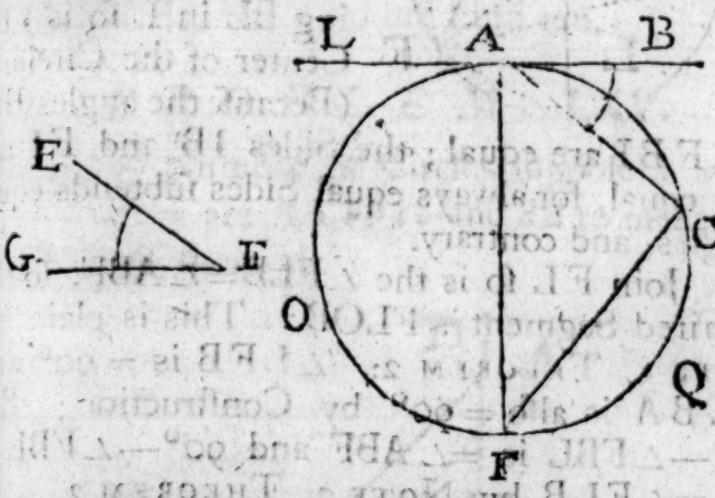


2. Where the Points F G and H touch the Circle, draw the Right-lines L *m*, *m n* and *n* L, so is the Angles of the Triangle L *m n* = the Angles of the Triangle ABC, (*viz.*) the Angle *m n* L = the Angle ABC and the Angle L *m n* = the Angle BAC, also the Angle L = the Angle C, 3, E. 4.

PRO-

PROBLEM LXIII.

From any given Circle to take away a Segment containing an Angle equal to a given Angle, as, EFG.



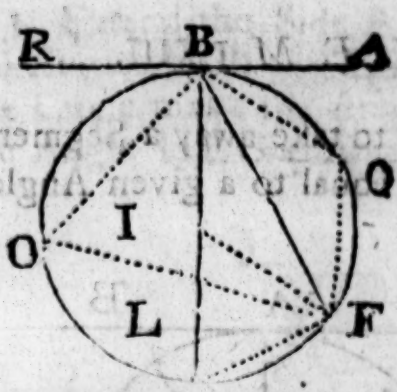
1. Draw the Right-line BAL perpendicular to the Diameter FA.

2. Make the Angle CAB equal the given one F, drawing AC till it cut the Circle in C; ſo is the Segment CQFOA that which is to be taken away, for the Angle CFA is equal to the given Angle BAC. 34. E 3.

PROBLEM LXIV.

Upon a given Right-line BF to make a Segment of a Circle in which the Angle ſhall be eqnal any given Angle as ABF.

1. Upon



1. Upon the angular Point B erect the Perpendicular BL and upon F (the extremity of the Line B F) make the ∠ BFI = the ∠FBL cutting BL in I, so is I the Center of the Circle. (Because the angles BFI and FBI are equal; the Sides IB and FI are also equal, for always equal Sides subtends equal Angles, and contrary.

2. Join F L so is the ∠FLB = ∠ABF, so the required Segment is FLOB: This is plain per NOTE 3. THEOREM 2. ∠LFB is = 90° and ∠LBA is also = 90° by Construction; But 90° — ∠FBL is = ∠ABF and 90° — ∠FBL is also = ∠FLB by NOTE 2. THEOREM 2.

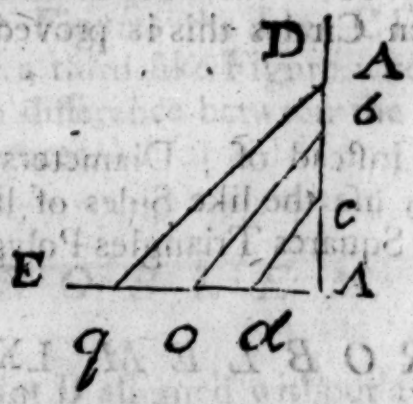
NOTE, All plane Angles are equal which stand upon the same Chord, and have their Angular points in the Periphery of the same Segment so the Angles F B and F L B are equal for they both stand upon the same Chord FB and their Angular Points and L are both in the same Segments Periphery therefore you can draw no two Right-lines from F and B to meet in the Arch FLOB but they will form an Angle at the said Arch = the ∠ ABF.

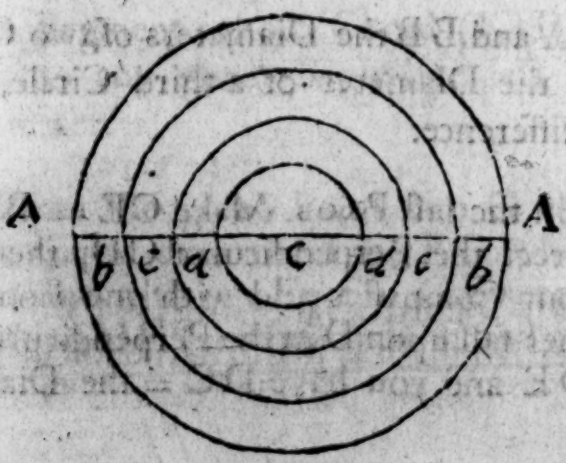
2. All four-sided Figures inscrib'd in a Circle have the sum of their two opposite Angles = 180° that is ∢ O + ∢ Q = 180°: also

also $\angle QF o + \angle QB o = 180°$ for QFOB is a Quadrangle, hence the Segment BQF contains an Angle = the Angle RBF, (*viz.*) Angle BQF = Angle FBR. Here you may observe that all such Quadrangles as are mentioned in this *Note* may be inscrib'd in a Circle, but no other kind can.

PROBLEM LXV.

Given any Number of Circles suppose 4 whose Diameters are AA, *bb cc* and *dd* to make one Circle of them all.





1. Upon

1. Upon any point as A by PROB. 10 make a Square DAE then take the Semi-diameter *c d* of the leaſt Circle and ſet from A to *d* alſo make A*e* = C *c* the next Semi-diameter join *c d* which will be the Semi-diameter of a Circle whoſe Area is = the Areas of both the Circles *dd* and *cc*.

2. Make A*o* = *d o*, and make A*b* = C*b* join *b o* which is the Radius of a Circle = the ſum of your 3 Circles *d d c c* and *b b*.

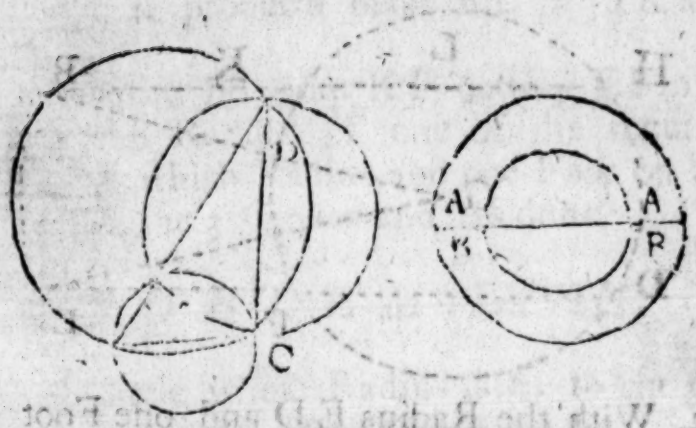
3. Make A*q* = *b o* and A A = C A and join A*q* which will be the Radius of the Circle required that is the Area of that Circle whoſe Radius is A*q* will be = the Sum of all the Areas of the four given Circles this is proved by THEOREM 2.

NOTE, that inſtead of , Diameters of Circles you may uſe the like Sides of like Figures whether Squares Triangles Polygons *&c.*

PROBLEM LXVI.

Given A A and B B the Diameters of two Circles to find the Diameter of a third Circle, equal their difference.

1. As in the laſt PROB. Make C E = B B and upon C erect the Perpendicular C D, then take A A in your Compaſſes and with one Foot in E let the other fall upon D in the Perpendicular C D ſo draw D E and you have D C = the Diameter



of the Circle requir'd by THEOREM 2.

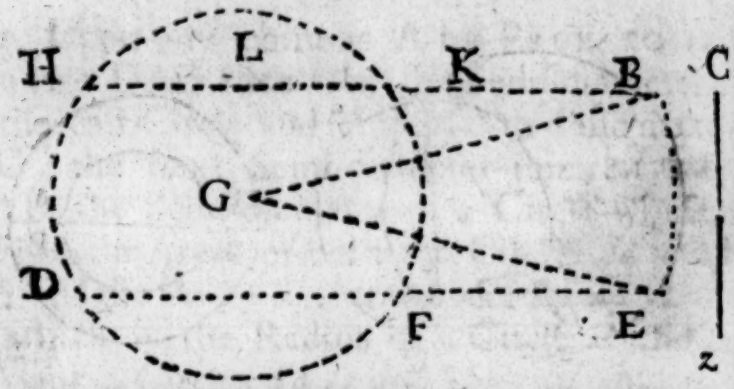
If upon the Lines BA and AA you make any two like Figures the Line CD will be the like Side of a third like Figure whoſe Area will be equal the difference between the Areas of the ſaid two Figures.

PROBLEM LXVII.

From a Point B aſſigned without a Circle to cut off a Segment whoſe Chord (KH) ſhall be equal to any given Right-line leſs than the Diameter.

1. From the given Point B to G the Center of the Circle draw the Line BG which make Radius and with one Foot in G Sweep an Arch BE cutting DF (extended) in E and make DF = Gz.

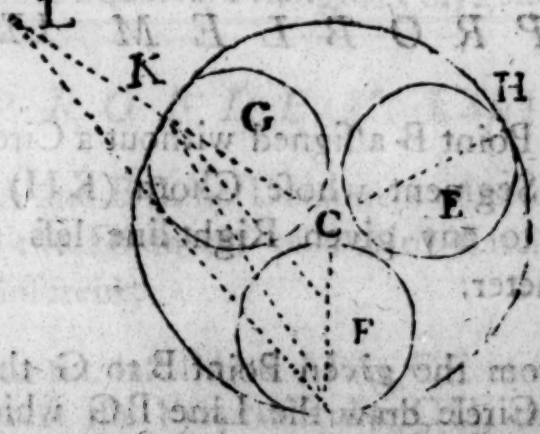
2. With



2. With the Radius E D and one Foot in B ſweep an Arch croſſing the Circles Periphery in H, join B H ſo is K L H the Segment, required for K H = C z is plain by conſtruction.

PROBLEM LXVIII.

In a given Circle H I K to deſcribe three of the greateſt Circles that can be.



1. From the Center of the given Circle draw the three Right-lines E H C I and C K dividing it into three equal Parts in the points H I and K join I K.

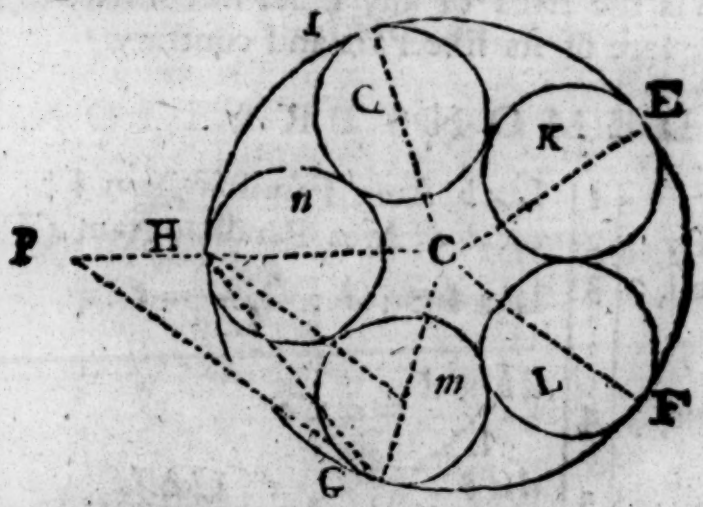
2. On

2. On CK produc'd make KL = ½ IK and draw IL.

3. Parallel to IL draw KF cutting CI in F so is FG the Radius of one of the requir'd Circles with which Radius and one Foot on FE and G, draw the 3 Circles and 'tis done.

PROBLEM LXVIII.

Out of a Circle whose Radius is 61 to cut 5 of the greatest equal Circles that can be.



1. From the Diagonal Scale take 61 and with that extent on C as a Center sweep the Circle EFGHI whose Perlphery divide into five equal Parts and to each of those Parts draw the Semi-diameters CE, CF CH &c.

2. Draw a Right-line between any of those two equal Parts as HG and on CH produc'd by HP = ½ HG and draw GP.

3. Thro' H Parallel to GP draw H*m* cutting CG in *m* so is G*m* (found equal about 22

by the Diagonal Scale) a Semi-diameter of one of the five equal Circles requir'd.

In this manner you may describe any Number of Circles in any given Circle the truth of which may be prov'd from similar Triangles.

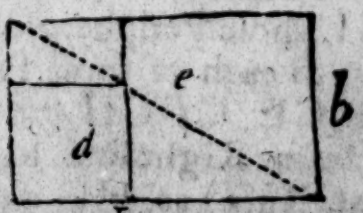
THEOREM 3.

Like Figures are in a duplicate Ratio of their like Sides 19, E, 6. that is as the Area of any Surface is to the Square of any Part thereof; so is the Area of any other like surface to the Square of its like Part and contrary.

DEMONSTRATION.

	1	$L \times b$ Area Parallelogram bL
By	2	$e \times d$ = Area Parallelogram ed
Theo. 2	3	$L : b :: d : \frac{db}{L} = e$
2 and 3	4	$\frac{db \times d}{L} = d \times e$
But	5	$\frac{ddb}{L} : dd :: Lb : \frac{Lbdd}{bdd} = LL$ &c.

Q. E. D.

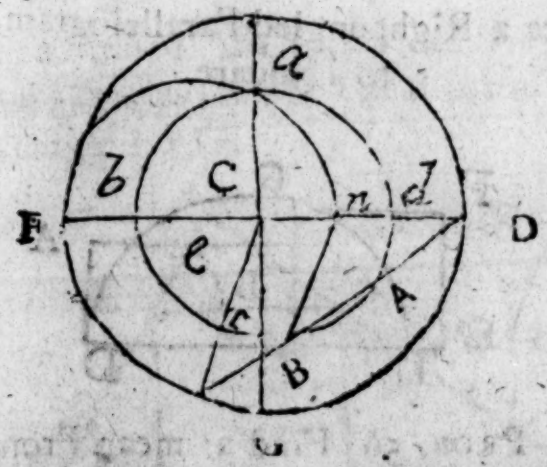


This THEOREM holding good in like Parallelograms will also hold good in like Triangle (the

(they being but the halves of Parallelograms) nd conſequently will hold good in any ſuperfical Figure whatſoever; for the Area of every ſuperical Figure is made up of ſome Number of Triangles.

PROBLEM LXX.

To divide a Circle into any proportion ſuppoſe as A to B, by another Circle, *a b c d* deſcribed upon the ſame Center.



1. By PROB. 32, Divide the Semi-diameter C of the given Circle DEEG as A to B, that as A : B : : D*n* : *n* C, and upon *e* the Middle of *n* F, Sweep the Semi-circle *n a* F cutting the Diameter E G in *a*, where *Note* that G E muſt be at Right-angles to F D

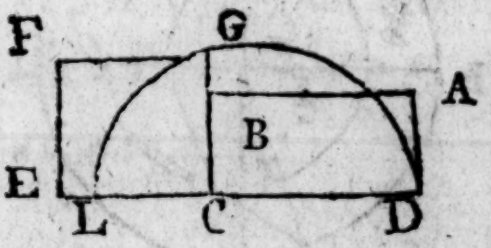
2. Upon C with the Radius C*a* deſcribe the Circle *a b c d*; and 'tis done, for as A : B : : the Area of the Ring D*d* E*a* F*b* G*c* to the Area of the Circle *a b c d* as was required you may prove this

this by the above THEOREM, for A + B is = or Proportional to the Area of the Circle DE FG thus A + B : □ DC (or □ DF) : : B : □ *d* or (*d b*).

CIRCUMSCRIPTION *and* INSCRIPTION *Of Figures.*

PROBLEM LXXI.

To reduce a Right-angled Parallelogram ABCD to a Square.



1. BY PROB. 56, Find a mean Proportional between the length and breadth of the given Parallelogram thus produce DC to L making CL = CB or AD and upon DL as a Diameter ſweep the Semi-Circle DGL cutting CB extended in G ſo is CG a Side of the required Square which being compleated becomes the Square CEFG whoſe Area is = Area, Parallelogram ABCD.

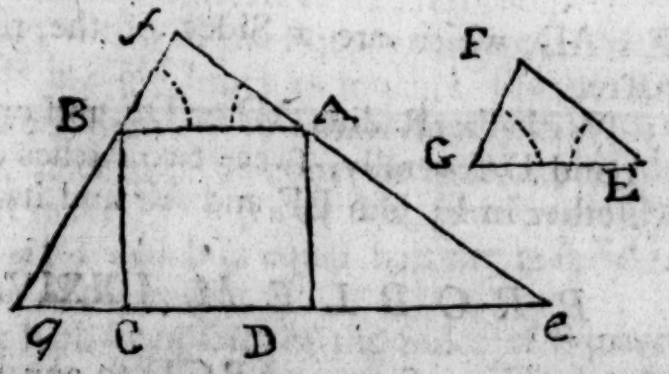
NOTE, 1. In this manner you may reduce any Number of Parallelograms to one Square, for

for it is but reducing every ſingle Par. to a Square and then adding all thoſe Squares together as is taught in PROB. 65.

2. If you conſider what two things multiply'd together will give the Area of any Figure then a mean Proportional between the ſaid two things is the Side of a Square whoſe Area will be equal to the Area of this Figure and thus you may reduce any Figure or Figures to a Parallelogram.

PROBLEM LXXII.

About a Square ABCD, to deſcribe a Triangle whoſe Angles ſhall be equal to the Angles of a Triangle EFG.

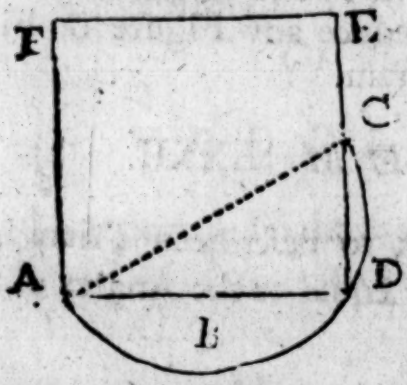


1. Upon any Side of the Square as AB make an Angle AB*f* = the ∠ G and upon A make an Angle BA*f* = the ∠ E continuing the Sides B*f* and A*f* till they meet in *f* and alſo cut the oppoſite Side of the Square *viz.* DC produc'd in *g* and *e* ſo is the Triangle *efg* that required, for becauſe *eg* is Parallel to AB the Angles *g* and B are equal, and ∠ *e* = ∠ A by the *Note* in PROB. 24 conſequently

consequently the ∠ F = ∠ *f* per 1 *Note* to THEOREM 2 hence ∠ *e* = ∠ E, ∠ F = ∠ *f* and ∠ G = ∠ *g*.

PROBLEM LXXIV.

Thro' any three Points A, B, and C, to draw a Square.



1. Thro' any two of the given Points, as A and B, draw a Right-line ABD.

From the third point C by PROB. 3. Let fall the Perpendicular CD, upon ABD and make DE = AD, which are 2 Sides of the required Square.

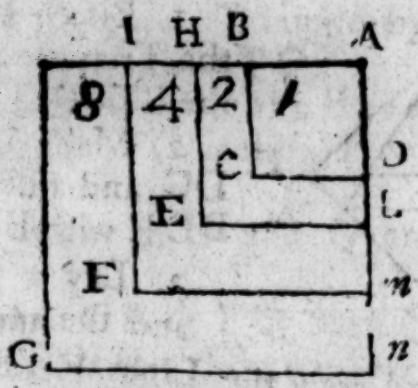
3. With the Radius AD or DE and one Foot in A and D severally, sweep two Arches crossing each other in F, join EF and AF and its done.

PROBLEM LXXIV.

To Augment a Square ABCD to any Proportion, Suppose to an Octuple Proportion.

1. This is founded upon Theorem 2. Therefore produce any two Sides of the Square as AB and AD any sufficient Length and draw the Diagonal AC any Length at pleasure.

2. Make AL and AH = AC and compleat the Square AHEL, which is double to the given Parallelogram



Parallelogram ABCD; for by THEO. 2. □AC = 2 □ of any of the Sides AB, BC, CD, DA, becauſe thoſe Sides are all equal.

3. Make A*m* and AI = AE and compleat the Square AI F*m*, which is double the □AHEL, or four times as much as the □ABCD.

4. Make AK and A*n* = AF and compleat the □AKG*n*, which is the Square required; for the □AKG*n* is eight times as much as the given □.

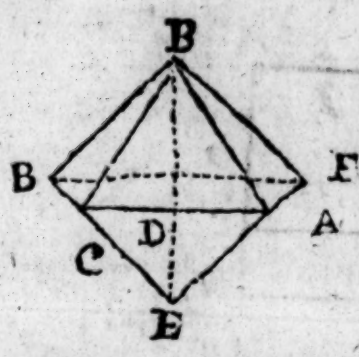
If it were required to Diminiſh any Square as AKG*n*, it is but the converſe work to this, for you muſt make AF = A*n* or AK, and ſo compleat the □A*m*FI which is equal half the □AKG*n*, and ſo on of the reſt.

What hath been ſaid of the Sides of Squares, alſo holds good in the Diameters of Circles &c. as in PROB. LXV and LXVI.

PROBLEM LXXV.

About any Equilateral Triangle ABC to circumſcribe a Square.

1. Biſect



1 Biſect any Side of the Triangle as AC with the Line BED.

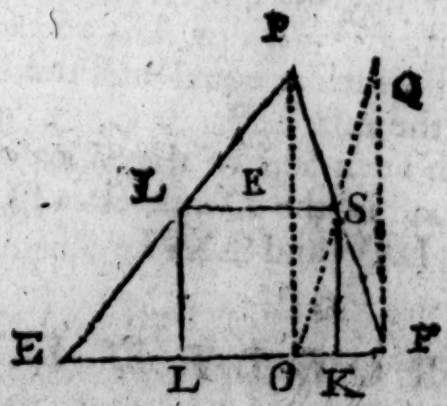
2. Make DE=DA or DC, and biſect the Line BDE with BF.

3. Lay a Ruler to E and the angular Points A and C, and draw the Lines EAF and ECB cutting thr Line FB in F and B, join BB and FB and you'll have the required Square FBBE.

PROBLEM LXXVI

Within any Triangle FPE to inſcribe the greateſt Square that can be.

For by ſimilar Triangles we have
FO : FQ (FE) :: KO (ES) : SK
And, FO : FE :: SE : LS



Therefore, KS = SL, conſequently KLLS (being a Rectangle) is a Square. Q.E.D.

1. With

1. Within the given Triangle by PROB. III. Let fall the Perpendicular PO.

2. Upon either End of the Baſe as at F, raiſe the Perpendicular FQ making it equal the Baſe FE.

3. Lay a Ruler to O and Q and it will cut FP in S.

4. Parallel to OP or FQ draw SK which is one Side of the required Square, by which you may compleat the whole Square.

Otherwiſe ARITHMETICALLY.

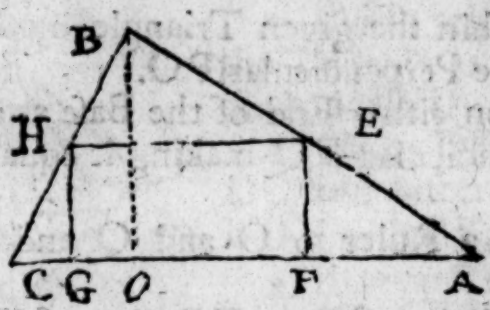
Suppoſe the Baſe EF = B = 30 and the Perpendicular OP = P = 10, SK a Side of the required Square = A = OE = SL = KL = LL.

By *Theo.* I.	1	$P - A : A :: P : \frac{AP}{P-A} = B$, for the Triangles PSL and PFE are ſimilar
I.	2	$AP = BP - BA$, that is $A \times P + B = BP$
2÷	3	$A = \frac{BP}{B+P} = \frac{300}{40} = 7{,}5 = A$, the Side of the Square required.

PROBLEM LXXVII.

Required the greateſt Parallelogram Right-angled that can be cut out of any Triangle as ABC.

1. Thro' the middle of the Perpendicular BO draw a Line EH Parallel to the Baſe AC, of the Triangle ABC.



2. Thro' H and E draw HG and EF each Parallel to the Perpendicular BO so have you EFGH for the Parallelogram required, for if any Number be divided into two equal Parts and those Parts or halves be multiply'd together they will be the greatest Product that can be made out of that Number being any otherwise divided.

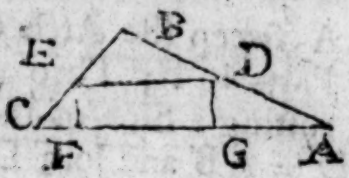
PROBLEM LXXVIII.

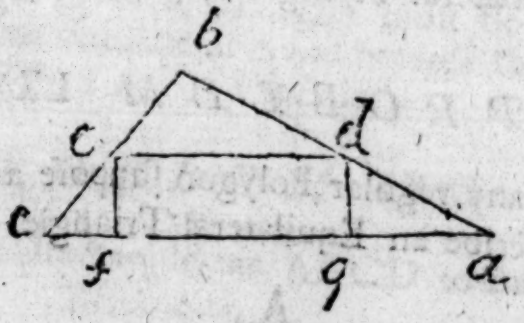
Within any Triangle ABC to inscribe a Rectangle whose length may be to its breadth in a given Ratio as E to F.

1. Of the given Lines E and F by PROB. XIX. make the Rectangle *defg* about which by PROB. 72 describe a △ *abc* having equal Angles with the given △ ABC then the three Triangles *ebd*, *abc*; and ABC are similar.

2. To any three like Sides, as *ed* AC and *cb* of those 3 Triangles by PROB. 34 find a fourth Proportional BE *viz.* as *bc* : BC : : *eb* : EB.

3. Thro' E and Parallel to AC draw ED the length of the requir'd Parallelogram and from D and E let fall the Perpendicular DG and EF

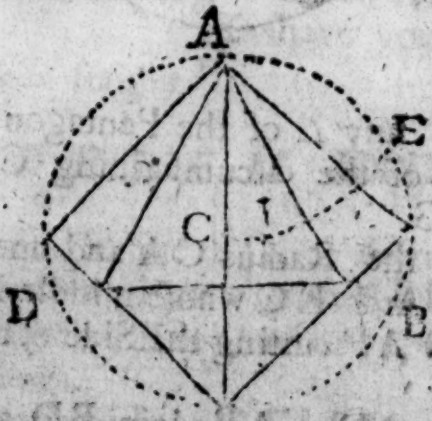




EF and 'tis done, for as E : F : : D E or F G : D G or E F. This may also be done by Prob. 76.

PROBLEM LXXIX.

Within a Square to inscribe an Equilateral Triangle.

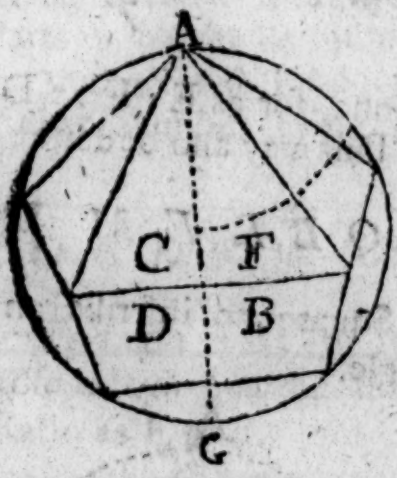


1. With the Radius A C of the circumscribing Circle and one Foot in any Angle of the given Square

Square as in A ſweep the Arch CE which divide into 2 = Parts in F; Thro' A and F draw the Line AB cutting a Side of the Square in B, then make AD = AB and join DB ſo is ADB the Equilateral Triangle required.

PROBLEM LXXX.

Within any regular Polygon ſuppoſe a Pentagon to inſcribe an Equilateral Triangle.



1. Thro' any ∠ of the Pentagon as A and C the Center of the circumſcribing Circle draw a Line ACG.

2. With the Radius CA and one Foot in A Strike the Arch EC whoſe middle is at F.

2. Draw AF cutting the Side of the Pentagon in B.

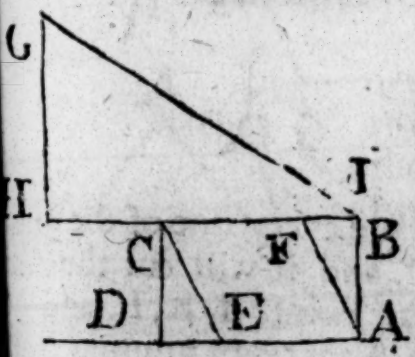
4. Make AD = AB, join BD and 'tis done.

The reaſon the 2 laſt PROBLEMS is plain, for draw the prick'd Line AE which is the Side of a Hexagon (becauſe the Side of any Hexagon

gon is equal to the Radius of its circumscribing Circle, and AE is equal AC *per* Fig.) now a Side of an Equilateral △ being the $\frac{1}{3}$ of a Circles Periphery in Chords and the Side of a Hexagon thereof an ∠ of a Hexagon must be double to an ∠ of an Equilateral △ and because the Arch FC is bisected the ∠ FAC is = $\frac{1}{2}$ ∠ EAC Q.E.I.

PROBLEM LXXXI.

Within any Rectangle as ABCD to inscribe a Rhombus.



1. By Prob. 34. Make it as BG (= 2 BC) : BH (= BC + CD) : : BI (= BC − CD) : BF.

2. Make DE = BF and draw AF and EC so have you AFCE for the Rhombus sought.

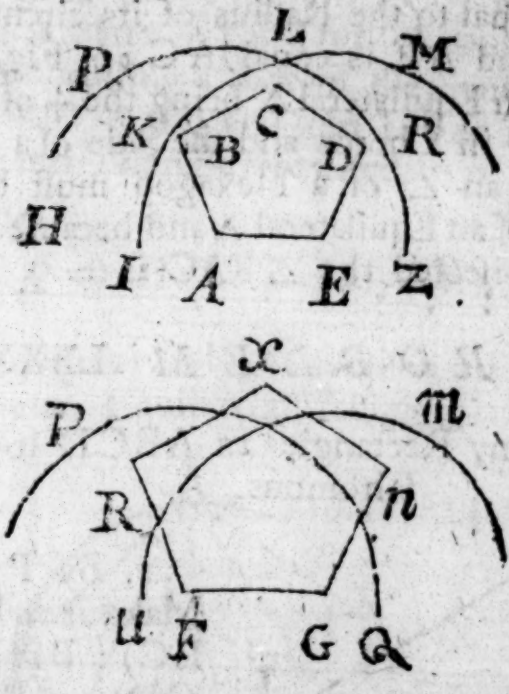
PROBLEM LXXXII.

Upon a given Right-line FG, to describe a Figure like unto a given Figure ABCDE.

This may be done by Prob. 16 or 17, Or otherwise as follows.

1, Produce any Side as AE of the given Figure ABCDE, and with any Radius and one Foot in E sweep the Arch IKLM, also with the same Radius and one Foot in A, sweep the Arch RLP.

2. Lay



2. Lay a Ruler to E and every one of the angu
lar Points A, B, C, D, and it will cut the Arch
(ſtruck upon E) in the Points I, K, L, and M
alſo a Ruler laid from A to every angular Poin
in the Figure, will cut the Arch (ſtruck upon A
in the Points Z, R, L, and P,

3. Produce the given Line F G on both Sides
and with the Radius A Z or E I and one Foot i
G, ſweep the Arch *u x m*, with the ſame Radiu
and one in F, ſweep the Arch *Q n x p*.

4. Take IK and lay it from *u* to R, alſo mak
R *x* = K L and *x m* = L M, in like manner mak
Q n = Z R and *n x* = R L, alſo *x p* = L P.

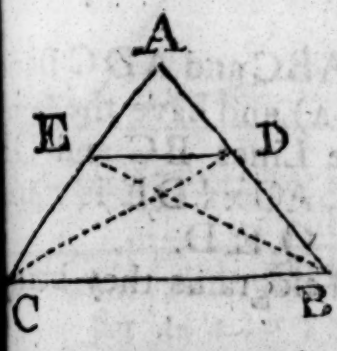
5. Lay a Ruler from G to every of tho
Marks in the Arch (ſtruck upon G) and dra
Lines as per Figuer; alſo lay a Ruler from F
each of the Marks *n*, *x*, *p*, and draw Lines whi
w

ill cut the Lines drawn from G in the Points , *x*, *n*, which are the Angles of the required Fi-ure.

With a little Consideration on equal Angles, he truth of this will appear for it depends on ROB. VI.

PROBLEM LXXXIII.

Within any Isosceles Triangle ABC to draw a Line DE Parallel to its Base CB, so that the said parallel Line and two Segments BD and CE may be all three equal.



By PROB. 7, Bisect the two Angles B and C at the Base, and where these two bisection Lines cut the Sides as in D and E, draw the Right-line DE which is the Line reqd.

For DE=DB=EC om equal Sides and equal Angles.

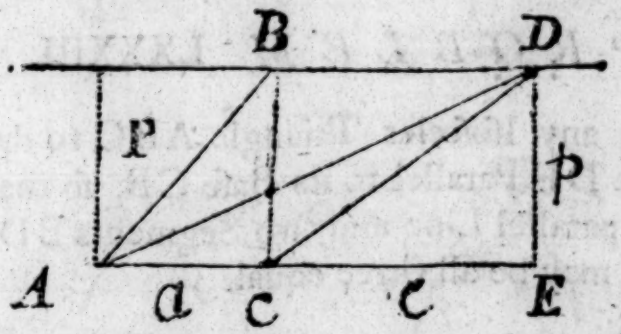
ROBLEM LXXXIV. *or, rather* THEOREM 4

All Parallelograms or Triangles which stand on the same, or equal Bases, and have the same eight, have their Areas equal. 36, 37, 38 E 1.

DEMONSTRATION

DEMONSTRATION.

	1	$\frac{1}{2}$ PE = Area △ DCE
	2	$\frac{1}{2}$ PE + $\frac{1}{2}$ PA = Area △ ADE
2 − 1	3	$\frac{1}{2}$ PA = Area △ ADC = △ ABC



That is, the Triangles ABC and ADC ſtand upon the ſame Baſe AC (= A) and have the ſame Height DE (= P); for the Lines BC and DE are Parallel; but by STEP 3, AC × $\frac{1}{2}$ DE is equal the Area of each Triangle. Q.E.D.

This alſo holds in Parallelograms they being the double of Triangles.

PROBLEM LXXXV, *Or*, THEOREM V,

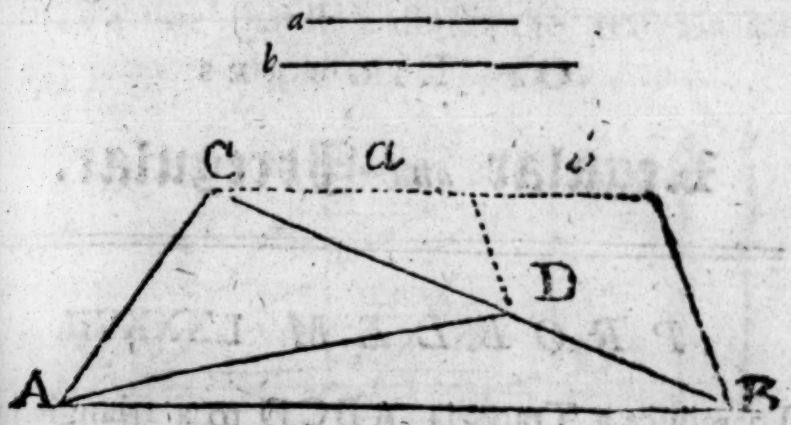
All Triangles (and conſequently all Parallelograms) that have the ſame height or Perpendicular, are in Proportion as their Baſes: For AC : × $\frac{1}{2}$ BC ($\frac{1}{2}$ PA) = Area ∠ ABC, and BD × $\frac{1}{2}$ − BC ($\frac{1}{2}$ PE) = Area ∠ DCE or BCD; But as $\frac{1}{2}$ PA : $\frac{1}{2}$ PE : : A : E, *&c*,

Theſe Theorems include Right-angled Parallelograms, as well as Oblique-angled ones,

PRO

PROBLEM LXXXVI.

To divide a Triangle ABC into any Proportion as *a* to *b* by a Line drawn from one of its Angles A to the oppoſite Side BC.



1. Divide the Side BC upon which the dividing Line is to fall as *a* to *b* that is (by PROB. 2) as *a* : *b* : : CD : DB, draw DA and 'tis one, for as *a* : *b* : : : Area △ ACD : Area △ ADB, this is plain by the laſt PROB. for the Triangles ACD and ADB ſtanding on one Line C, and both Metting in A, have the ſame Height,

☞ The XXXIV following PROBLEMS are chiefly for SURVEYING DIVIDING, *&c.*

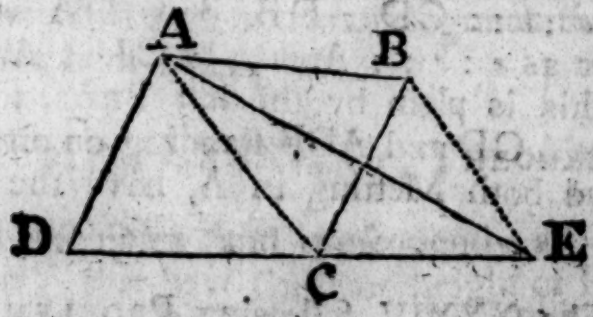
Reduction and *Division*

Of Figures

Regular *and* Irregular.

PROBLEM LXXXVII.

To reduce a Trapezia ABCD to a triangle by Lines drawn from any of its angular Point as A.

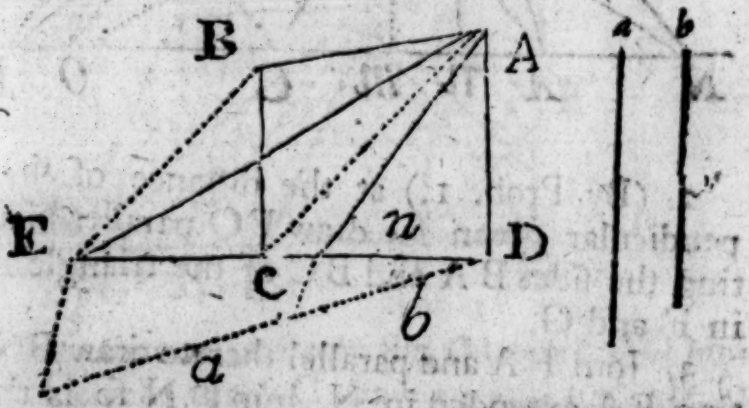


FROM A draw the diagonal AC and from B draw BE parallel thereto cutting CD (produc'd) in E join AE so is the △ AED = the trapezia ABCD as was &c. This is true (by PROB. 84) the Triangles BAC and AEC

AEC are equal for they both ſtand upon AC and are intercepted between the Parallels AC and BE.

PROBLEM LXXXVIII.

From a given angle A in a trapezia ABCD to draw a line that ſhall divide the trapezia into any proportion poſſible ſuppoſe as *a* to *b*.



From the angle A by the laſt Prob. reduce the trapezia to the triangle AED whoſe baſe DE divide into ſuch proportion as *a* to *b* (by Prob 2) that is make it as *a* : *b* : : E*n* : *n* D join A*n* and 'tis done, for as *a* : *b* : : area A*n* E (= area A*n* CB) : area A*n* D by Prob. 86.

PROBLEM LXXXIX.

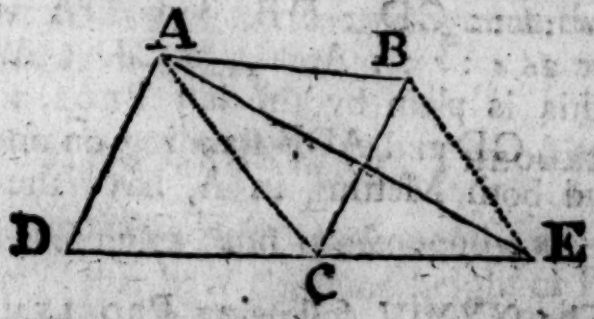
Several Triangles of different kinds (ſuppoſe two ABC and CDE) being given to make one Triangle equal to them whoſe height or Perpendicular ſhall be equal a given Line *a b*.

1. On

Reduction and *Division* Of Figures Regular *and* Irregular.

PROBLEM LXXXVII.

To reduce a Trapezia ABCD to a triangle by Lines drawn from any of its angular Points as A.

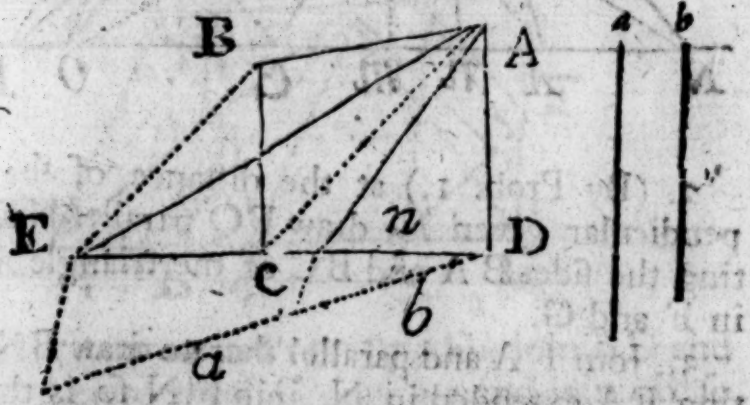


FROM A draw the diagonal AC and from B draw BE parallel thereto cutting CD (produc'd) in E join AE so is the △ AED = the trapezia ABCD as was &c. This is true (by PROB. 84) the Triangles BAC and AEC

AEC are equal for they both ſtand upon AC and are intercepted between the Parallels AC and BE.

PROBLEM LXXXVIII.

From a given angle A in a trapezia ABCD to draw a line that ſhall divide the trapezia into any proportion poſſible ſuppoſe as *a* to *b*.



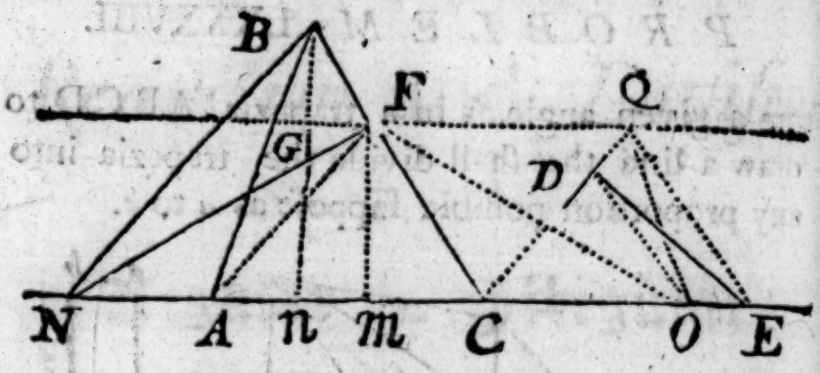
From the angle A by the laſt Prob. reduce the trapezia to the triangle AED whoſe baſe DE divide into ſuch proportion as *a* to *b* (by Prob 32) that is make it as *a* : *b* : : E*n* : *n* D join A *n* and 'tis done, for as *a* : *b* : : area A*n* E (= area A*n* CB) : area A*n* D by Prob. 86.

PROBLEM LXXXIX.

Several Triangles of different kinds (ſuppoſe two ABC and CDE) being given to make one Triangle equal to them whoſe height or Perpendicular ſhall be equal a given Line *a b*.

1. On

1. On the same right-line NE join the bases of the given triangles or any other Sides (AC and CE) thereof.



2. (By Prob. 1.) at the distance of the perpendicular given *ba* draw FQ parallel NE cutting the sides BA and BC of the triangle ABC in F and G.

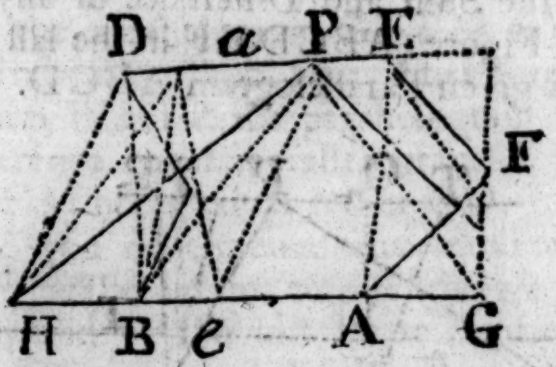
3. Join FA and parallel thereto draw BN cutting EA extended in N, join FN so is the triangle FNC = △ ABC; in like manner produce DQ till it cut FQ as in C join QE and parallel thereto draw DO cutting AE in O join OQ so is the triangle OQC = CDE join QN or FO and its done, for the triangle NFO = △ ABC + CDE as was *&c.* —— For draw the perpendiculars B*n* and F*m* now, because BN is parallel to FA the triangles ABC and AFC are similar (having both one angle at C) therefore by Theorem 1. As B*n* : NC : : F*m* : AC therefore NC × F*m* = AC × B*n*, consequently triangle ABC = NFC the same may be prov'd that triangle CDE = CQO, also = triangle FOC

by

by Theo. 4. and thus may you reduce any number of triangles or parallelograms to one, of any given perpendicular.

PROBLEM XC.

From a Point (P) in the side of any right-lin'd Figure (ABCDEF) to reduce it to a Triangle.



This is done as in Prob 87, thus join BD and parallel thereto thro' C draw *a e* and join *a* B so is the triangle BCD reduc'd to the triangle BAD for they both stand upon the same side BD and have both an angle in the parallel *e a*.

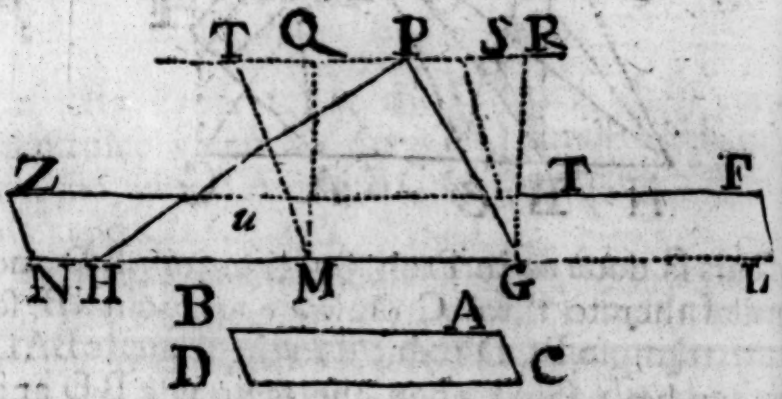
2. join PB and parallel to it draw *a* H so is the triangle A B P = △ PHB for they both stand upon BP and have each an angle in the parallel *a* H or if you join *e* D the triangle BC D is reduc'd to the triangle D *e* B for they both stand upon BD and have each an angle in *a e* join P *e* and draw DH parallel thereto draw P H as before, so is the triangle P D *e* reduc'd to the triangle P H *e* for they both stand upon P *e* and each an angle in the parallel D H so that you may see there is no difference if you have both triangles in one and the

same

ſame parallel; in like manner by drawing AE and a line thro' F parallel to AE &c. as before we get the line PG ſo is the triangle GPH equal the figure ABCDEF as was &c. and ſo may any figure be reduc'd to a triangle.

PROBLEM XCI.

To find the Sum and Difference of any Right-lined Figure (ABCDEF in the laſt Prob.) and a given Parallelogram ABCD.



1. By the laſt Prob. the irregular figure ABDCEF, is reduced to the triangle GPH whoſe baſe GH biſect with the Line QM; parallel to which draw the line RG, thro' P and parallel to the ſaid baſe GH draw the line RT cutting GR and MQ in R and Q, ſo is the triangle GPH reduced to the right-angled parallelogram GRQM.

2. Make the ∠TMN = ∠B or C of the given parallelogram ABCD, and parallel to MT draw GS; ſo is the rectangle GRQM (and conſequently the triangle GPH,) by Prob. 84. reduc'd

to

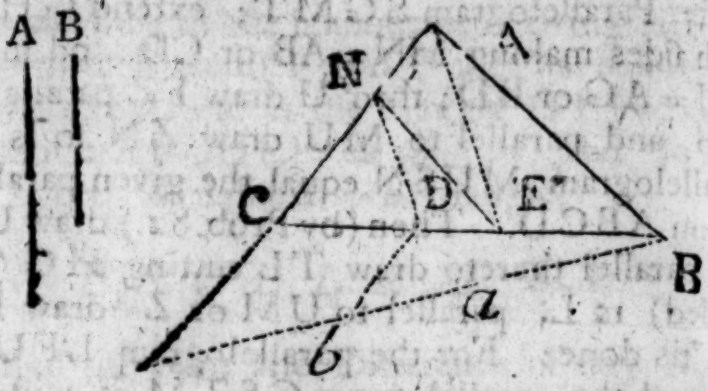
to the Parallelogram SGMT; extend GH on both ſides making MN = AB or CD, and make NU = AC or BD; thro' U draw FZ parallel to GH, and parallel to MU draw ZN ſo is the parallelogram MUSN equal the given parallelogram ABCD. Then (by Prob. 82.) draw UG and parallel thereto draw TL cutting HG (extended) in L, parallel to UM or ZN draw LF and 'tis done: For the parallelogram LFUM is equal the parallelogram GSTM as will appear by inſpecting Prob. 89. Therefore the required ſum is equal the parallelogram LFZN and difference equal parallelogram FWTL. WU or FM being equal AB or CD, ſo that by Prob. 89. you may reduce any number of triangles or parallelograms to one triangle or parallelogram, and by the laſt PROB. any irregular figure to a triangle, and by this Prob. any triangle to a parallelogram of any given height, or upon any given line and to have any propoſed angle, &c. which is of great uſe in dividing of Land geometrically as will appear hereafter.

PROBLEM XCII.

To divide a Triangle (ABC) into any proportion by a Line drawn from a given Point (E) in any of its Sides (as BC) ſuppoſe as A to B.

1. From the given point E to its oppoſite angle A draw the line EA, then (by Prob. 32) divide BC as A to B; that is, as A is to B ſo is BD to DC.

2. Thro'



2. Thro' D and I parallel to EA draw DN, join EN and its done: For as BENA is to ENC ſo is A to B.

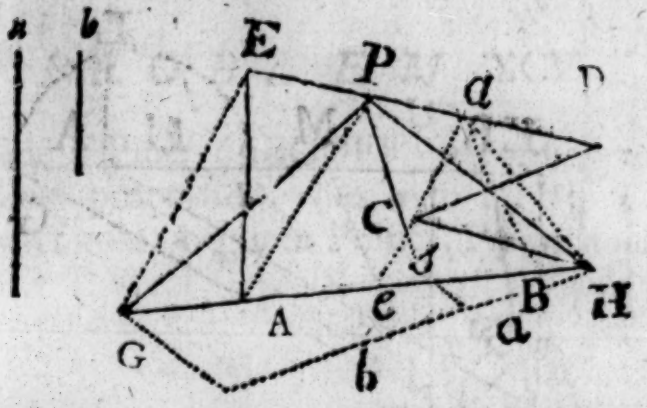
If you join AE it will be, as A is to B ſo is triangle BAD to triangle DAC; now per figure, triangle DAN + triangle NDC = triangle DAE, alſo triangle CDN + NDE = triangle ECN; but triangle DAN = triangle EDN by Prob. 84 for they both ſtand upon ND and have each an angle in AE (*viz.* the angle A and E) and AE is parallel to ND by conſtruction: conſequently the triangle ENC = triangle DAC, ſo the line EN is truly known.

PROBLEM XCIII.

From any aſſigned Point (P) in the Side of any Right-lined Figure (ABCDE) to divide the ſame into any poſſible proportion, as *a* to *b*.

1. By Prob. 90. The given figure is reduced to the triangle GPH, whoſe baſe GH divide into ſuch proportion as *a* to *b* that is, as *a* is to *b* ſo is HS to SG, ſee Prob. 86 and 88.

2. From



2. From the given point P draw PS and its done: For as *a* is to *b* ſo is Area S PH (SP DCB) to Area PGS (PEAS,) this is plain by the figure, for by Prob. 87. the triangles PGA and ACP are equal; now PGA + PAS = PGS, as alſo = PEAS, becauſe AEP = PGA.

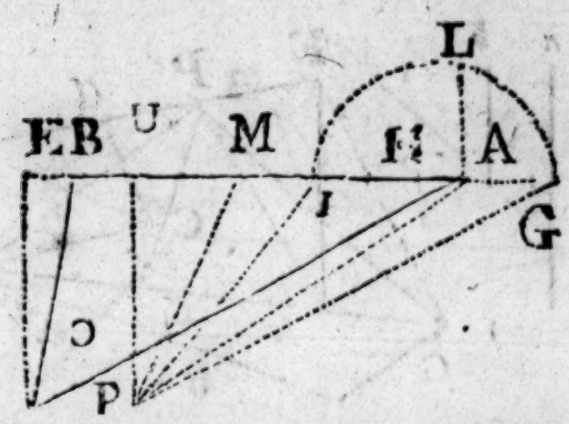
Here Note, That the triangles *a* BD and BCD being equal (by Prob 90) if either of them be taken out of the ſpace *a e b d* there will leave the ſpace or triangle *e* B *a* ſo that the two lines B C and C D are reduc'd to the one line *a b* ſee the fig.

PROBLEM XCIV.

To divide a triangle A B C into any Proportion (as *a b*) by a Line drawn from *a* Point (P) without the Triangle.

1. Thro' the given point P and parallel to A C draw P G cutting B A extended in G.

2. By Prob. 32, make it as a is to *b* ſo is AH to HB, alſo by Prob 34 make it as G P : AC : : AH : H I.

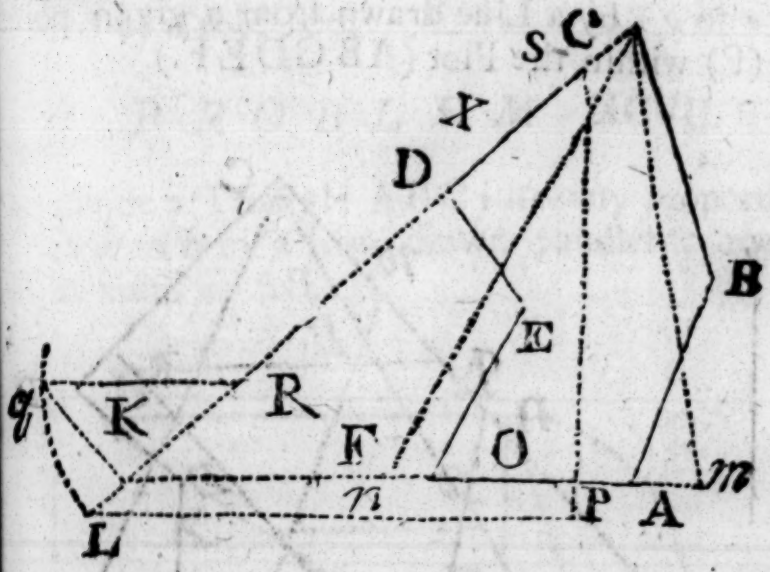


3. By Prob. 36, find a mean proportional between GA and AI which is found AL, take AL and lay it from H to M join PM and its done, for as $a : B :: AMD : MDCB$ draw the perpendiculars CE and PU there by Theorem 1. as $AC : CE :: GP : PU$ join CH then $CE \times AH = \triangle AHC$ (by Theo. 4.) and $PU \times AG$ = triangle AIP first join ID but triangle AIP = triangle AHC that is $PU \times AG = CE \times AH$ hence $GP \times AG = AC \times AH$, which last equation gives as $GP : AC :: AH : AI$ the same as above in the same manner may Prob. 92 be done, now the triangles AIP, AMP *&c.* have all one height and so are in proportion as their bases AI, AM *&c.* and therefore the areas of such triangles may be expressed in any proportion by their bases *&c.*

PRO-

PROBLEM XCV.

To divide a Plot (ABCDEF) according to any poſſible proportion (as *a* to *b*) by a Line drawn from a given Point (P) without the Plot.



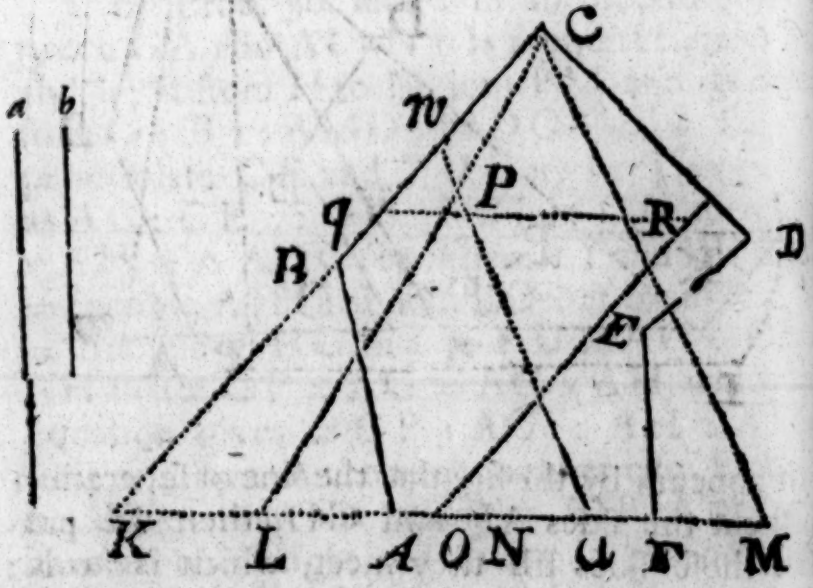
It appears by the fig. that the line of ſeperation muſt cut the ſides AF and CD, therefore produce thoſe ſides till they meet, which is at K; then from the angle C (oppoſite to the given point P) by Prob. 90, the figure will be reduc'd to the triangle MCN, whoſe baſe MN divide as *a* to *b*; that is, As *a* to *b* ſo is MO to ON, thro' P draw PL parallel to MN cutting CD extended in L, and make it as LP : KO :: KC : KX. Then by Prob. 36, a mean proportional between LK and KX is found equal K*q*; then (always) divide XK into two equal parts in R, and draw

Rq

R q which ſet from R to S, join SP and its done: For as $a : b : :$ ABCSP : PSDEF, this is proved per laſt Problem.

PROBLEM XCVI.

To divide a Plot into any proportion poſſible (as a to b) by a Line drawn from a given point (P) within the Plot (ABCDEF.)



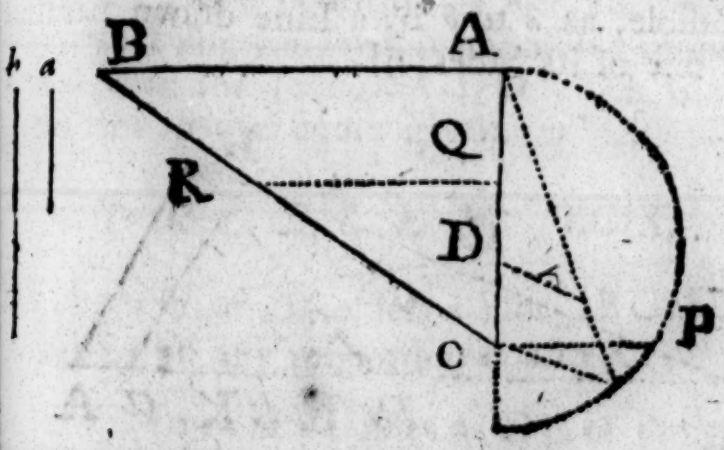
It appears that the line of diviſion will cut the ſide AF and CB, therefore produce thoſe ſides till they meet which is at K; by Prob. 90, the figure will be reduced to the triangle LCM, whoſe baſe LM make as $a : b : :$ LN. : NM, then take the neareſt diſtance between the given point P and AF (*viz.* a perpendicular from P to AF) and ſay as a perpendicular from P to AF is

to

to a perpendicular from C to LM so is $\frac{3}{4}$ KN to KO, thro' which O and parallel to KC draw RO; also thro' P and parallel to AF draw qPR crossing RO in R, then it must be $\sqrt{\square PR - \square qP}$ = a line S, which line S set from O to U, so thro' U and P draw the line WPU and its done: For as $a : b :: ABWU : UWCDEF$. This is proved by Prob. 109.

PROBLEM XCVII.

To divide a Triangle ABC into any proportion as *a* to *b* by a Line drawn parallel to one of its Sides as AB.



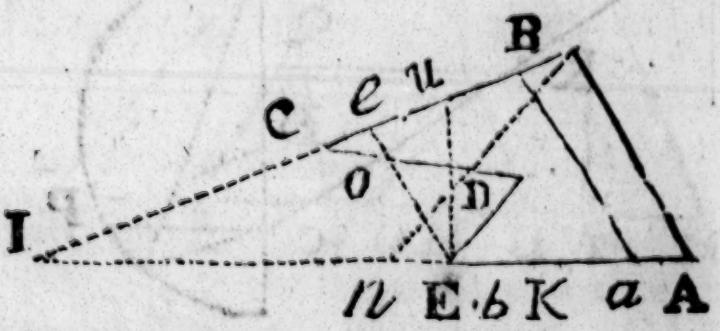
Divide either CA or CB, suppose CA into such proportion as *a* to *b*; that is, as $b : a :: AD : CD$: Then by Prob. 36, you'll find CP a mean proportional between the whole side AC and the segment DC, which CP lay from C to Q then thro' Q and parallel to AB draw QR and its done: For as $a : b :: CQR : ABRQ$, this

this way I have done for variety, but it may be done easier by Prob. 41, and so may the following Problem.

If the triangle is to be divided as a to b, then it is evident its area must be either equal or proportional to $a+b$, and AC being made as a to b we have by Theo. 3, Prob. 70: As $a+b$ (area $\triangle$ ABC) : $\overline{a+b}|^2$ ($\square$ AC) :: a (area $\triangle$ CQR) : $a \times a+b$ ($\square$ QC) but by construction $\square$ QC is $=$ AC $(a+b) \times$ DC $(a,)$ so the line QR is truly drawn.

PROBLEM XCVIII.

To divide any Plot ABCDE into any proportion possible, as a to b by a Line drawn parallel to one of its Sides AB.



Because the line of seperation is to be parallel to AB it must consequently cut BC and AE, therefore produce those two sides till they meet as in I, then the fig. will be reduced to the trapezia ABE, and then to the triangle ABN whose base NA make as $a : b$: : AK : KN: Then find a mean proportional between AI and KI which is found equal

equal to I*a*, ſo thro' *a* and parallel to AB draw *aw* and its done: For as *a* : *b* : : CB*wa* · *wa*EDC; If to the trapezia AB*u*C we add the triangle C*I**u* we have the triangle ABI now ſuppoſe *d* = the baſe of the triangle C*u*I and to bear ſuch proportion to its area as *a* × *b* to the area of ABC*u* then by Thorem 3, as *a* × *b* × *d* area triangle AIB) $\overline{a \times b \times d}^{2}$ (□ AI) : : *d* × *b* area triangle *wa*I): $\overline{a \times b \times d \times d \times b}$ = AI × I,K = □ I*a* ſo *wa* is truly drawn.

Note, that if the proportions be ſuch as that the the line of ſeparation cuts more ſides of the given figure than two in ſuch caſes it is not poſſible to perform the required diviſion, as in the above figure if C*e* were the required parallel line you ſee it is impoſſible becauſe it takes in the triangle CDO which triangle is not in the figure given.

PROBLEM XCIX.

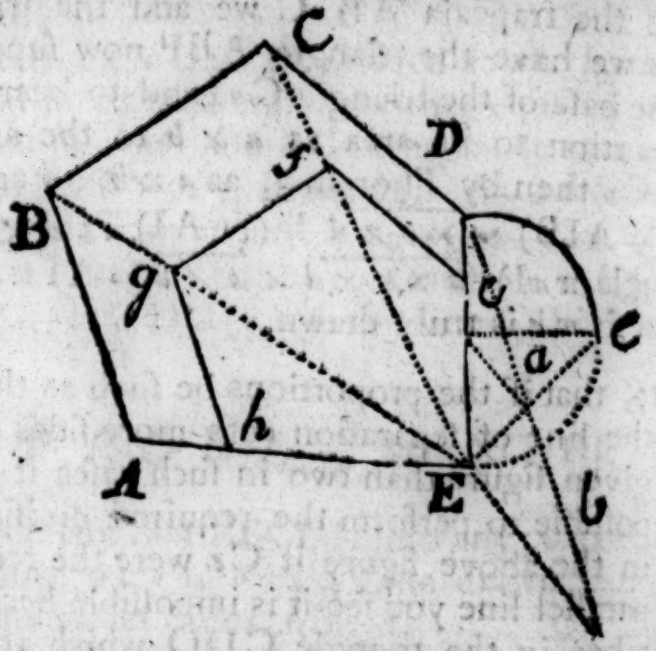
To diminiſh or make leſs a Plot ABCDE according to any proportion as *a* to *b*.

1. Take any of the ſides as DE of the given Plot and by Prob. 97 or 41 make it as *b* + *a* : *b* : : □ DE : □ *e*E and draw the diagonals EC and EB.

2. Thro' *e* and parallel to CD draw *ef* cutting EC in *f* thro' *f* and parallel to BC draw *fg* and

thro'

thro' g and parallel to BA draw gh and its done
or as a : b : : ABCDE : efgh E per Theorem 3.



PROBLEM C.

To enlarge a Plot (efgh E last Figure) ac-
cording to any proportion, as b to a.

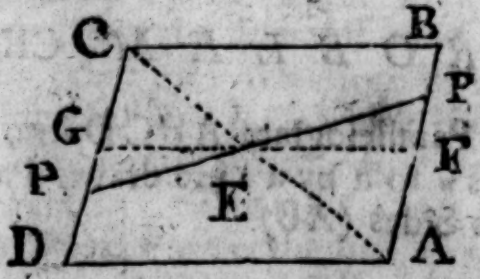
1. Draw the diagonals Ef and Eg and extend
the sides E h and Ce.

2. By Prob. 42, enlarge the side E e in powe
as b : to a that is make it as b : ab : : □ Ee :
□ ED, draw D C parallel to ef cutting fE in
C extended, likewise draw CB parallel to fg and
BA parallel to gh so its done, for as b : a :
efgh E : ABCDE by Theorem 3.

PRO-

PROBLEM CI.

To divide a Parallelogram ABCD into two equal Parts by a Line drawn from a given Point (P) any where within, without or in the Sides or Angles &c.



1. Thro' the middle of any two opposite sides draw a line, or which is the same thing draw a diagonal as AC whose middle is at E.

2 Lay a ruler to E and the point P wherever it be and draw the line PEP and 'tis done for PBCP is = PADP.

If thro' E the middle of the given parallelogram ABCD you draw FG parallel to AD or BC then *per* fig. angle GEP = ∠ FEP and because AB and CD are parallel the angles at F and G are equal also AD and BC being parallel the angles at B and D are equal and because E is in the middle of the line AC the sides of the triangles PEF and PEG as well as their angles are respectively equal consequently the triangles PEF and PEG are equal; hence it is evident that if the line PP move upon E as a center it will in any position divide the parallelogram ABCD into two equal parts.

After the ſame manner it may be prov'd that a diagonal AC in any parallelogram ABCD divides the ſame into two equal parts or triangles (ABC and ADC) hence it is that every triangle (ABC or ADC) is equal to half its circumſcribing parallelogram ABCD 41, E. 1.

PROBLEM CII.

To divide a Parallelogram ABCD into any Proportion as *a* to *b* by a Line drawn Parallel to one of its Sides (AB).



1. Becauſe the diviſional line muſt cut the lines AD and BC divide either of thoſe lines as BC by Prob. 32 that is make it as *a* : *b* :: BF : FC.

2. Make AE = BF and draw EF ſo its done for as *a* : *b* : : ABFE : FEDC the parallelograms ABFE and FEDC have the ſame height and are therefore proportional to their baſes BF and FC Prob. 35.

PRO-

PROBLEM CIII.

To divide a Parallelogram ABCD laſt Fig. into any poſſible Proportion as *a* : *b* by a Line drawn from a given Point (P) any where.

1. It appears by the figure that the line of diviſion muſt cut the ſides AB and CD therefore divide the other two ſides into two equal parts by drawing the line L G which divides the given parallelogram into two equal parts.

2. By Prob. 32 make it as *a* to *b* : : L *q* : *q* G then thro' *q* and P draw the line *u n* and 'tis done for as *a* : *b* : : *u* A D *n* : *n* C B *u* if thro' *q* and parallel to A D we draw *z y* then by the laſt Prob. it will be as *a* : *b* : : *y* A D Z : *y* B C Z and it may be prov'd as in Prob. 101 that the triangles *u q n* and *z q n* are equal ſo that the line P *q n* in any poſition (while it cuts the two oppoſite AB and D C will divide the parallelogram as *a* to *b*; this Prob. as well as the foregoing ones is of great uſe in dividing of Land for ſuppoſe the parallelogram were divided as 2 to 3 by the lines *y z* drawn parallel to the line or hedge A D and ſuppoſe the ſaid line *y z* for ſome reaſon is to be ſhifted 2 chains from C toward D here you muſt meaſure 2 chains from *z* to *n* and then go to the oppoſite ſide AB and meaſure 2 chains from *y* to *u* and ſo line out the hedge from *u* to *n* and 'tis done for *u* A D *n* is = *y* A D Z *&c.*

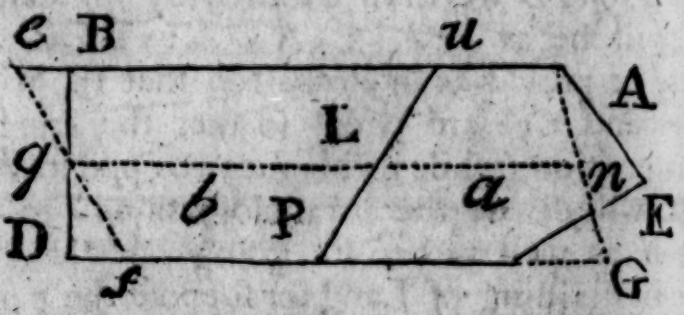
But here Note if the line of diviſion be ſuch an one as Q *b* which does not cut two oppoſite ſides then this diviſion is not true as may appear

per

per figure for the triangle D $q m$ wants the triangle $h m$ D to make it = the triangle Q $q u$ so that Q A h is too little by $h m$ D this is to be observed in all Problems that mention possible proportion.

PROBLEM CIV.

To divide a Plot (ABGDE) having two opposite Sides (AB and GD) Parallel into any possible proportion by a Line drawn from a Point (P) any where given so that the Line of Division pass thro' the Parallel Sides let the Proportion be as a to b.



1. By Prob 90 the figure is reduc'd to the trapezia GABD thro' the middle of whose sides GA and DB draw the line $n g$ which by Prob 32, make as $a : b :: n$ L : L g.

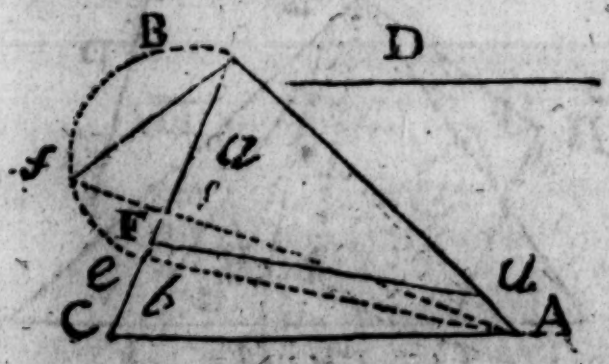
2. Thro L draw u L P passing thro' the given point P and and its done, for as $a : b :: u$ A E G P (= n A n G P) : D B P, draw $f e$ parallel to A G then by the last Prob. as $a : b :: u$ A n G P : P $f e u$ but the triangles B $g e$ and $f g$ D are equal by Prob. 101 consequently D B n P = n P $f e$ so the line u P is truly drawn. These IV, Problems

blems may be done as the foregoing ones but they are ſo eaſy done this way and are of ſo great uſe in ſurveying that I think it pity to omit them.

Note the line *u* P may vary upon L as a center any how (while it cuts the ſides AB and GD) it will always keep the figure in the ſame proportion ſee the laſt Problem.

PROBLEM CV.

To divide a Triangle ABC into any proportion as (*a* to *b*) by a Line making given Angles with the Sides of the Triangle; or which is the ſame thing, by a Line drawn parallel to a given Line D any where either within or without a Triangle.



1. By Prob. 32, Make one of the ſides which the diviſional line is to cut, as *a* to *b*, that is, as *a* : *b* : : B*g* : *g*C; join A*g* and from A parallel to D draw A*e*; then by Prob. 41, make it as B*e* : Parallelo. B*e* : : B*g* : □B*f*.

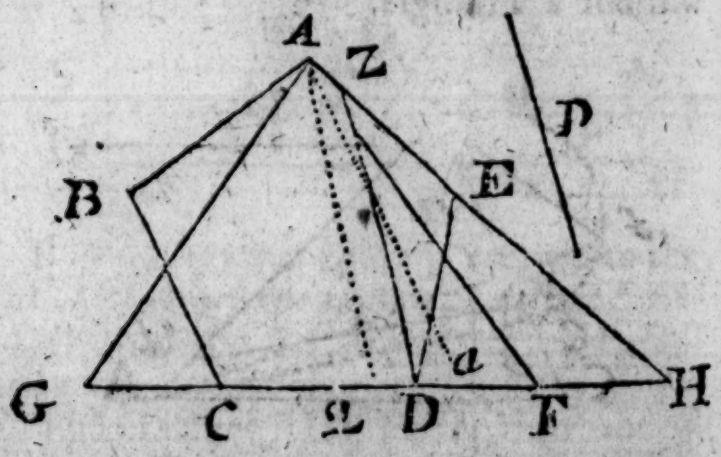
2. Thro'

2. Thro' F and parallel to D or Ae, draw FU and its done: For as $a : b$: : BUF : UACF.

Or as in Prob. 97, A mean proportional between eB and gB, will be found equal BF, by which Problem the truth of this will appear; or thus, The triangles BAg and BAe having the same height are as their bases (*per* Theo. 5,) and per Prob. the triangle BUF must be equal triangle BAg, therefore by Theo. 3, as eB ($\triangle BAe$: Parallelo. eB : : Bg ($\triangle BAg = \triangle BUF$) : □ BF

PROBLEM CVI.

To divide a Plot ABCDE into any possible Proportion (as $a : b$) by a line drawn parallel to a given line (D) any how situated.



It appears by the figure that the line of sepcration must cut the sides AE and CD, therefore produce those sides till they meet in H, and the figure is reduced to the triangle AGF, whose base GF make as a ; b : : Ga : aF ; join Za and draw

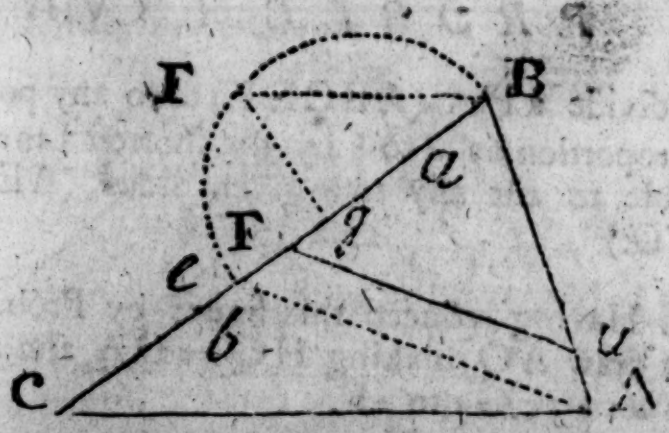
raw AQ parallel to the given line D, then make
t as HQ : □HQ : : Ha : □Hd = HD; ſo
arallel to D or AC draw ED and its done, for
s a is to b ſo is ZABCD to DZE.

As in the laſt Prob. the triangles HaA and
HQA are as their baſes Ha and HQ and ZD is
arallel to AQ, therefore by Theo. 3, as HQ :
□HQ : Ha (area triangle HaA = △HFZ)
□Hd which make HD equal to.

Note, If a had been a little nearer towards F
he Prob. would have been impoſſible, for ZD
would cut ED &c. ſee the Figure.

PROBLEM CVII.

To divide a Triangle ABC into any proportion as a to b by the ſhorteſt Line poſſible, cutting two given Sides BA and BC.



1. Draw Ae making eB equal the ſide BA, and by Prob. 32, divide the ſide BC according to the given proportion, as a : b : : Bg : gC.

2. By

2. By Prob. 41. Make it as $eB : \square eB :: Bg : \square BF$, (thus upon eB as a diameter, describe the semi-circle, and upon g raise the perpendicular gF, and join BF;) Thro' F and parallel to Ae draw Fu and its done: For as $a : b :: BFu : FCAu$; and no line shorter then uF can do it. This is but a repetition of the last Prob. for the line of division uF is to cut the side BA and BC (which it passes thro') in equal angles; that is, angle $BuF = \angle BFu$, for which reason you make $eB = AB$.

The reason may be thus defined; a circle contains a greater area than any other figure made of its periphery can do, therefore an equilateral triangle contains more space than any other triangle made out of the sum of the same.

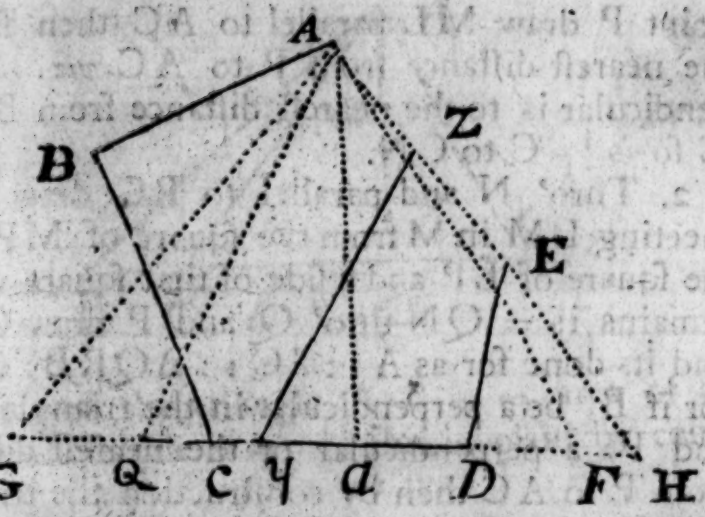
PROBLEM CVIII.

To divide a Plot (ABCDE) into any possible Proportion as a to b by the shortest Line possible to cut any two given Sides (AE and CD.)

1. Having reduced this figure by Prob. 106 next draw AQ making $HQ = HA$ and make it as $a : b :: Ga$ to aF.

2. By Prob. 41, Make it as $HQ : \square HQ :: Ha : \square Hy$, so thro' y and parallel to AQ draw

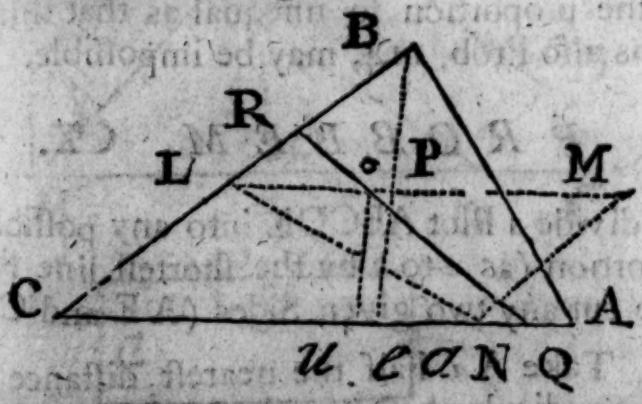
y



z and its done, for as $a : b :: zABCy : yDEz$ nd is prov'd exactly as problem 106.

PROBLEM CIX.

To divide a Triangle ABC into any Proportion as a to b by a Line drawn from a given Point (P) within the triangle.



1 The dividing line being to cut AC and BC divide either of these lines as AC by Prob. 32, hus as $a : b :: Aa : aC$; and thro' the given

point P draw ML parallel to AC then ſay as the neareſt diſtance from P to AC *viz.* *a* perpendicular is to the neareſt diſtance from B to AC ſo is ½ *a* C to C N.

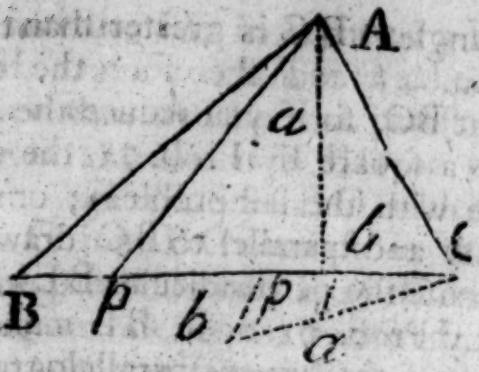
2. Thro' N and parallel to BC draw NM meeting LM in M from the ſquare of MP take the ſquare of LP and a ſide of that ſquare which remains is = QN thro' Q and P draw QPR and its done for as A *a* : *a* C : : AQRB : CRQ for if B *e* be a perpendicular in the triangle ABC and P *u* a perpendicular or the neareſt diſtance from P to AC then by conſtruction the triangle NLC = ½ triangle CQR = QRLN that is P *u* × NC = ½ C × *e* B, which gives, as *e* B : P *u* : : ½ *a* C : CN as above ſo that the line QR it to paſs thro' P in ſuch a manner as to make the ſpace QRLN = triangle NLC now MN = LC and ML = NC becauſe of their being parallel to one another *&c.*

Note, P may be ſo far within the triangle and the proportion ſo unequal as that this Prob. as alſo Prob. 114, may be impoſſible.

PROBLEM CX.

To divide a Plot ABCDE into any poſſible Proportion (as *a* to *b* by the ſhorteſt line Poſſible to cut any two given Sides (AE and CD)

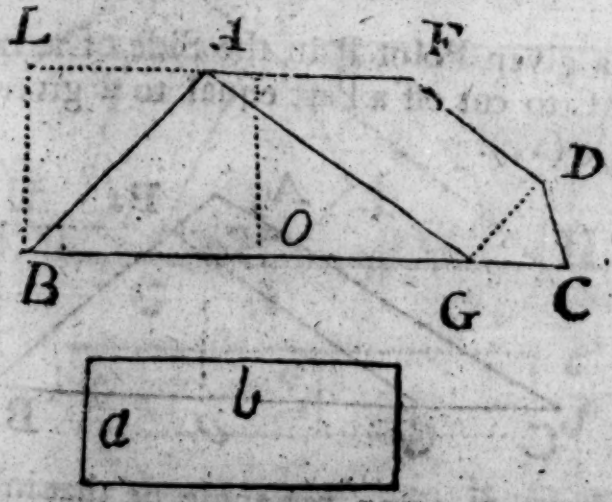
1. Take A *a* half the neareſt diſtance (*viz.*) a perpendicular between the angle A and its oppoſite ſide BC and by Prob. 33, make it as A *a* to *ab* ſo is *ab* to B *f*, join A *f* and its done for the



the area of the triangle A$_p$B is = area of a ſquare whoſe ſide is = ab, this is plain by conſtruction for A$a \times {}_p$B = $ab \times ab$.

PROBLEM CXI.

By a Line drawn from a given Angle (A) in a Right-lin'd Figure ABCDE to cut off a Part towards the Angle B that ſhall be equal to a given Rectangle ($a b$.)



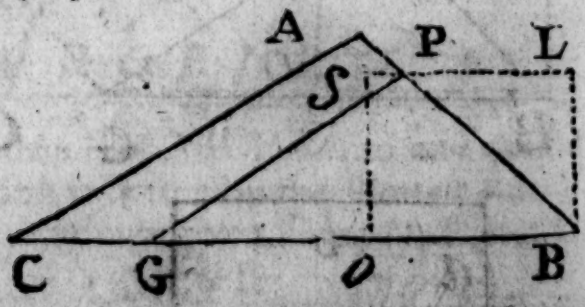
1. If the line of diviſion cut the line CD then extend CD and AE till they meet σ

But

But the triangle ABC is greater than the given parallelogram *ab* and therefore the diviſional line muſt cut BC, ſo if you reduce the parallelogram *ab* to a ſquare by Prob. 71 the work will be the ſame with the laſt problem; or otherwiſe thus, thro' A and parallel to BC draw LS and upon CB raiſe the perpendicular BL cutting SL in L then by Prob. 91 upon BL make a parallelogram equal the given parallelogram (*viz*) parallelogram BLSO equal parallelogram *ab* then make OG = OB and draw BA ſo is the triangle ABG = the parallelogram *ab* as was *&c.* This is plain by conſtruction for the parallelogram BLSO and triangle ABG have the ſame height and the baſe of the triangle is double the baſe of the rectangle *viz.* AG = BO.

PROBLEM CXII.

From a given Point P in the Side of a triangle ABC to cut off a Part equal to a given Rectangle (*ab*).



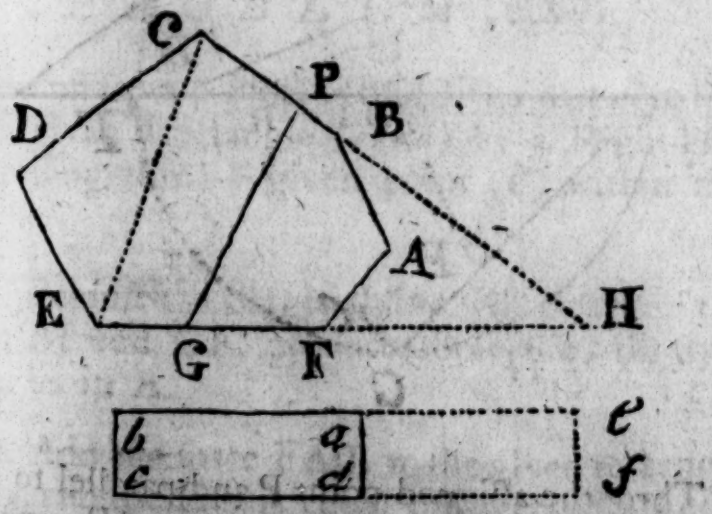
1. Let the given rectangle be that in the laſt Prob. *viz. ab.*

2. Thro' P and parallel to its oppoſite ſide BC draw LS and from B perpendicular to BC draw

draw BL meeting SL in L then by Prob. 91 or 89 upon BL make a rectangle equal to the given rectangle *ab* thus reduce the parallelogram *ab* to one whose altitude is = BL or make it as BL : *aa* : : *ab* : LS = BO which BO lay from O to G join PG and so is the triangle BPG = the parallelogram LSOB consequently = parallelogram *ab* as was required; the truth of this is made manifest in Prob. last.

PROBLEM CXIII.

To cut off from a given Plot or Polygon (ABCDEF) a Part = to a given Rectangle (*abca*) by a Right-line passing thro' a given Point (P) in one of its Sides (BC.)

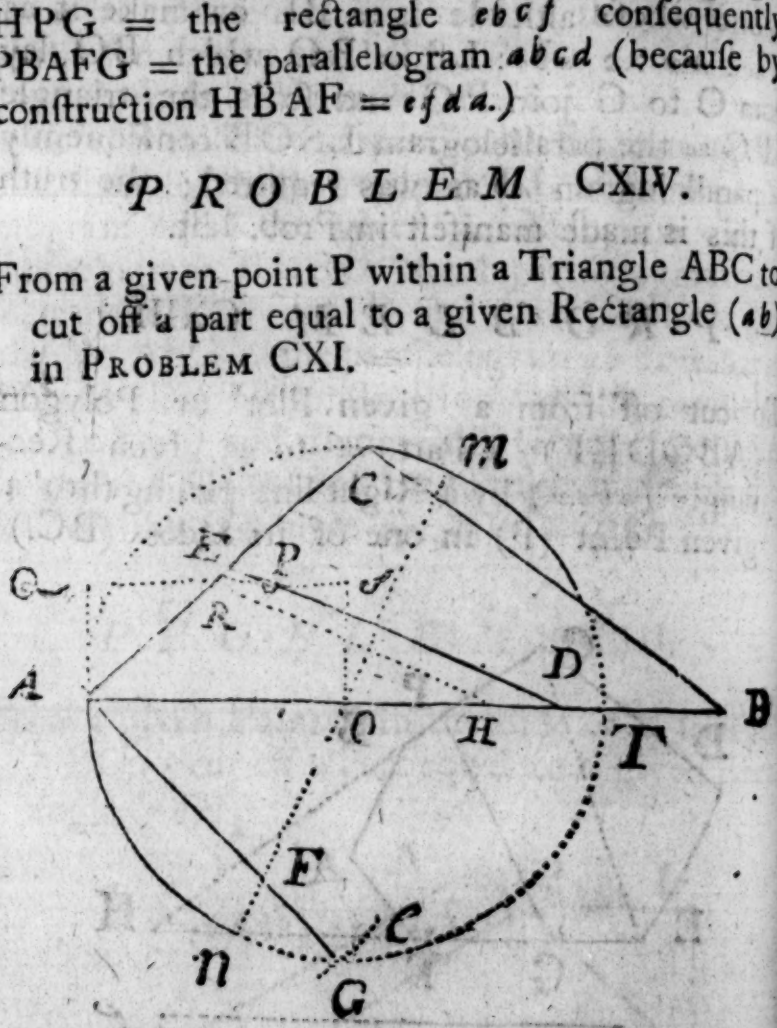


1. Because the dividing line is to cut the sides EF and CB produce those sides till they meet which is at H then by Prob 91 add the figure HBAF to the given parallelogram *abcd* and

and you'll have the rectangle *ebcf* to be cut from the triangle HCE thro' P with which work as in the last Prob. and you'll find the triangle HPG = the rectangle *ebcf* consequently PBAFG = the parallelogram *abcd* (because by construction HBAF = *efda.*)

PROBLEM CXIV.

From a given point P within a Triangle ABC to cut off a part equal to a given Rectangle (*ab*) in PROBLEM CXI.



Thro' the assigned point P and parallel to AB, draw QPS cutting AC in R, upon AB erect the perpendicular AQ meeting SQ in Q; upon AQ make the parallelogram AQSO equal the parallelogram *ab*; with the distance AG (=AO+2RP) and one foot in A sweep an arch G, and with the distance

diſtance AO and one foot in O croſs it with another arch *ee* in G, ſo draw AG thro' which AG draw OF making AF = AO: Make OD = FO thro' D and P draw DPE, ſo is the triangle AED = parallelogram AQSO as was required. The proof of this is as follows; Upon O with the radius OA deſcribe the circle, and extend FO till it become the diameter *nom* and parallel to ED thro' R draw AH; then AD is = MF becauſe AF = AO, FG = 2RP and DO = FO, and for the like reaſon DT = NF, (now per 35, E 3.) AF×FG = NF×MF = AD×DT = MF×NF = AO×2RP = (2AO) TA×RP: whence AD×DT will be = AT×HD, becauſe HD = RP. (for RP and HD are parallel, alſo PD and RH are parallel.)

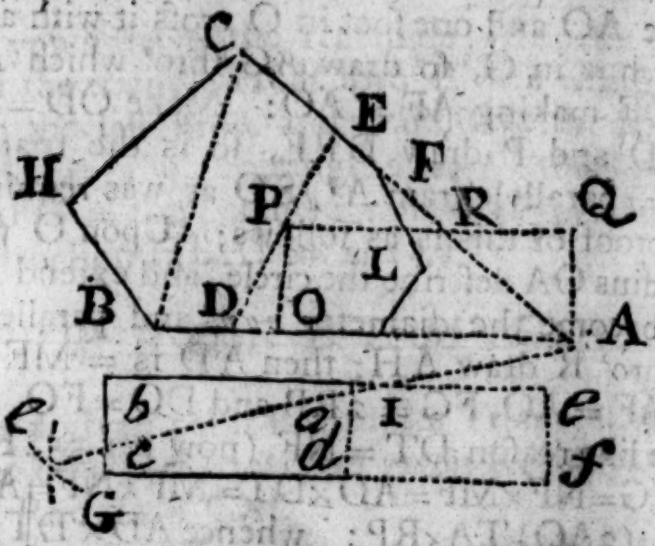
PROBLEM CXV.

To cut off from a Plot (HBCLIF) a part equal to a given Rectangle (*acbd*) by a Right-line paſſing thro' a given point (P) within the Plot.

1. Becauſe the diagonal line will paſs thro the ſides BI and CF, produce thoſe ſides till they meet as in A.

2. Add the ſpace FAIL to the given rectangle *abcd* and the ſum is equal the rectangle *befc*; now the work is the very ſame as in laſt Prob. for we have the parallelogram *befc* to cut off the triangle ABC; and therefore the directions in the laſt Prob. alſo works this Prob. becauſe the letters

are



are the ſame in both &c. EFLID = *abcd*, for the triangle AED = parallelogram *befc* = parallelogram AQSO.

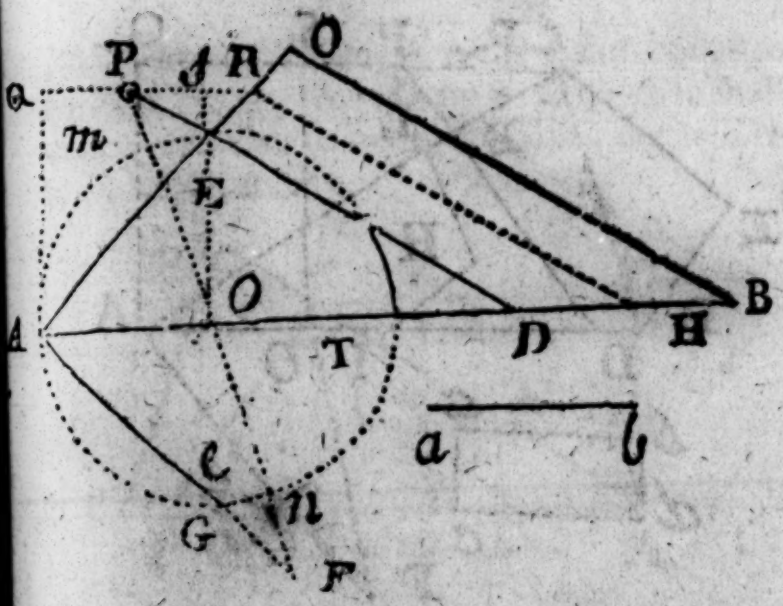
PROBLEM CXVI.

To cut off from a Triangle ABC a Part equal to a given Square whoſe Side is = *ab* by a Line drawn from a Point P without the Triangle.

1. As in Problem 114 make the rectangle AQSO = the given ſquare whoſe ſide is = *ab* and with the radius AGC = 2 AO — RP and one foot in A ſweep an arch with the radius AC and one foot in O croſs it with another arch *ecm*G ſo draw AG produc'd to F making GF = RP join FO and make OD = FO.

2. Thro' D and P draw DEP making the triangle AED = the required ſquare. To prove this draw RH parallel to ED upon O with the radius

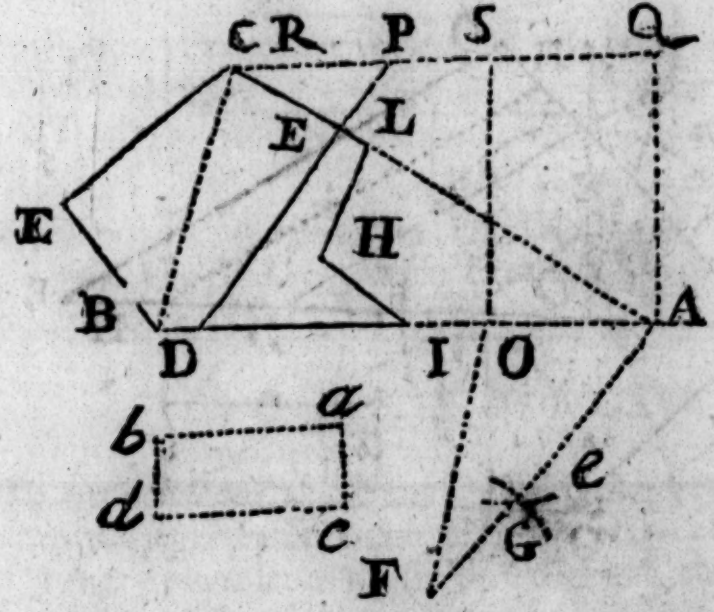
adius OA ſweep the circle and extend F O to *m* making the diameter *n o m*, then *per* conſtruction GF = RP, AF = AT and F O = OD therefore is AD = *m* F, DT = *n* F and AD × DT = *m* F × *n* F = AF × GF (36, E. 3.) = AT × BP whence AD × DT = AT × DH, be-



uſe DH = BP (*per* figure) now if to each of oſe equal rectangles be added the rectangle D × AT we ſhall have AD × DT + AT = T × DH + AD or parallelogram AD = AT AH *&c.* as in Prob. 114 the triangle ADE prov'd = the rectangle AQSO *&c.*

PROBLEM CXVII.

To cut off from a given Plot IHLCEB a part equal to a given Rectangle *abcd*, by a Line drawn from a given point P without the Figure.

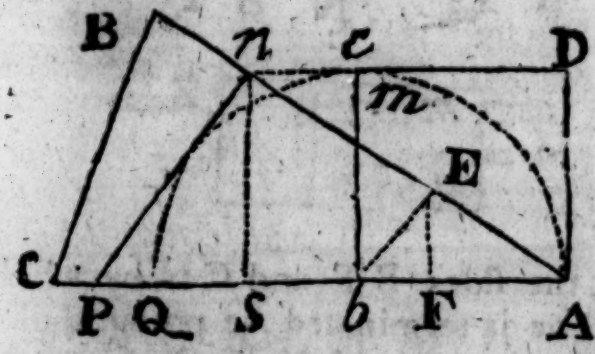


Becauſe the line of diviſion is to cut the ſide CL and BI, extend thoſe ſides till they meet in A and draw BC; then it appears by the figure that if to the given parallelogram *abcd*, you add the figure ALHI and take that ſum from the triangle ABC as in the laſt Prob. it will be done: Therefore on A raiſe the perpendicular AQ, thro' the given point P and parallel to AB draw QR cutting AC in R; upon AQ make the parallelogram AQSO equal the parallelogram *abcd* + ALH the

hen from twice AO take RP and with the remainder and one foot in A, ſweep an arch G; with the radius AO and one foot in O croſs it in *r*, draw AG till FG = RP make OD = OF and draw DP and its done: For LEDIH = parallelogram *abcd* per laſt Prob.

PROBLEM CXVIII.

To cut off from a Triangle ACB a part equal to a given Square A*b*cD by a Line that ſhall make given Angles with the Sides of the Triangle which it cuts.

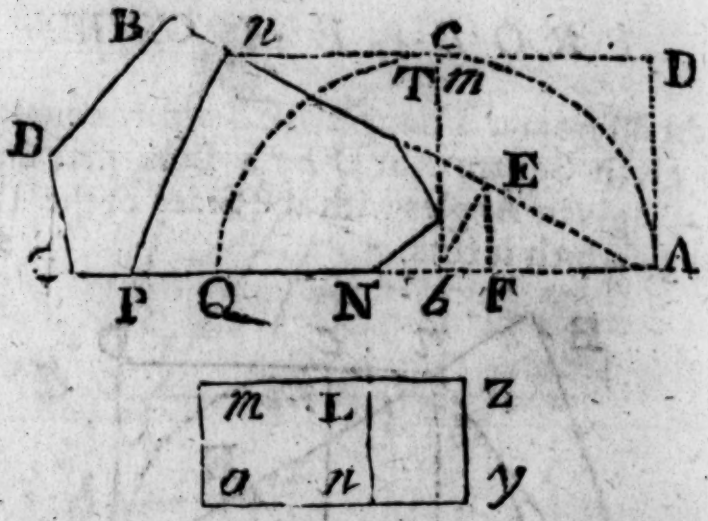


Suppoſe the dividing line is to cut the ſides AB and AC and to make equal angles therewith equal to the angles E*b*A and *b*EA; from E let fall the perpendicular EF and make *b*Q = 2 EF, then upon AQ deſcribe the ſemi-circle A*m*Q cutting *bc* in *m*, thro' *m* and parallel to AC draw *mn* meeting AB in *n*, thro' *n* and parallel to E*b* draw *n*P, ſo is the triangle A*n*P equal the ſquare A*bc*D, as was *&c.*

PRO-

PROBLEM CXIX.

From a given Polygon ABCDEF to cut of a Part equal to a given Rectangle L *mno* by the ſhorteſt Line poſſible.



1. Let the ſides BT and CN by which the dividing line is terminated be produc'd till they meet in A, now it is plain by the figure that if to the given parallelogram L *mno* (by Prob. 91 you add the ſpace TANE that ſum (*viz*) the parallelogram *zmoy* being taken from the triangle ABC as in the laſt Prob. it will be done therefore 2, By Prob. 71, the parallelogram *zmoy* is reduc'd to the ſquare A *bcd* then by Prob. 107 make AE = A*b* and draw BE, from E let fall the perpendicular EF make *b*Q = twice EF and upon AQ deſcribe the ſemi-circle or which is all one make *bm* = ½ AQ thro' *m* and parallel to AC draw *mn* meeting AB in *n*

draw

draw nP parallel to EB ſo is, triangle AnP = parallelogram $zmoy$ therefore nENP = the given rectangle Lmno as was required.

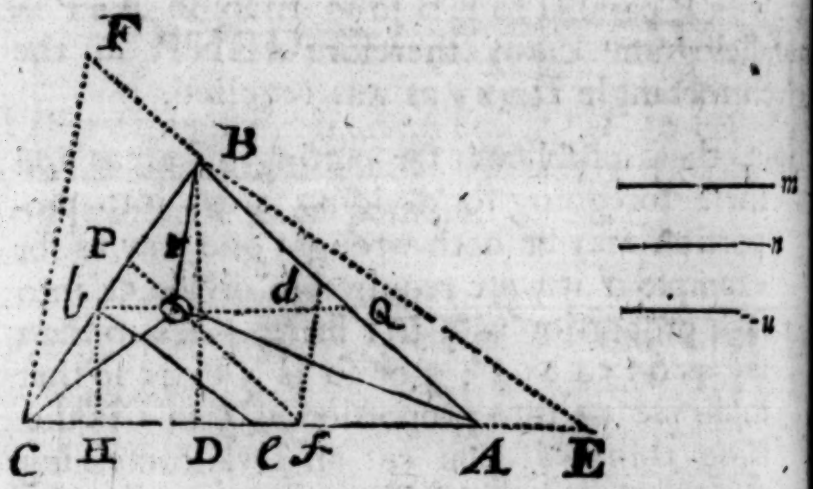
Note, theſe problems for cutting off areas and theſe foregoing for dividing areas into proportion may be both wrought one way as for example if it were required to divide 54 into ſuch proportion as 5 to 1 firſt 5 + 1 = 6 than ſay as 6 : 54 : : 1 : 9 or ſo is 5 to 45 ſo that to divide 54 into proportion as 5 to 1 is the ſame thing, if you cut an area containing 54 into 9 for the leſſer part or 45 for the greater part, be the figure what it will: alſo if you would cut 9 acres from ſome figure whoſe area is 54 acres and chuſe to do it by the problems of proportion then 54 − 9 = 45 ſo that proportion will be as 9 : 45 or lower as 1 to 5, thus have you two ways for theſe hard problems.

PROBLEM CXX.

To find a Point (O) in a given triangle ABC from whence if three Right-lines be drawn to the three Angles ABC ſhall divide the whole Triangle ABC into three other Triangles (AOB) BoC and CoA whoſe Areas ſhall be as m n and u reſpectively.

1. In CA and AB produc'd if need be make CE and AF each equal to $m + n + u$ joining EB and FC make Ce = m, Ad = n and draw eb and df parallel to EB and CF.

2. Thro'



2. Thro' *b* and *f* draw *b*Q parallel to AC and *f p* parallel to AB and at *o* the interſection of thoſe two lines is the point required for draw *b*H and BD perpendicular to AC then the triangles CBE, C*b e* as alſo CBD, C*b*H are ſimilar therefore it will be as *m* (C*e*) : *m* + *n* + *u* CE :: C*b* : CB : : *b*H : BD : : triangle A*o*C : triangle ABC *per* Theo. 5 for the triangles A*o*C and ABC have one baſe AC in the ſame manner it may be prov'd that as *n* : *m* + *n* + *u* : : triangle AOB : triangle ABC conſequently as *u* : *m* + *n* + *u* : : triangle BOC : triangle ABC : QED.

Note, The 8 following Problems ſolve all the Caſes in plane Trigonometry, Geometrically.

RIGHT-

RIGHT-ANGLED PLANE

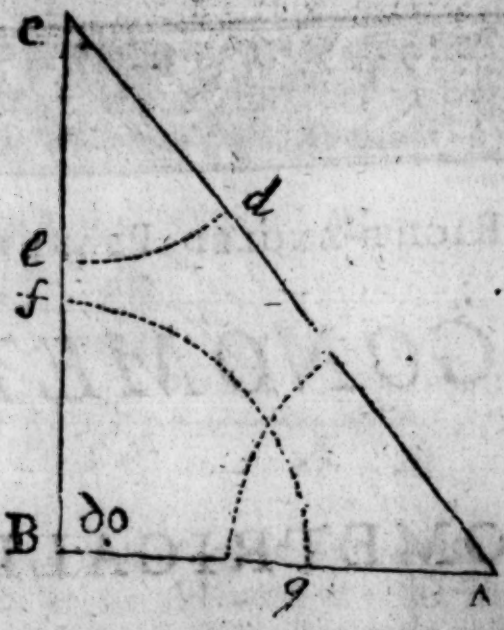
TRIGONOMETRY

GEOMETRICALLY.

PROBLEM CXXI.

Two Legs $\left\{\begin{matrix} AB = 90 \\ \text{and} \\ BC = 120 \end{matrix}\right\}$ given, to find the Hypothenuse AC and the Angle A and C.

DRAW a Right-line BC, which make = 120 by the DIAGONAL SCALE: Upon one end as B raise the perpendicular BA, or upon B make a right-angle; and from the same Diagonal Scale take 90 and lay it from B to A, join AC and the triangle is constructed. Take the chord of 60°



60° in your compaſſes and one foot in either A or C; ſuppoſe C, ſtrike an arch *de* which laid on the ſame is = 37° for the angle C and by the ſame diagonal ſcale A C is found = 150.

Definitions *continued from Page* 17.

Trigonometrical.

73. If one of the acute angles of a right-angled plane triangle be given the other acute angle is alſo known *per* Note Theo. 2, by taking the one from 90° there leaves the other ſo in the laſt Prob. the angle C *viz.* the arch *de* being found 37° this 37° being taken from 90° leaves 53° = the ∠ A.

74. Complement

74. Complement to 90° is what any arch or
ngle wants of 90° ſo the complement of 37°
90 degrees, is 53 deg. the like is to be un-
erſtood of any angles complement to 180 deg.

75. In all triangles the greateſt ſide ſubtends
r is oppoſite to the greateſt angle and contrary.

76. The meaſure of any plane angle is an
ch ſtruck from one of the ſides that makes the
ngle to the other ſide by the chord of 60 deg.
pon the angular point as in the laſt Prob. the
rch *g f* is the meaſure of the angle at B the ſame
tting the two containing ſides B A and B C
nd is alſo drawn with the chord of 60 deg. and
ne foot in B and becauſe the arch *f g* is = 90 deg.
a quarter of a circle the ſides B A and B C
e perpendicular to each other.

77. What ſcale of equal parts or diagonal
ale ſoever you begin to lay down the ſides or
ght-lines of any figure by that ſame ſcale you
uſt uſe to all the right-lines in that figure which
ou are concern'd with otherwiſe your work will
falſe, whether you lay down, or meaſure, or
th.

78. What line of chords ſoever you take ra-
us 60 deg. from that ſame ſcale of chords
ou muſt lay down or meaſure the arch by.

79. The ſides of all plane triangles are laid
wn or meaſured by ſome ſcale of equal-parts
diagonal ſcale.

80. The angles of every plane triangle are laid down or measured by some scale of chords Prob. 8.

81. If the sides of any angle be ever so long or ever so short the measure of the angle is still the same as in definition 8, the measure of the angle $b\,C\,b$ is the arch $b\,k$ now be the sides $C\,b$ and $C\,k$ longer or shorter, so long as they are streight, they still point from C towards b and k and therefore the measure of the angle $b\,C\,k$ is the arch $b\,k$ &c.

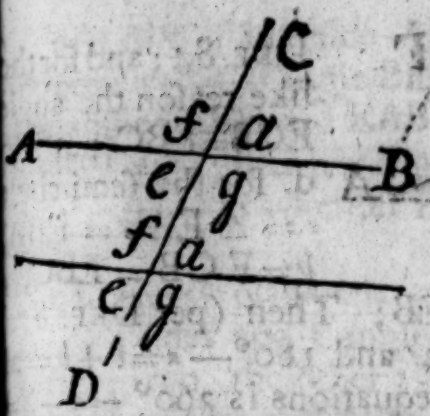
82. The reason why we always sweep an arch with the chord of 60 deg. (when we make or measure plane angles) is because 60 deg. in chords is the radius of that circle whose circumference or periphery is 360 deg.

83. A diagonal scale is but a scale of equal parts only you can take great numbers whereas from a scale of equal parts you can take but small numbers see Prob. 21. and 23.

84. If any two sides of a triangle be equal the angles opposite to those sides will be equal per definition 75, and contrary.

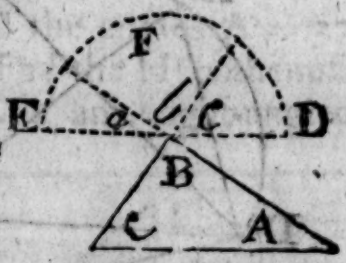
85. If two right-lines be drawn so as any how to cross each other they will make 4 angles the two opposite of which are equal (15 E. 1) as in the following figure AB and CD cut each other making $\angle\, a = \angle\, e$ and $\angle\, p = \angle\, g$ this is plain by the figure.

86.



86. If a right-line CD cut two parallel lines AB and AB, it will make = angles with both parallel line (29, E 1.) *viz.* angle a = $\angle a$, $f=f$, $g=g$, $e=e$, alſo, $\angle f=g$, $a=e$, &c. This is alſo ſo plain it needs no demonſtration.

87. The NOTES to THEO. 2*d*, being of ſo great uſe in plane Trigonometry, it may not be amiſs to prove them in this place: To Note 1 and 2; firſt thro' the angular point B (triangle ABC) and parallel to AC.



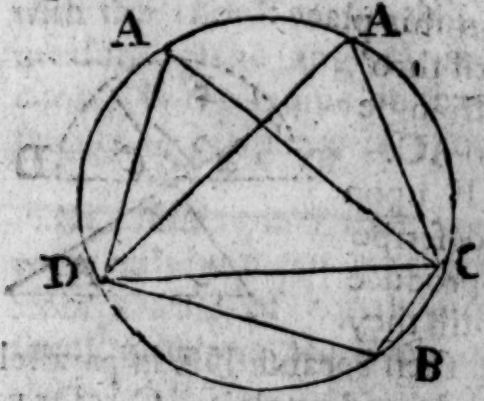
Draw a line D E upon which as a diameter deſcribe the ſemicircle DFE, produce the ſides AB and CB till they meet the ſemi-circle; then becauſe DE is parallel to AC the angle a = A, and angle c = C, (DEF. 86;) alſo, angle b = B, (DEF. 85.) Therefore angle $a+b+c=\angle A+B+C$: But angle $a+b+c$ = the ſemicircle or 180 degrees by the figure, *Q.E.D.* this alſo proves Note 2d.

88. To prove Note 3d Let ABC be an angle whoſe angular point B is in the ſemicircle, draw the radius EB, then in the triangle AEB the ſides AE and BE being equal the angle EAB = EBA,

DEF.



DEF. 84; and for the like reason the angle ECB=EBC, put 180 d. for the semi-circle $e = \angle EAB = EBA$, $b = ECB = EBC$, a =AEB and $a = \angle CEB$; Then (per DEF. 87.) $180° - \angle d = e + e = 2e$ and $180° - a = b + b = 2b$ the sum of these two equations is $360° - a - d = 2e + 2b$, but by figure $a + d = 180$ deg. which put in the last Equation gives 360 deg. $- 180 = 2e + 2b$; hence 180 deg. $= 2e + 2b$, that is $90 - e + b$, *Q.E.D.*



89. By this it is easy to conceive that any plane angle, as angle A is acute if it be in a segment ACD greater than a semicircle; and also any angle B is obtuse if it be in a segment less than a semicircle; and also that all angles in the same segment are equal: Likewise angle B + ∠A = 180 deg. and angle C + D = 180.

90. An angle at the center of a circle is double to one at the periphery, if they both stand on one chord; this is already proved, for angle $d = 2b$ and angle $a = 2e$; also, the angles A and F both stand upon the chord BC and are both in the circle's periphery, so angle F = A = ½BEC &c.

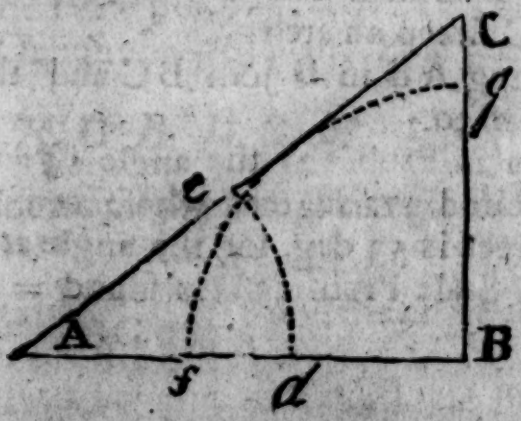
91. Al

91. Alſo if any ſide AE of a triangle AEB, be produced, the angle BEC will be equal to both the angles ABE and EAB, this is alſo proved for angle $a=e+e$; this per Figure, $180°-\angle a=d$, and $180°-\angle d=e+e$ whether EA and EB be equal or not.

92. Every plane triangle conſiſts of ſix parts; (*viz.*) 3 ſides and 3 angles, and any 3 of theſe 6 being given, the other 3 may be found: There muſt always be 3 of thoſe 6 parts given, otherwiſe the triangle will admit of various anſwers. See more after PROB. 135.

PROBLEM CXXII.

Given a Leg AB = 44 and the angles A = 37 Degrees G = 53 deg to find the Hypothenuſe A C and other Leg B C in any right-angled Plane Triangle.



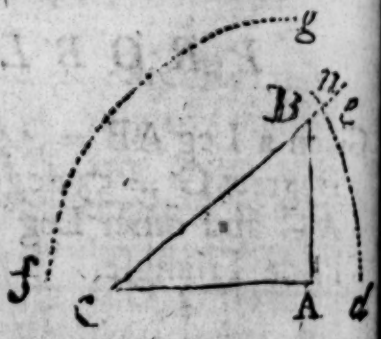
1. From a ſcale of equal parts (Prob. 121) Make AB = 44 and upon B with the chord of 60 deg. ſweep the arch *f g* lay 90 deg. from *f* to *g* and thro' *g* draw B C ſo is ∠ CBA a right-angle alſo

alſo with the chord of 60 deg. and one foot in A ſweep the arch *d e* and lay 37 deg. of chords from *d* to *e* thro' A and *e* draw the line A *e* meeting B C in C ſo is the triangle made or laid down.

2. B C laid on the ſcale of equal parts is found = 33 and likwiſe AC is found = 55 as was reqd.

PROBLEM CXXIII.

Given the Leg A C = 44 and the Hypothenuſe BC = 55 to find the Leg A B and angles B and C in a Right-angled plane triangle.



1. By a ſcale of equal parts (Prob. 23) make A C = 44 upon A by Prob. 10 make a Right-angle CAB from the ſcale Prob. 23 take 55 and with one foot in C ſweep an arch *r r* croſſing A B in B join B C and the triangle is protracted.

2. As in Prob. 121 the angle C *viz.* the arch *d e* is found 37 degrees whoſe complement to 90 degrees is 53 deg. for the angle at B alſo A B (by the Scale Prob. 23) is found = 33 as was &c.

PRO-

PROBLEM CXXIV.

In a Right-angled Plane Triangle (ABC last Figure) given the Hypothenuse BC = 55 and the Angle C 37 degrees consequently Angle B 53 (*per* Def. 73) to find the base AC and Perpendicular AB.

1. From Prob 23, make BA = 55 by Prob. 9, Make an angle at B of 53 deg. and an angle at C of 37 deg. draw CA and BA till they meet in A and its made; otherwise make an angle at either end of BC suppose at C of 37 deg. and from B by Prob. 3, let fall the perpendicular BA cutting CA in C and its constructed.

2. Take CA in the compasses and lay it upon the scale (Prob. 23) and you'll find it = 44 in like manner AB is found = 33 as *&c.*

☞ The IV. following PROBLEMS belong to Oblique-angled plane Triangles

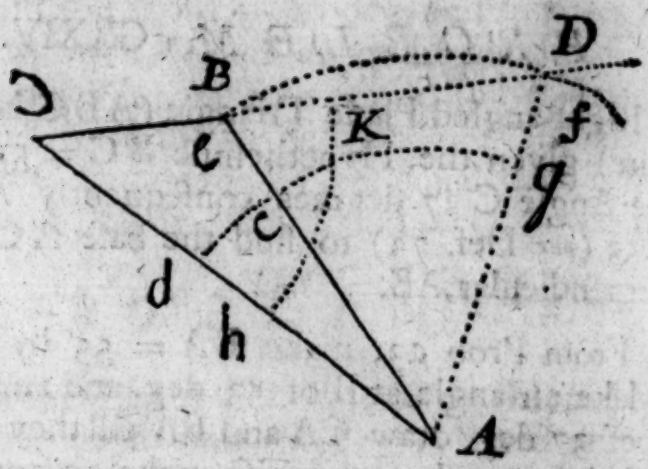
PROBLEM CXXV.

In the oblique-angled plane Triangle ABC there is given,

the two sides $\left\{\begin{matrix} AB\ 270 \\ AC\ 380 \end{matrix}\right\}$ and the angle at C 45°

reqd. the angles $\left\{\begin{matrix} A \\ B \end{matrix}\right\}$ and third side BC.

1. Draw a right line AC which make = 380, upon C make an angle BCA = 45d. drawing CB at large; take 270 in your compasses and with one



one foot in A ſweep an arch *e f*, which cuts CB in two places (viz) in B and D: And therefore this Problem is doubtful, viz. whether ABC or ACD be the triangle ſought that is if one of the unknown angles be obtuſe then ABC is the triangle, but if both the unknown angles be acute then ACD is the triangle ſought.

2. If one of the unknown angles be obtuſe with the chord of 60 d. and one foot in A ſweep an arch *d c* which by the ſame chords is found = 38 d. for the angle CAB then ∢ C 45 d. + ∢ BAC 38 d. = 83 d whoſe complement to 180 d. is 97d. for the ∢ ABC and by the diagonal ſcale BC is found = 235.

3. But if both the unknown angles be acute then the arch *d g* by the chords (Prob. 8) is found = 52 d. for the angle DAC and angle ACD 45 d. + ∢ DAC 52 d. = 97 d. whoſe complement to 180 d. is = 83 d. for the ∠ CDA and by the diagonal ſcale CD is found = 301 d.

Becauſe

Becauſe the numbers 270 and 300 are pretty
ge I make AB = half 270 and AC = half
o and ſo call CD and CB double to what I
d them &c.

PROBLEM CXXVI.

the oblique angled Plane Triangle ABC
(laſt Figure) is given the two Sides AB =
135 AC = 190 and their contained angle
CAB = 38d. to find the third Side BC and
the other two Angles B and C.

1. Draw a right-line AC which make equal to
o (Prob. 23) upon A make an angle BAC =
d. making AB = 135, Prob. 23 join BC and
e triangle is conſtructed.

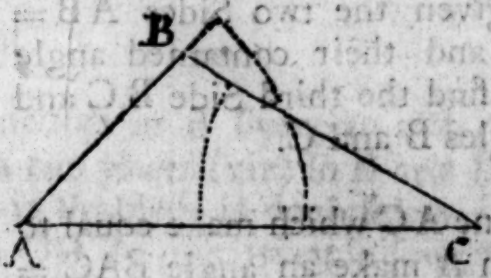
2. With the chord of 60d. and one foot in
her B or C ſuppoſe C, ſweep an arch *hk*
ich meaſured on the ſame chords is found =
d. for the angle at C, then 45 d. added to 38d
equal to 83d. the ſum of the angles A and C
oſe complement to 180d is 97d. for the an-
ABC (*Def* 87) and by the diagonal ſcale
C is found = 102 $\frac{3}{5}$ as was required.

PROBLEM CXXVII.

In the oblique-angled plane Triangle ABC is given, the Side AB 29,

and the angles { A 45° and B 105° } consequently C = 30

to find the Sides BC and CA.



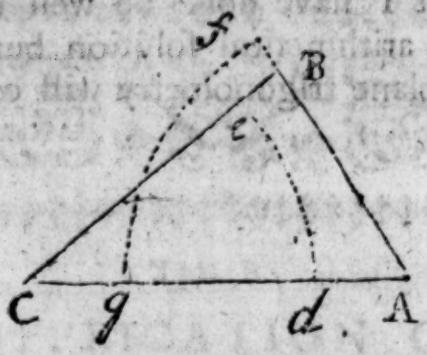
1 (By Prob 9) Make the ∠ CA = 45d. and from a ſcale of equa parts Prob. 22 make AB = 2 upon B (by Pro 11) make an angle ABC = 105 d. continuin AC till it meet BC in C ſo is ABC the triangl

2 The baſe BC by the ſcale of equal par (Prob. 22) will be found = 56 and the ſide B = 41 as was required

PROBLEM CXXVIII.

Given { AB 29, BC 41, AC 56 } the 3 Sides of an Obliqu angled plane Triangle, to find the angles.

How to make this triangle is ſhewn in Pro 14 thus make AC = 56 (Prob. 23) and wi 41 in your Compaſſes and one foot in A, ſwe



n arch, next with 29 in your compaſſes (all from rob. 23) and one foot in A ſweep another rch to croſs the former in B, join AB, BC and s conſtructed.

2. With the chord of 60° and one foot in C weep an arch *de* cutting CA and CB extended *d* and *e* likewiſe with the chord of 60° and ne foot in A ſweep the arch *fg* by the ſame nords (Prob. 8) you'll find the arch *de* = 30° or the ∠ BCA alſo *fg* is found = 45° for the ngle BAC then 45 d. + 30 d. = 75 d. whoſe omplement to 180 d is equal 105 d. = ∠ B hus with two true ſcales (*viz.* one of chords ne of equal parts) and a nimble hand you may ork any caſe in plane trigonometry exact e- ough for any ordinary uſe the arithmetical ſo- tion I here omit for theſe reaſons.

1. For want of room to demonſtrate the Axi- ms, explain Gunter's ſcale the lograthims &c.

2. There are more books ſhew the arithmeti- l than geometrical ſolution conſequently more erſons acquainted with the one than are with the ther.

3. If

3. If what I have done be well underſtoo
not only the arithmetical ſolution but even t
myſtery of plane trigonometry will come eaſ



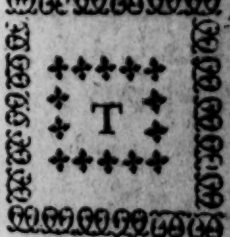
THE

THE APPLICATION OF TRIGONOMETRY.

To the MEASURING *of* *Heights* and *Distances.*

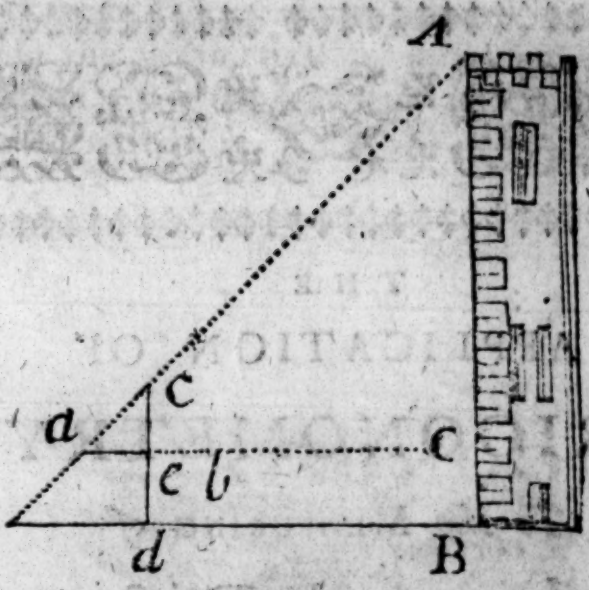
PROBLEM CXXIX.

To find the Height of any TOWER, TREE, STEEPLE, *&c.* as AB by two ſtreight Sticks laid croſs each other.



1. MAKE two ſtreight ſticks of any convenient length as *ab* and *cd*, and tye them exactly croſs each other in their middle as at *e*; that is, Make $eb = ed = ea = ec$, and let the 4 angles at *e* be all right ones, or *ab* and *cd* perpendiculars to each other, ſo have you a croſs *acbd*.

2. Set any end of the croſs as *a*, as near your eye as you can, (not to hinder ſight) and move it



it ſo as the ſtaff *c d* may be parallel to the Tower AB, with the croſs in this poſition, move backward or forward till you can juſt eſpy the top A of the Tower; over the end *c* of your croſs and at the ſame time obſerve the point C which the ſtaff *a b* points to in the Tower.

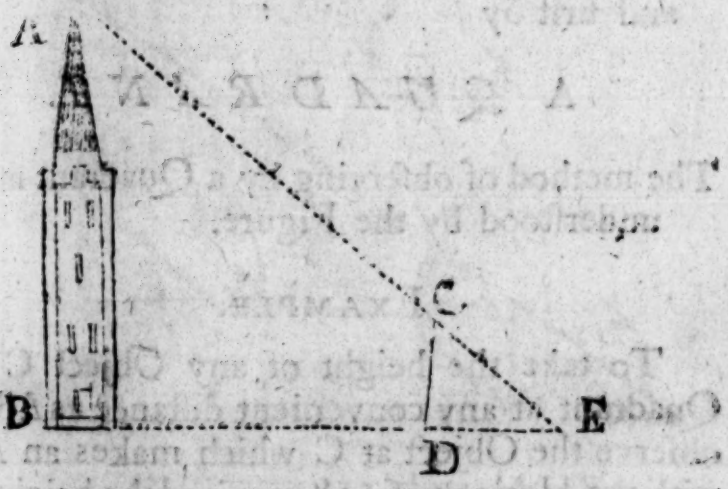
3. Meaſure in a right-line from your eye *a*, to the mark C in the Tower; which ſuppoſe 30 Y: and ſo much (viz) 30 yards is the height of the Tower above the point C, to which add CB and you will have the height of the whole Tower AB as was required.

The reaſon of this is plain, for if *d c* be parallel to AB the triangles *a c e* and *a* AC will be alike; therefore (per Theo: 3.) As *a e* : *e c* : : *a* C : CA; but *a e* = *e c*, therefore *a* C = CA: QED.

PRO-

PROBLEM CXXX.

To find the Height of any Object as AB by the ſhining of the SUN or MOON.



1. Set up a ſtreight ſtaff as CD of any length parallel to the object AB.

3. Suppoſe the ſhadow of the ſtaff to be 6 Feet =ED and the length of the ſtaff CD = 3 F; the ength of the ſhadow of the object 17 F = EB: Now CD being parallel AB the triangles DEC nd BEA are ſimilar; therefore ſay,

As ED 6 : DC 3 : : EB 17 : AB 8½ Feet the ltitude required.

3

6) 51 (8½
48

3

PRO-

PROBLEM CXXXI.

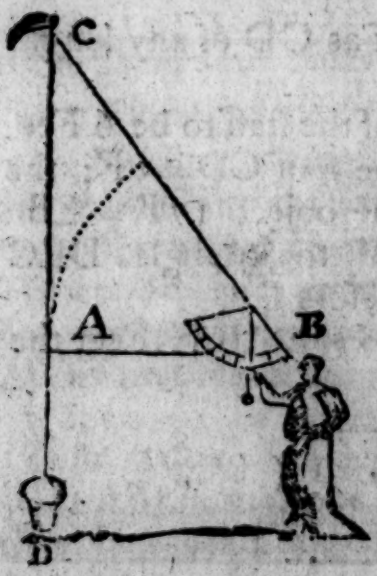
To find the Altitude of any Object as AB by the QUADRANT, or better by the SEMICIRCLE; and first by

A QUADRANT.

The method of obſerving by a Quadrant may be underſtood by the Figure.

EXAMPLE.

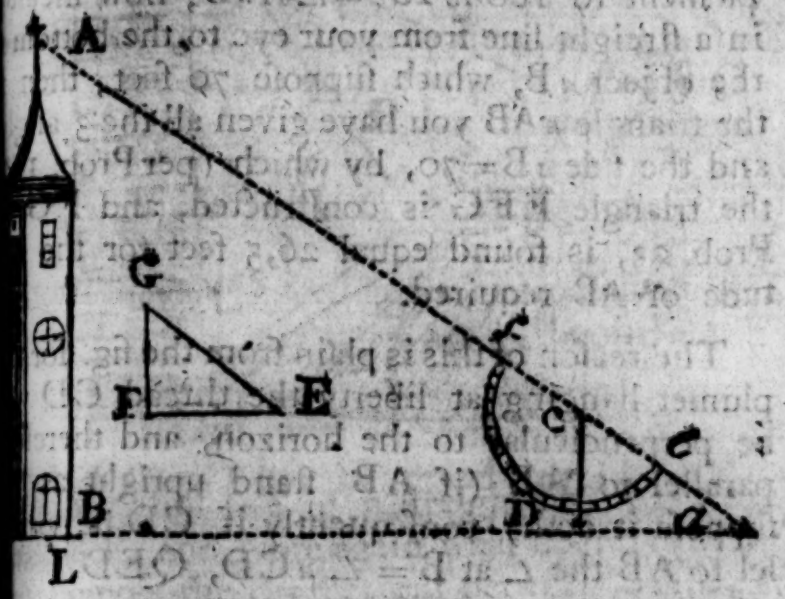
To take the height of any Object C by a Quadrant at any convenient diſtance as AB, 8 F. obſerve the Object at C which makes an Angle with the Horizon of 62° required the height.



1. Draw the line BA
2. Raiſe the perpendicular AC.
3. Make the angle ABD 62° which drawn cuts the line AC in C, the meaſure of AC 15 F. to which add the height of the eye 6 F. gives 21 F. the whole perpendicular height DC

II. A

II. *By a* SEMI-CIRCLE.



1. To the center C (of the ſemi-circle uſed for ſtances Def. 17,) faſten a thread CD with a umet D at the other end thereof; then ſet the rner *a* to your eye where the degrees begin, d thro' the ſights upon the diameter (*a* and *f*,) y any part (as A) of the tree whoſe height you uld know, and the limb of the ſemi-circle hang- g downwards with the thread and plumet at erty, the thread will cut 65°, in this example ich is the meaſure of the angle *a*AB: Next fore you ſhift your feet, obſerve in like manner bottom B of your object, and ſuppoſe then thread to cut 85°, (here you may conclude the ject ſtands on higher ground than you do, herwiſe the thread would cut at leaſt 90°.) Now s 85d. is the meaſure of the angle ABL, whoſe

complement 180d. is the angle ABa=95, there
fore angle aBA° + ∠ aAB65 = 160° whose com
plement to 180 is 20° = ∠ AaB; now measure
in a ſtreight line from your eye to the bottom o
the object aB, which ſuppoſe 70 feet; then in
the triangle aAB you have given all the 3 angles
and the ſide aB=70, by which (per Prob. 127
the triangle EFG is conſtructed, and FG by
Prob. 23, is found equal 26,5 feet for the alti
tude of AB required.

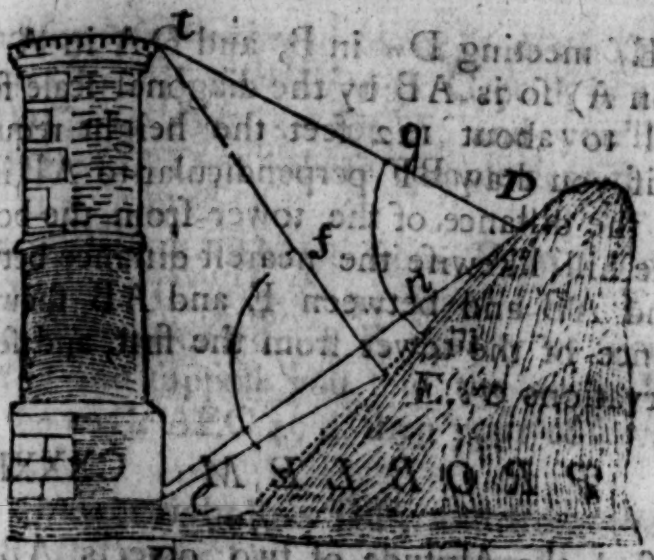
The reaſon of this is plain from the fig. for the
plumet hanging at liberty the thread CD will
be perpendicular to the horizon, and therefore
parallel to AB (if AB ſtand upright as we
ſuppoſe it doth) conſequently if CD be paral
lel to AB the ∠ at B = ∠ aCD, QED.

PROBLEM CXXXII.

To find the Altitude of any Object (as AB)
when by Reaſon of ſome Wood, water, &c.
you cannot come near it.

For varieties ſake, we ſhall take the height o
the Tower AB from the ſide CF, of a Hil
FEDG.

1. As in the laſt Prob. upon any convenient
place of the hill's ſide as at D, obſerve the top
A of the tower and mark what degrees the
thread cuts upon the limb of the ſemi-circle
alſo look thro' the ſights to the bottom B o
the Tower and obſerve what degrees the thread
cuts as before, likewiſe look thro' the ſights
dow



own the hill ſide in a ſtreight line for ſome nark as F, and Note again what degrees are cut ow ſuppoſe the degrees cut between looking at A and B, to be 52, 5° and between B and F o be 15°.

2. Move in a ſtreight line towards F, any iſtance ſuppoſe 58 Feet from D to E there nake obſervations to AB and F as before, and uppoſe the degrees cut between looking at A nd B to be 66 d. and between B and F to be 20 d. ſo you have done your obſervation, then o protract the ſame.

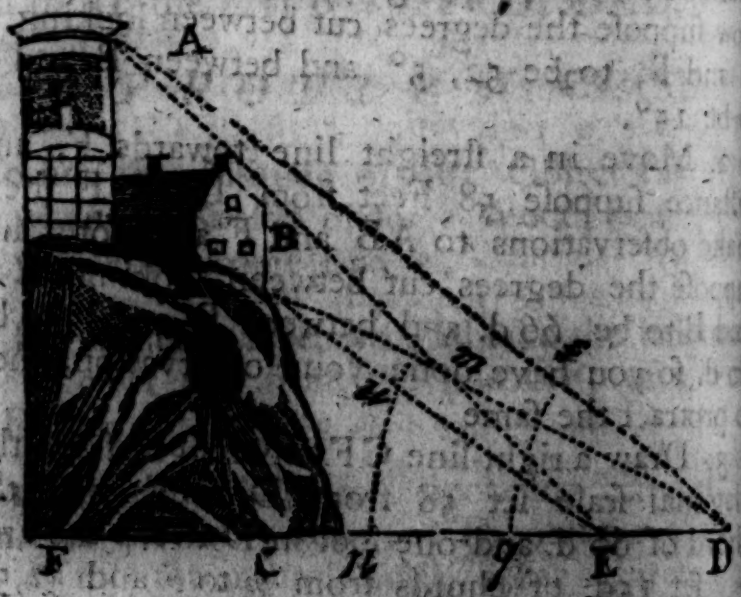
3. Draw a right-line CF any length from the iagonal ſcale ſet 58 from D to E with the hord of 60 d. and one foot in D ſweep the arch *m g* ſet 15d. of chords from *m* to *n* and 52,5d. f chords from *n* to *g* draw D*n* and D*g*.

4. With the chord of 60 d. and one foot in E ſweep the arch F *ſ* ſet 20 d. of chords from F o *e*, and 66 d. of chords from *e* to *ſ* draw E*e* and

and E*f* meeting D*n* in B, and DA in A, (viz. D*g* in A) ſo is AB by the diagonal ſcale found equal to about 122 feet the height required, alſo if you draw BF perpendicular to AB it will ſhew the diſtance of the tower from the bottom of the hill, likewiſe the neareſt diſtance between D and AB and between E and AB ſhews the diſtance of the tower from the firſt, and ſecond obſervations *&c.*

PROBLEM CXXXIII.

To find the Altitude of two objects AB and BC ſtanding the one upright upon the other and come near neither of them.



At any diſtance from the foot of the object as at D obſerve C the bottom of the object and ſuppoſe the thread to cut juſt 90 d. Prob. 131 ſo is the angle at C a right-angle alſo look to B

B and ſuppoſe the thread to cut 66 d. whoſe complement to 90 d. becauſe angle C 90 d. is 24° = ∠ BDC; ſo the like for A, and you'll find the angle CAD 52 whoſe complement to 90 d. 38° = ∠ ADC.

Meaſure any diſtance from D in a ſtreight line towards C ſuppoſe 44 from D to E then make obſervations upon E exactly as you did upon D and ſuppoſe you find the angle BEC 33° the angle AEC 49 d. then your obſervation is ended.

1. Draw a right-line DF, and from the diagonal ſcale make DE = 44, upon D make the angle ſ Dg 38 d. and the angle ε Dg 24 d. by Prob 9.

2. Upon E make the angle *m* E*n* 33 d. drawing the lines E*n* and E*ν* till they meet Df in A and De in B then becauſe the angle at C was a right-angle at both obſervations, from A and B let fall (upon DF) the perpendiculars AF and BC which by the diagonal ſcale are found AF 132 and BC 69 nearly for the altitudes required, alſo you'll find EB 121 then between your ſecond obſervation and the foot of the tower.

The reaſon of the two laſt Problems are evident from conſtruction.

PROBLEM CXXXIV.

To find the perpendicular Height of any hill or mountain as ABC.

1. As in the laſt Prob. at any diſtance from the bottom of the hill as at D obſerve the top thereof



thereof as A and ſuppoſe the thread cut 52° 30' whoſe complement to 90 d. if you ſtand upon level ground for then let fall a perpendicular from A upon a right-line drawn from your eye ſuppoſe the line DC will make a right-angle is 37° 30' = ∠ ADC.

2. Meaſure in a ſtreight line any diſtance from D towards the bottom of the hill ſuppoſe DE 530 yards then upon E make a ſecond obſervation as you did upon D and ſuppoſe you find the angle AEC 55 d. then to lay this down.

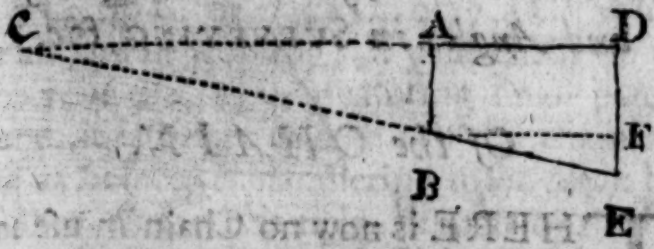
3. Draw a right-line DB making DE 530 from the diagonal ſcale, but becauſe 530 is a great number I divide it by 10 and ſo make DE equal the quotient, *viz.* 53 upon D by Prob. 9, make the angle *g* DE 37 ½ alſo upon E make the angle *n* E *u* 55 d. drawing E *u* till it meet D *e u* A, from A by Prob. 3 let fall the perpendicular AC which by the diagonal ſcale is found 83, but 83 × 10 becauſe 530 was ÷ 10 = 830 for the altitude required in yards.

This much I judge ſufficient for Heights.

PRO-

PROBLEM CXXXV.

To find the Diſtance of ſome Place a far off (as C) by three ſtreight Sticks.



1. Suppoſe you can come no nearer the mark C, than the point A; ſo from A go back any diſtance as to D, then cauſe ſome to ſet up a ſtreight ſtaff at A, or by A in ſuch a place as that you may ſee the mark C and A in a ſtreight line from D; alſo ſet up a ſtaff at D and meaſure the diſtance DA which ſuppoſe 55 yards:

2. From A move in a perpendicular line to AD any number of yards, ſuppoſe from A to B 26 Y: there ſet up a ſtaff at D; come to D and move in a perpendicular line from D, (*viz.*) move parallel to AB till over the ſtaff at B you can juſt eſpy the mark C; then meaſure the diſtance DE which ſuppoſe to be 40 yards, now it will be, as 14 the difference between DE and AB is to 55 the diſtance between D and A, ſo is AB 26 to 102, Y the diſtance AC required.

If you draw BF parallel to AD, then becauſe DE and AB are parallel the triangles DCE, ACB and FBE are ſimilar, therefore (per Theo. 1) as FE (=DE—AB) : FB (=DA) : : AB : AC, as before &c. alſo : : DE : DC.

DEFI-

DEFINITIONS

Of INSTRUMENTS *used for taking Distances and Angles in* SURVEYING, *&c.*

Of the CHAIN.

93 THERE is now no Chain in use among Surveyors but that of Mr. GUNTER's so I need discribe no other; This Chain is in length 66 feet, or 22 yards, or 4 poles, perches or rods, and is divided into 100 equal links (each link being 7,92 Inches long,) so that the links are decimal parts of a chain, hence chain and links may be wrought together as decimals. In measuring with the chain to prevent mistakes at first setting forward the person that goes with the fore end of the chain, should have 10 sticks each about 2 foot long sharp at one end, and at the end of every chain stick down one stick in the ground, and he that follows with the hind end of the chain to take it up; and thus do till all the sticks be gone if your line be so long, ever observing to move in a streight line from mark to mark; next let him that goes behind see that he have all the 10 sticks, and if the line be not measured out give them again to him that go before, and so proceed on till the line be out remembering to prick down how often you change sticks that no error be made.

At the end of every 10 links there is common-
a braſs ring, and ſome have marks at every 10
nks, which are for the more ready numbering of
e links: chains and links may be ſet down thus,
Let it be required to ſet down 7 chains and 40
nks) 7 C: 40 L. or 7,40 C. or 7,4 C. Which
ill be all one in working: If your chain be too
ng for your uſe, as in meaſuring ſmall parcels
ground, it is better to take half the chain and
eaſure as before, remembering to ſet down half
e number, as for Example, ſuppoſe by the half
ain you had put down 11 ſticks and have 25
d links; it muſt be ſet down 5 C: 75 L. or 5,75
and then in plotting or finding the area, it will
the ſame as if you had meaſured with the whole
ain

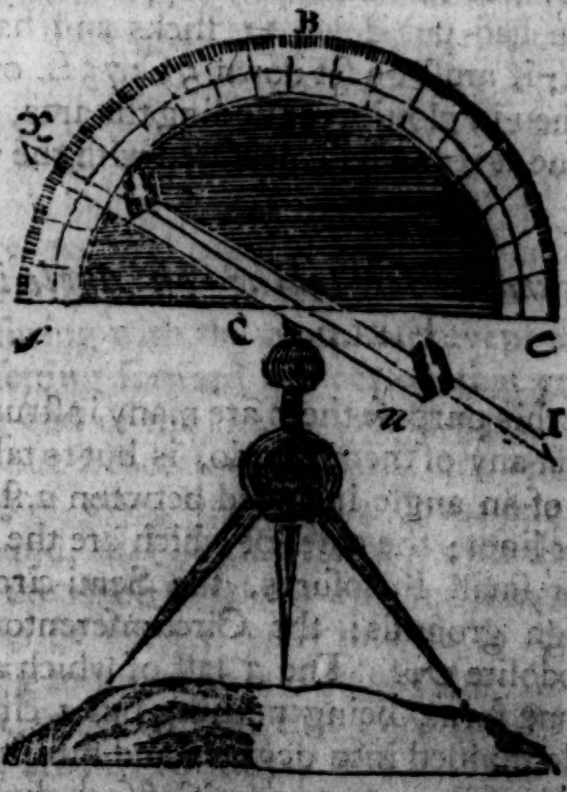
f Inſtruments for taking an Angle in the FIELD.

94 For this purpoſe there are many inſtruments
t all that any of them can do, is but to take the
antity of an angle included between 2 ſtreight
dges or lines; the chief of which are the Plain
ble for ſmall Incloſures, the Semi-circle for
ampaign grounds; the Circumferentor, and
Theodolite *&c.* The 3 laſt of which are all
ar of one form, being nothing but a circle or
mi-circle divided into degrees and minutes, (if
figure be ſo large as to admit of ſuch diviſion.)

95, The Plain Table, Semi-circle or Theodo-
, are what I ſhall here make uſe of.

96. The PLAIN TABLE, is no more but a board, or rather 3 pieces put together making a parallelogram ſufficient to hold a ſheet of paper, with ledges to keep the table together and hold the paper faſt; with a looſe index and ſights thereon, to lye upon the Table &c. Its uſe is more for taking the plot of a field, than for diſtances; ſo I ſhall reſerve it till we come to ſurveying.

97. The SEMI-CIRCLE or THEODOLITE, is



a half-circle or two quadrants divided into degrees &c. as is taught in Prob. 8.

Upon AD the diameter of the ſemi-circle are two ſights (e and f) but ſome ſemicircles wants thoſe ſights,

ſights, for in taking diſtances they may be ſuppli-ed by the ſights, (*a* and *u*) upon the index or pointer IX; this index moves round the center C to any degree on the limb ABD, and when the one end (X) falls off at 180°, the other end (I) comes on at 181° *&c.*

This inſtrument (as alſo the plain table) is made faſt upon the head of a ſtaff by a ſcrew, and the other end of the ſtaff ſtuck into the earth, ſo that the inſtrument lies flat with the degrees &c. facing the zenith.

98. As the THEODOLITE is an Inſtrument for taking an angle in the field ſo the Protractor is for laying the ſame down upon paper with more eaſe than by a line of chords the figure thereof is like that of the theodolite the uſe is lay the center C againſt the degrees that your angle is to contain, this is plain by inſpecting the figure.

99 Circumferentor, is a circle drawn upon a card fixed in a box, a needle touch'd by the load-ſtone moves flat upon its center this is uſed in ſurveying mines, *&c.*

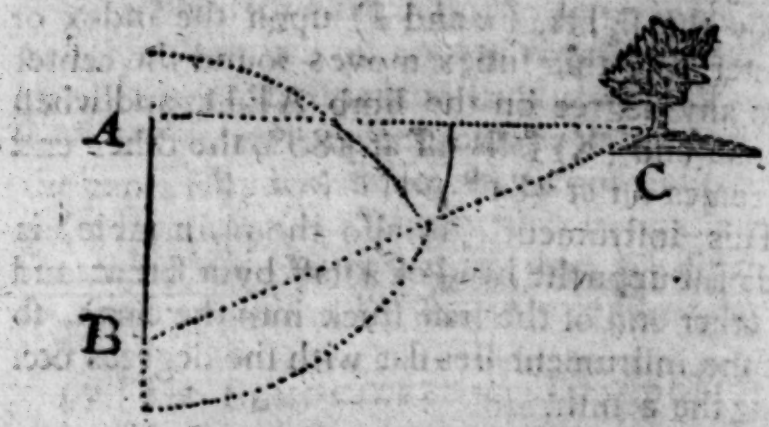
100. See Prob. 141, for water level.

PROBLEM CXXXVI.

To find the diſtance of an object C which you can come no nearer to than A by the Theodolite.

1. Set down your ſtaff at A with the theodolite flat thereupon then turn it till thro' the ſights along the diameter you can eſpy your object at C there ſcrew the inſtrument faſt and

move



move the index thereupon to 90 d. then thro' the ſights upon the ſaid index eſpy ſome mark as at B, ſo have you C A B a right-angle.

2. Meaſure the diſtance A B which ſuppoſe 76 links then upon B fix your inſtrument as you did upon A and thro' the ſights along the diameter, eſpy your firſt ſtation A and then holding the inſtrument faſt move the index till thro' the ſights thereupon you can eſpy the object at C and then ſuppoſe the index to cut 62 d 45' ſo is the angle A B C = 62 d 45'.

3. Then in the right-angled plane triangle, ABC you have given the leg A B = 76 and all the angles by which (and Prob. 122.) the triangle is conſtructed, and by the diagonal ſcale the required diſtance AC is found = 152 links or 152 chains alſo B C is your diſtance from C the ſecond ſtation.

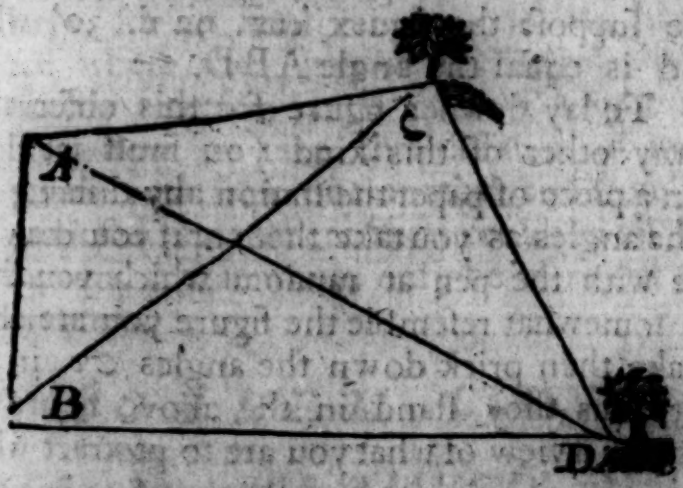
If what is here ſaid with a view of the inſtrument and the nature of an angle be well underſtood I think that the taking of an angle will appear ſo plain that I need give no more directions concerning it.

Note,

Note, If you move ſo far upon the line AB till the angle at B be 45 d. ſo the mov'd diſtance AB will be equal to your required diſtance AC and ſo in this caſe you need not have trouble to lay down the triangle.

PROBLEM CXXXVII.

To find the diſtance of ſeveral Objects, ſuppoſe two as C and D from a propoſed Place A, by any Inſtrument that will take an angle



1. You may work this Prob. like the laſt only here becauſe you have two objects you muſt have two angles at each ſtation therefore place the inſtrument at A and laying the index upon the diameter where the degrees take their beginning thro' the ſights on the index eſpy the object C the inſtrument ſo reſting move the index till thro' the ſights you can eſpy the object D and there ſuppoſe the index cuts 30 d. ſo 30 d. is the meaſure of the angle CAD again move the index till thro' the ſights you eſpy the ſecond ſtation

ſtation at B, and then ſuppoſe the index cuts 96 d. which 96° = the ∠ CAB, meaſure the diſtance AB which ſuppoſe 92 links or yards &c.

2. Placing your inſtrument at B, by the index upon the diameter where the degrees begin and move the body of the inſtrument till thro' the ſights upon the index you can eſpy your firſt ſtation at A there ſcrew the inſtrument faſt and move the index till thro' the ſights you eſpy your firſt object C and ſuppoſe the index cuts 50 d. ſo the angle ABC is = 50 d. laſtly move the index till thro' the ſights you can eſpy D and there ſuppoſe the index cuts 92 d. 30' which $92\frac{1}{2}$ d is equal the angle ABD.

3. To lay down a figure for this obſervation or any other of this kind you muſt ſet down upon a piece of paper the ſtation any diſtance and all the angles as you take them or if you draw a figure with the pen at random which you think will ſomewhat reſemble the figure you are about to take then prick down the angles &c. in that figure (as they ſtand in the above figure) ſo have you a view of what you are to protract which do thus.

Draw the line AB making it = 92, by (Prob. 23) then by Prob. 9 and 11 upon A make the angle CAD = 30 d. and angle CAB = 96 d. likewiſe upon B make the angle CBA = 50 d angle DBA = 92,5 d. and the meeting of theſe lines at C and D, point out the required objects which by the ſcale Prob. 23, is found AC = 11 C. the diſtance of C from A, AD = 204 C the diſtance of D from A join CD and you'll find it = 121 C. the diſtance of the objects

from

from each other, likewiſe BC and BD ſhews the diſtance of the objects from B.

Note, in the laſt Prob. I have uſed ſights upon the diameter, where you muſt always mind to turn that end of the diameter where the degrees begin toward the object which you are to eſpy but in this Prob. I have made the ſights upon the index ſerve for all.

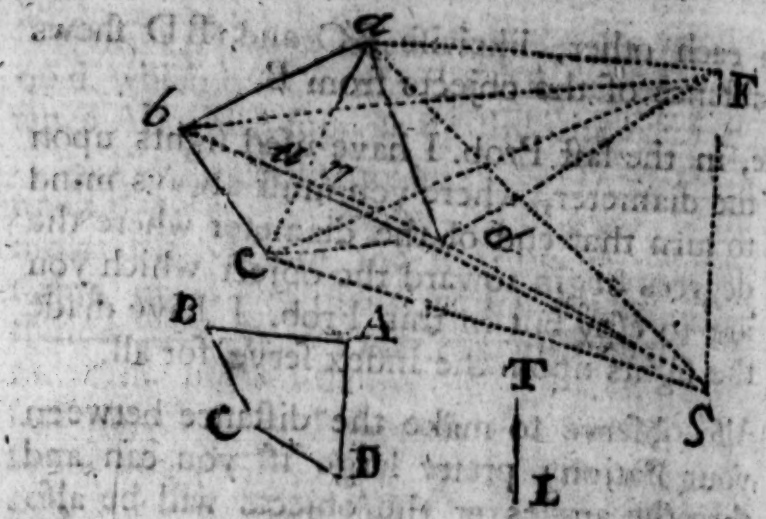
☞ Alſo obſerve to make the diſtance between your ſtations pretty large if you can and then the angles at the objects will be alſo large, for in ſmall angles one can hardly tell where the lines at the angular point exactly interſect each other I might alſo ſhew how to take the diſtance of many places both from one another and from any propoſed place where they could be all ſeen but doing that would only be to repeat this problem.

PROBLEM CXXXVIII.

To meaſure any level field (ABCD) which can be ſeen all at once, and not come near it.

This is no more but to take the diſtance from two convenient ſtations (F and L) and every corner (A B C and D) of the field *per* laſt prob. therefore placing your inſtrument at T obſerve L and every corner of the field alſo from L obſerve T and every corner ABCD of the field exactly *per* laſt Prob. and ſuppoſe the angles and ſtationary diſtance to be as is here ſet down.

The



	d. m.		d. m.
The ∠s at the 1 ſtation on T, are	ATB 12:30	The ∠s at the 2 ſtation on L, are	DLC 46:20
	ATC 22:30		DLB 65:00
	ATD 37:00		DLA 68:03
	ATL 96:15		DLT 70:00

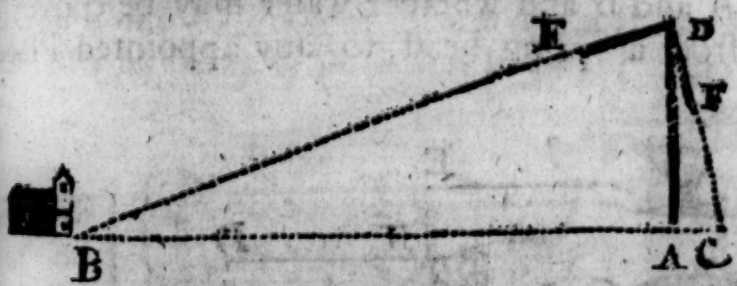
The ſtationary diſtance is, LT = 83 chains.

Now to lay down this upon a large ſheet of paper draw a line FS making it = 83 chains but in this example for want of room FS is only 83 links) and upon F with the chord of 60d. ſweep an arch and lay off the ſeveral angles *a*F*b* = 12d. 30′ *a*F*c* = 22d. 30′ *a*F*d* = 37d and *a*F*s* = 96d. alſo upon *s* make the ſevetal angles F*sa* = 46d. 20′ F*sd* = 65d. F*sb* = 68d. and F*sc* = 70d. the angles taken at the ſecond ſtation, and you'll find thoſe lines or ſides of the angles to meet in the points *abc* and *d* which points join with right-lines, ſo have you the field protracted *viz.* *abcd* according to its true form now by the diagonal ſcale that F*s* was taken

taken from, you may measure the distance of every corner of the field *abc* and *d* from both stations F and S as also the distance between each corner, Also if you draw the diagonal *bu* you'll find it = 74.5 chains and the perpendicular *au* = 42 chain *cu* = 8 chain now 42 + 8 = 50 half whereof is = 25 chain, and *bd* 74.5 C. multiply by 25 C. is equal to 1862.5 square C. or 186,25 acres the area of the field ABCD as was required.

PROBLEM CXXXIX.

To take a distance AB as the breadth of a River by a Carpenter's or Mason's Square



1. Take a streight staff of any suitable length as DA and stick it down at the river's side perpendicular to the horizon and upon the staff's head hang the square as you see per figure.

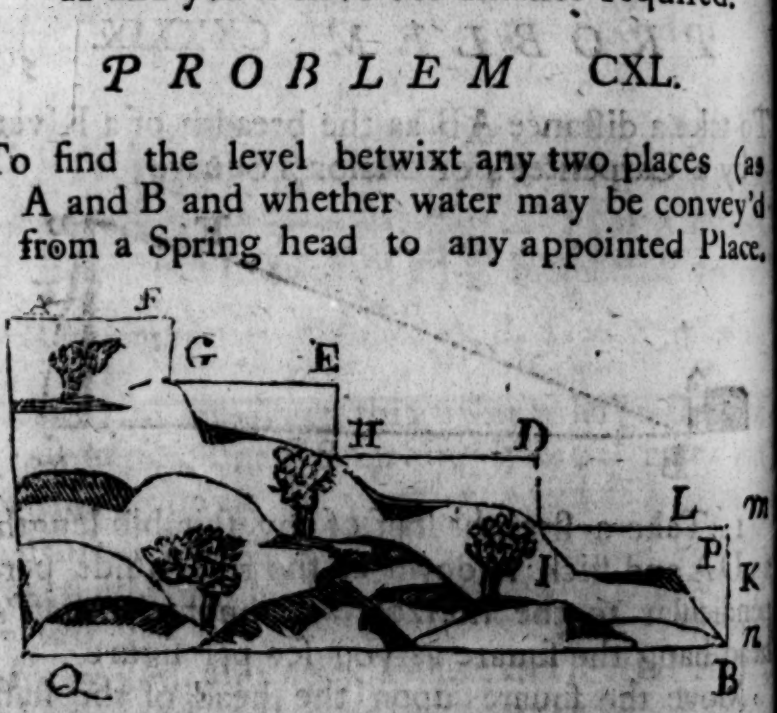
Move the square upon the head of the staff till along the side DE you can see B the other side of the river there hold the square fast and see where the upper side DF points the ground which is at C measure CA which suppose = 2 feet and AD the length of the staff and thickness of the square which suppose = 8 feet then say by the rule of three as CA 2 : AD 8 so is

1 : AD8 : AB 32 feet the breadth of the river required, this is grounded on Prob 36.

Note, If you ſlit the head D of an upright ſtanding ſtaff D A and in that ſlit put a ſmall ſtick D E bending it up or down in the ſlit till along the ſide D E you can juſt ſee the far ſide B of the river: then turn the ſtaff D A about and mind where the ſtick D E points, on the land which point meaſure from A and you'll have the diſtance required.

PROBLEM CXL.

To find the level betwixt any two places (as A and B and whether water may be convey'd from a Spring head to any appointed Place.



1. For this purpoſe you may make an inſtrument by tying a ſquare K L *m* upon Q, the head of the ſtaff L *n* and at the angle L of the ſquare faſten a thread L P and plumet P, now it is evident when this ſtaff L *n* ſtands upright the thread LP will (hanging at liberty juſt touch the ſide L K of the ſquare L K M.

2. At

2. At B ſet down this inſtrument making it ſtand upright which you may do by the line and plumet as before directed and placing your eye at *m* look along the ſide *m*L of the ſquare (or which is better have two ſights to look through) and mind well what part of the hill you ſee which in this figure is the point I, then at I cauſe ſome aſſiſtant to ſet a mark meaſure the height L *n* (of your ſtaff above the ground which ſuppoſe five feet ſo is five feet the height of the part I B and a ſtreight line meaſured from L to I the level thereof as appears *per* figure.

3. Fix your inſtrument at I and do exactly as you did at B and you'll obſerve the point H *&c.* till you come to obſerve A and ſuppoſe the dimenſions to be as is ſet down below.

	Feet.		*Feet.*
The ſeveral heights are	L B = 5	*The ſeveral levels are*	L I = 66.3
	I D = 5.1		DH = 30.5
	E H = 5.3		E G = 20.26
	F G = 6.2		A F = 35.8

The ſum is 21.6 = AQ and 152.86 = BQ as appears *per* figure.

Hence it appears that if you digg a pit at A 21.6 feet deep, there will be juſt level for the water to ſtand and that level will be 152,86 feet long: But here, Note, the common allowance for the fall of water, is 4½ inches in a mile or 1760 yards, that is if a level be 1760 yards along one end of that level muſt be 4½ inches

inches lower than the other end, that ſo the water may have a current, and ſo in proportion for any other diſtance.

PROBLEM CXLI.

It is required to know, whether water can be carried from A to B?

This may be done as in the laſt Prob. but for varieties ſake take the water level which is only a ſmall pipe of glaſs made cloſe at each end with wax, *&c.* and fill'd with water, excepting a very ſmall vacuity which will ſhew itſelf in the middle when it lies level but upon the leaſt leaning either way, you will find it move accordingly now this laid in a piece of wood having ſights at each end is a very fit inſtrument for this purpoſe.



1 Take two ſtreight ſtaves of any proper length and divide each of them into inches and decimal parts of an inch.

2. Set your two ſtaves upright at B one, and the other any where in a ſtreight line with AB, as at *n*, ſo that from your inſtrument being placed at L a point any where between B and *n*, you may obſerve both ſtaves;

3. Having

3. Having your inſtrument placed at L in a level order, cauſe ſome aſſiſtant to move a piece of white paper up and down the ſtaff at B, till from F thro' the ſights you can juſt eſpy the upper edge of the ſaid paper, which you'll obſerve to be at D there ſee what inches are cut; ſuppoſe 72,2 from B to D, then look from E for the moved paper at the ſtaff *n* (as before) and you'll obſerve the point G, ſuppoſe *n* G = 20,82 inches; next, letting the ſtaff ſtand at *n* move your inſtrument in a right-line with *n* A towards A any diſtance (not loſing ſight of the ſtaff *n*) ſuppoſe to O, there do as you did at L, and ſuppoſe *n* Q = 84,56 inches and looking towards A you juſt ſee the point A, ſo the obſervations are over; in this Example

	Inches.		Inches.
Now	BD = 72,2	and	*n* G = 20,82
	n Q = 84,56		A = 0,0
Sum =	156,76	and	20,8
Subtract	20,8		
	135,96		

That is, from A to B or B is 135,96 Inches below A.

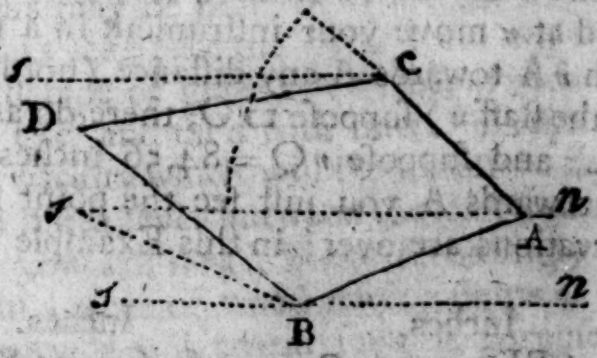
This will appear plain after a little conſideration.

SUR-

SURVEYING of MINES

PROBLEM CXLII.

To survey Mines and set out new Pits.



OVer the top of the pit lay a piece of wood from the middle of which let down a line and plumet to the bottom of the pit, by this you may know how to place your instrument so that the center thereof (Def. 99) may be in the same perpendicular line at both top and bottom of the pit which you are carefully to observe.

2. your instrument thus placed at the bottom of the pit, hold the end of a small line or thread over its center, and let some assistant go with the other end of the line as far as he goes streight *viz.* to the first corner or pillar there make a mark.

3. See what degrees are cut by the line, suppose 16°, measure the length of the line between the center of your instrument and the said mark, which

which ſuppoſe 65,2 feet, ſet theſe down and removing your inſtrument to the ſaid mark, work as you did before, and ſo go on as far as you have occaſion, and ſuppoſe the diſtance and degrees to cut as here ſet down,

	F.		D.	
Diſtances	65,2 51,72 84,1	Degrees	16 137½ 188	cut.

This done thoſe that know no better, do but place the inſtrument at the top of the pit with its center at the head of the foreſaid perpendicular line, and ſo meaſure out your diſtances by the degrees exactly as they were taken in *&c.* But this way may ſometimes create much trouble, in Leveling when the ground is uneven, and perhaps may have occaſion to go thro' wood, water, *&c.* which may cauſe great if not impracticable labour.

To remedy all which take the following method: But firſt Note, Wherever you place this inſtrument for obſervation, place it upon a ſtool, your hat, &c. ſo that it ſtands upon a level, and move it till the end of the Needle point the Flower de-luce, or the north point where the degrees begin and are numbered from north to eaſt 90°, from eaſt to ſouth 180° from ſouth to weſt 270°, and from weſt to north 360°: To protract this upon a ſheet of paper; draw a right-line NS repreſenting the north and ſouth line, upon any point whereof as at B, make an angle ABN of 16d. making BA=625 (from Prob. 23); now the next degrees cut were 137,5 which take from

180d.

180d. leaves 42,5d; that is, 42,5d. to the eaſt ſide of the ſouth, ſo thro' A and parallel to NS draw another north and ſouth line *ns*, upon A make the angle CAs=42,5d making AC=51,72; again the next number of degrees ſet down are 188d; that is 8d. from the ſouth weſtward, ſo thro' the C draw another north and ſouth line Cs parallel to *ns*, upon C make the angle SCD = 8d. and CD = 84,1; join BD and the figure of your obſervation is protracted. Meaſure the angle DBS and you'll find it = 37d. which by the figure lies on the eaſt ſide of the ſouth, therefore 180d. — 37d. = 143d. alſo meaſure BD Prob. 23 and you'll find it = 74.5: now placing your inſtrument at the top of the pit (centeral to the perpendicular line) and as directed in the above note, cauſe your aſſiſtant to go with the line 74,5 feet from the center of your inſtrument, the ſaid line cutting 143d. and there you have a place to begin your new pit which will fall perpendicular upon the end of your laſt taken diſtance in the mine.

PROBLEM CXLIII.

To work the laſt Problem, by the Semi-circle.

1. Lay a ſtreight piece of wood over the top of the pit to which faſten two lines and plumets at any ſmall diſtance from each other let this wood lie faſt and the plumets go juſt to the pit's bottom.

2 To theſe two plumets lay the diameter of your inſtument, hold a line to its center and let your companion go with the other end of the line

to

to the first corner, there let him make a mark and a small scratch along the side of the line, suppose the line cuts 16d. and its length between the mark and center of your instrument to be 65,2 yards set there down.

3. Lay the center of your instrument to the mark and its diameter along the scratch and proceed with the line as before and thus do till you have gone thro' the mine and you'll find the dimensions to be as is here set down.

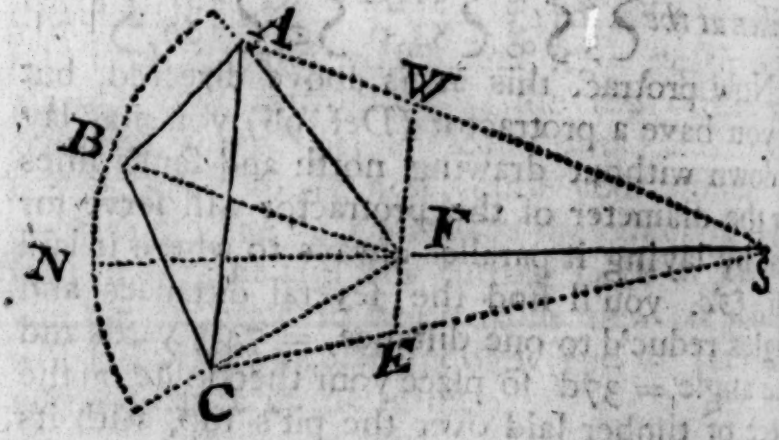
		Yards.	Deg.	
Distances taken at the	1	65,2	16	*cut.*
	2	51,72	137	
	3	84,1	188	
	Station		*Degrees*	

Now protract this as is above directed, but if you have a protractor (Def. 98) you may lay it down without drawing north and south lines for the diameter of the protractor will serve for that by laying it parallel always to where it laid last, *&c.* you'll find the several distances and angles reduc'd to one distance, = 74,5 yards and one angle = 37d so place your theodolite on the piece of timber laid over the pit's top, with its diameter to the two aforesaid lines, and measure out 74,5 yards over 37d. *&c.* as *per* last Prob.

NOTE, *The forementioned scratch is to be made parallel to the diameter of your instrument which you may easily do thus, remove the end of your line from the center of the theodolite to the end of the diameter which suppose 4 inches and let your assistant move his end 4 inches the same way and there make another mark, now a scratch made between these two marks will be parallel to the diameter of your instrument* &c. *as may easily be understood.*

PROBLEM CXLIV.

Coasting along the shore I see three Ships (viz.) A, B *and* C *at Anchor*; A *bears from me North* 57½° *Westerly*, B *North* 23° *Westerly and* C *East from the North* 47°. *Walking* 100 *Perches towards the South*, A *bears from me North* 24 *Westerly*, B *North* 10 *Westerly and* C *North* 9 *Easterly, how far are those Ships distant from each other.*



1. Draw N S for the north and south line at right-angles, to which draw E W for the east and west line, and because I mov'd 100 towards the south; produce NS till FS be equal 100 so doth F represent the place of my first observation and S the second: Now because A and B laid to the west side of the north and C to the east thereof, upon F make an angle NFA=57½ d. and an angle NFB=23d. (both from N towards W) also an angle NFC=27d (on the east side of N;) likewise upon S make the angle NSA=24d, the

angle

angle NSB=10d. and the angle NSC=9d. drawing SA, SB, SC till they meet FA, FB and FC in the points A, B and C ſo doth A, B, C repreſent the three ſhips, which join with right-lines and its done.

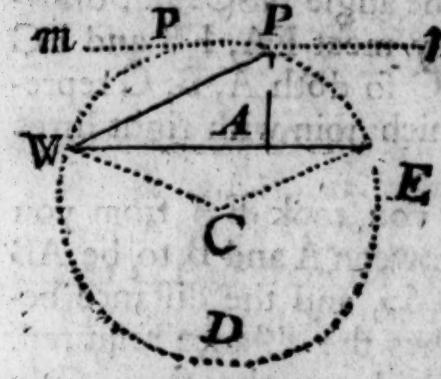
2. By the ſcale that you took FS from you will find the diſtance between A and B to be AB =42, the diſtance BC=60, and the diſtance between A and C=86 perches the diſtance required. Alſo FA, FB, FC, laid on the ſame ſcale will ſhew you the diſtance from each ſhip at the firſt obſervation, and SA, SB, SC your diſtance from each ſhip at the ſecond obſervation; likewiſe if you draw lines thro' A, B and C parallel to NS you may eaſily find the bearing of thoſe ſhips from one another *&c.*

This Prob. is of the ſame kind with Prob. 138 only obſerv'd by different inſtruments, as the circumferentor or ſeamens compaſs, the theodolite will alſo do the very ſame if you have the needle inſtead of an index.

PROBLEM CXLV.

Three Ships AB *and* C, *ſail from one Port,* A *ſails due South* 24 *miles* B *ſails South Weſterly, and* C *South Eaſterly, each a diſtance unknown, but* B's *diſtance is greateſt; and then all the three Ships are in the ſame Eaſt and Weſt Line, the Sum of the Courſes of* B *and* C *are* 98d. Query *their Courſe and Diſtance ſailed* B *and* C *being* 78 m. *aſunder.*

1. Draw



1. Draw EW for the east and west line making it equal 78.

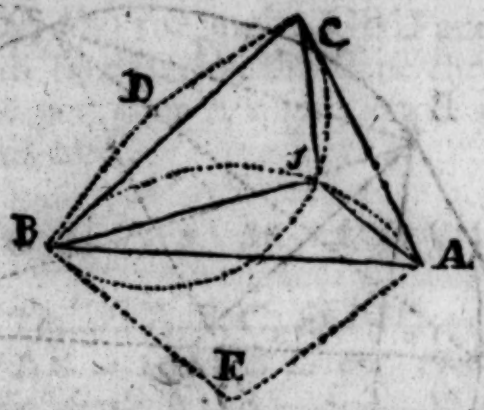
2. Make the angles CWE and CEW each 8d. and with the radius CE or CW, upon C sweep the circle at the distance of 24 from EW and parallel thereto draw *m n* cutting the circle in P; join PE and PW, draw PA perpendicular to EW, so doth P represent the port departed from, PW B's distance sail'd, the angle APW her course, PE C's distant sail'd, and the angle APE her course, PA = 24 A's distance sail'd which by the scale PW is found 69,3, PE=27,2 miles, and the angles by the chords are found south 69½ d. west equal angle WPA and south 28½ d. east equal angle APE C's course as was required.

If the angle WPE (the sum of the courses B and C) be = 98d. then *per* Def. 90 the arch WDE must be equal twice 98d. = 196d and if WDE = 196d. then the arch WPE *viz.* the ∠ WCE must be = (360d. — 196d.) 164d. and because CW = CE the angles CWE and CEW (*per* Def. 87 and 84) are each = 8d.

PROBLEM CXLVI.

A Ship at S observes 3 Capes AB *and* C *whose distances are known to be* AB = 99 AC = 7[illegible] BC = 86 *and the angles of Observation are* ASB = 127d. ASC = 128d. *so the angle* BSC *must be* = 105d. *how far is she distant from each Cape.*

1 (B

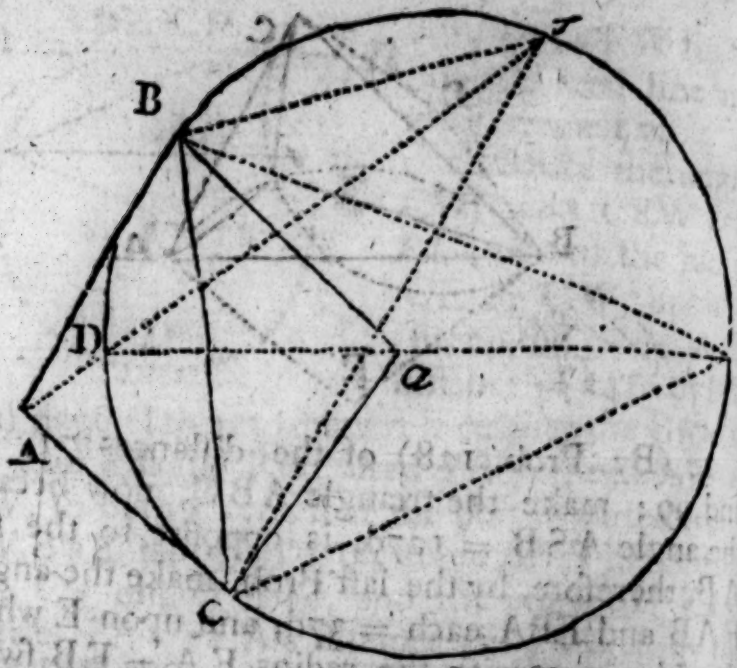


1. (By Prob. 128) of the diſtances 71, 86, and 99; make the triangle ABC, now becauſe the angle ASB = 127d. is oppoſite to the ſide AB, therefore, by the laſt Prob. make the angles EAB and EBA each = 37d, and upon E where the ſides meet with the radius EA = EB ſweep the arch ASB likewiſe angle CSB being = 105d. by what is ſaid in the laſt Prob the angles DBC and DCB muſt each be = 15d, upon D with the radius DB = DC ſtrike the arch CSB cutting the laſt arch in S ſo doth S repreſent the place of the ſhip at the time of obſervation whoſe diſtance from the 3 Ports (by the ſcale Prob. 23) is found SA = 40,5 PB = 67 and SC = 42 as &c. For proof ſee the laſt Problem.

PROBLEM CXLVII.

A Ship at S. obſerves 3 Ports AB and C *whoſe diſtances are known to be* AB = 80 AE = 72 *and* BC = 120 *the angles under which ſhe obſerves each Port are* CSA = 25d. AB = 19d. *conſequently* CSB = 44d. *Query her diſtance from each Port.*

1. (By



1. (By Prob. 128) of the 3 ſides 120, 80 and 72 make the triangle ABC upon B make the angle DBC = 25d. alſo upon C make the angle DCB = 19d. drawing CD and BD 'till they meet each other D.

2. About the triangle BDC (by Prob. 57,) deſcribe the circle, lay a ruler to D and A, and draw the right-line ADS cutting the circle in S, ſo is S the place of the ſhip at the time of obſervation join SB and SC and the figure is made.

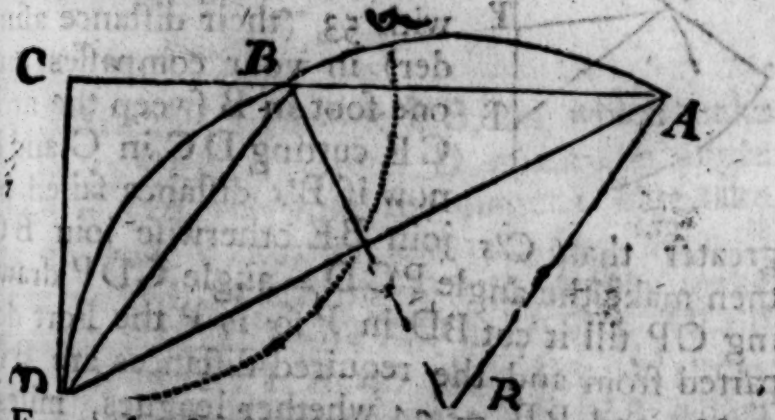
3. Thoſe lines meaſured on the ſame ſcale of equal parts that you laid down the triangle ABC from, will be found *s*A = 164 SC = 158, and SB = 95, the diſtances ſought if the port A be fareſt from the ſhip, but if *a* be neareſt to the ſhip then S*a* is found = 96, SC = 149, and SB = 167, this conſtruction is plain from Def. 89 for angles DSB and BCD ſtand both on the ſame

chord

chord DB &c. alſo the angles CSD and CBD are equal both ſtanding on the ſame chord CD ſo that be the port *a* or A where it will the conſtruction holds true.

PROBLEM CXLVIII.

There are 4 *Towers* A, B, C, *and* D, *whoſe diſtances are* AB = 100 BC = 50, *and* CD = 83; *and the Angle* ADB = 29d. 58′ *required the diſtances* DA *and* DB, A, B *and* C *being all in a Right-line.*



From the ſcale (Prob. 23) make AC = 150 AB = 100 BC = 50 upon AB make an Iſoceles triangle ABR whoſe angle R may be = 2 angle BDA thus, make the angles RAB and RBA each = 60d. 2m. upon R with the radius RB or RA deſcribe the black circle with 83 in your compaſſes and one foot in C ſweep the arch DE cutting the circle in D, ſo will the diſtances DB be found 99 and DA = 172 as was &c. For the angles BRA being in the center, and BDA in the periphery of the ſame circle the angle BDA = 2 ∠ BRA *per* Def. 90, *Note, the diſtances would be* QB *and* QA *if the given angle were obtuſe.*

PRO-

PROBLEM CXLIX.

Two Ships B *and* C *ſail from one Port* P *ti*
their diſtance aſunder B C *be* 53 *and the ſu*
of their diſtances ſail'd P C *and* P B 63, *th*
ſum of their courſes or the angle B P C *i*
110°, 30', *What is their diſtances* P C *and* P
ſeverally?



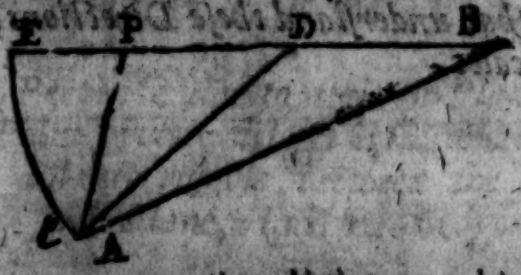
1. Make an angle R D C = 55° 15' making B D = 6 the ſum of their diſtance with 53 (their diſtance aſun der) in your compaſſes an one foot in B ſweep the arc C E cutting D C in C and now if B's diſtance ſailed b greater than C's join B E otherwiſe join B C then make the angle P C D = angle C D P draw ing C P till it cut B D in P ſo is P the Port de parted from and the required diſtances are B = 39, and P B = 24 whether leagues, mile furlongs, yards *&c.*

For if the angle B P C = 110 d. its comp ment to 180d. is = 69 d. = angle D P C the fore *per* Def. 87, if P C equal P D the ang P D C equal angle P C D each = 55d. 15m. this example ſee Def. 84.

PRO

PROBLEM CL.

Two Travelers set out at the same Time from a Place P, *A goes South* B *goes* 96d. 30m. *from the South, and when their distance asunder is* 125, *miles* B *hath traveled* 53 *miles more than* A; *how many miles hath each gone.*

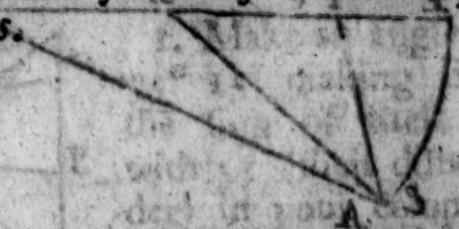


1. Draw the right-line EB, on which make D = 53 Prob. 23 upon D make the angle DA = 41d. 35′ from the diagonal scale take 25 in your compasses and one foot upon B keep an arch cutting DA in A then make the angle PAD = angle ADE drawing AP till it cut EB in P so is P the place departed from and the required distances are PB, PA which by the diagonal scale are found PB = 106,7 miles for B's distance travell'd and PA laid on the same scale is found = 53,7 miles the distance travelled by A, as was required.

If PD be made = PA then (per def. 87,) the angles ADP and PAD must be each = 41° 35′ per Definition 84.

☞ *That a Learner may become well acquainted with the Scale used in what goes before, I have applied the Scale in* Prob 8, 20, *and* 23, *to the*

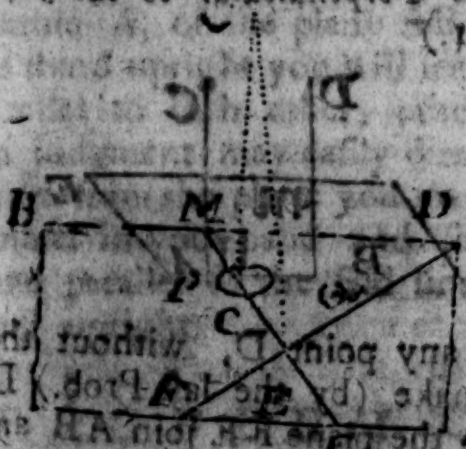
foregoing Problems that treat of Numbers; and if any apply these Scales to such Problems, they will find it answer the Direction: But if such Problems are laid down and measured, by Scales more exactly divided, the Answer will perhaps differ somewhat from the Answers here given. But since the directions for making and applying those Scales are certainly true, such small Errors may easily be rectified by those that understand these Directions, and have better Scales.



SOLID GEOMETRY.

PROBLEM CLI.

To draw a Perpendicular to a given Plane, (AB from a Point given without it C) or to find the shorteſt diſtance poſſible between the Point C and the ſaid Plane, (11 E. 11.)



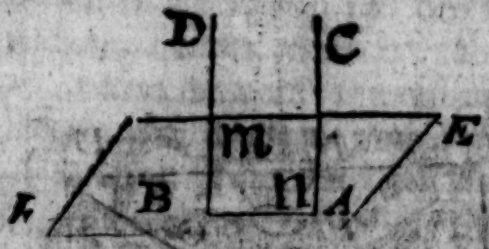
IN The plane AB draw any right-line as DF, upon which from C let fall the perpendicular CE, then in the Plane AB thro' E draw GEM perpendicular to the ſaid DF then from C upon GM let fall the perpendicular CG which CG is the perpendicular requir'd, or if you cannot eaſily let fall perpendiculars take the nearer diſtance with a chord or rule from C, to DF, and GM.

2. In

2. In practice, you may faſten one end of a chord or rule to the given point C, and move the other end thereof round upon the plane, which kept at a conſtant ſtretch, will deſcribe a circle QP whoſe center G (found by Prob. 44) is the neareſt diſtance from C as before; the truth of both theſe ways will appear if you conceive the point C to be the vertex of a cone or pyramid and AB the plain of its baſe.

PROBLEM CLII.

From a given Point A, in any Plane EF, to erect a line Perpendicular to the ſame Plane (12, E. 11.)



1. From any point D, without the given plane FE, make (by the laſt Prob.) DB perpendicular to the plane FE, join AB and parallel to BD draw AC which AC, is the perpendicular required.

Or if you take a carpenter's ſquare as *m*A*n* and turn it round upon the given point A, (ſee the figure) it will direct the perpendicular line.

PRO-

PROBLEM CLIII.

To know if any Plane as a wall, a piece of Wainscot &c. AB, *ſtand upright and not come near it.*

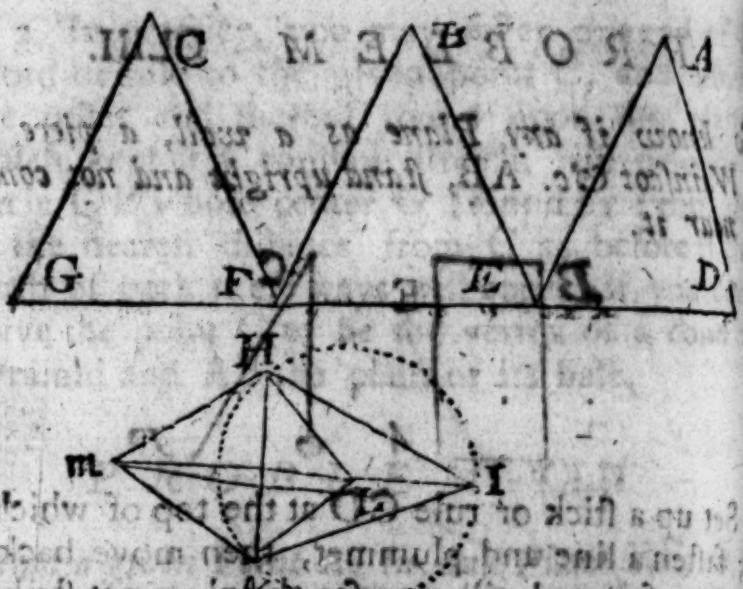


Set up a ſtick or rule CD at the top of which C, faſten a line and plummet, then move backward or forward till you ſee the plummet ſtrike at the bottom A, of the plane AB and then if the plane ſtand upright you will ſee the line and plane parallel to each other, otherwiſe not as your own judgment may eaſily direct you or at any diſtance from the plane you may hold a line and plummet in your hand and ſo ſee if the plane ſtand parallel to the ſaid line, as it will if it ſtand upright.

PROBLEM CLIV.

To make a Solid Angle (HMIK) *of three Plain Angles* A, B *and* C, *whereof any two (however taken) are greater than the third, but the three given Angles muſt be leſs than four Right-angles.* (*ſee* Def. 62)

Make AD, AE, BE, BF, CF, CG; equal to one another, and of their ſubtendent lines



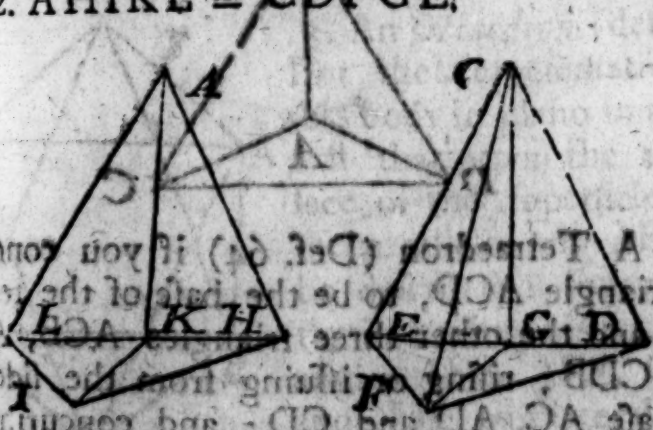
lines DE, EF, FG; make the triangle HIK (viz. IK = DE, IH = FG, and HK = EF) about which describe a circle HIKL; now because DA (EA, EB &c.) is greater than HL let it be □ AD = □ HL + □ LM and let LM be perpendicular to the plane of the circle HIK draw HM, LM and KM; and it is done.

For since the ∠ HLM is a right-angle, therefore (*p* 1 Theo. 2) □ HM = □ HL + □ LM = □ AD consequently AD = HM for the same reason MK, ML, and so forth (that is AE, BE &c. are equal) therefore since HM = AD, MI = AE &c. the ∠ A = ∠ HMI, ∠ B = ∠ IMK and the angle C = ∠ KML wherefore a solid angle is made at M, of the 3 given plane angles as was required (23 E. 11.) this will appear more plain if you conceive the triangles ADE, BI, and CFG to stand upon their respective bases

bases IK, KH and HI; and their tops AB, and C o meet in the point M then ML will be the xis of the Pyramid or Solid angle, *&c.*

PROBLEM CLV.

Upon a given Right-line (AL) *and at a given Point* (A) *to make a Solid Angle equal to a given one,* viz. AHIKL = CDFGE.

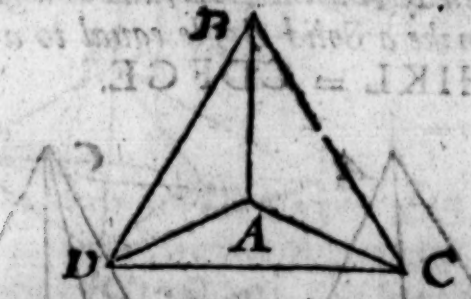


From some point as F, of the given angle raw FG perpendicular to the plane DCE, nd draw DF, EF, EG, DG and CD; make L = CE add the angle LAH = angle ECD nd AI = GF, and in the plane HAL make the ngle HAK = DCG then ereet KI perpendicu- r to the plane HAC, and let KI be equal to FG raw AI, AHIL a solid angle equal to CDFGE he solid angle given as was required, (26 , 11.)

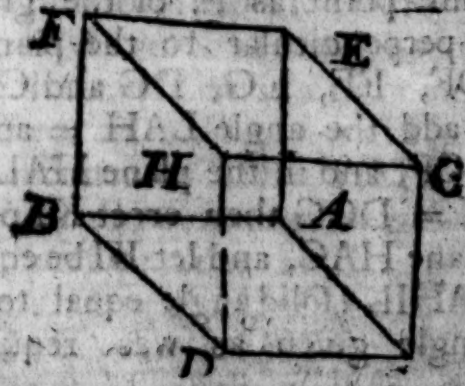
PRO-

PROBLEM CLVI.

To represent the five Regular or ordinate Bodies in Plano, called the Pythagorean, or Platonick Bodies.

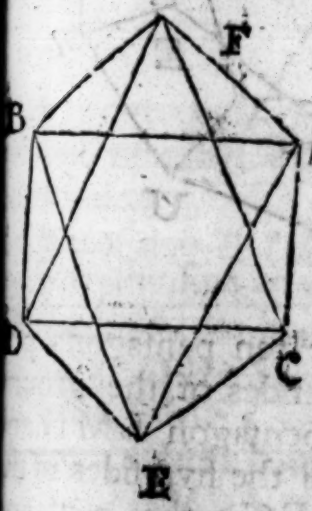


1. A Tetraedron (Def. 64) if you conceive the triangle ACD, to be the base of the tetraedron and the other three triangles ACB, ADB and CDB; rising or issuing from the sides of the base AC, AD and CD; and concuring in the point E, you have the representation requir'd.



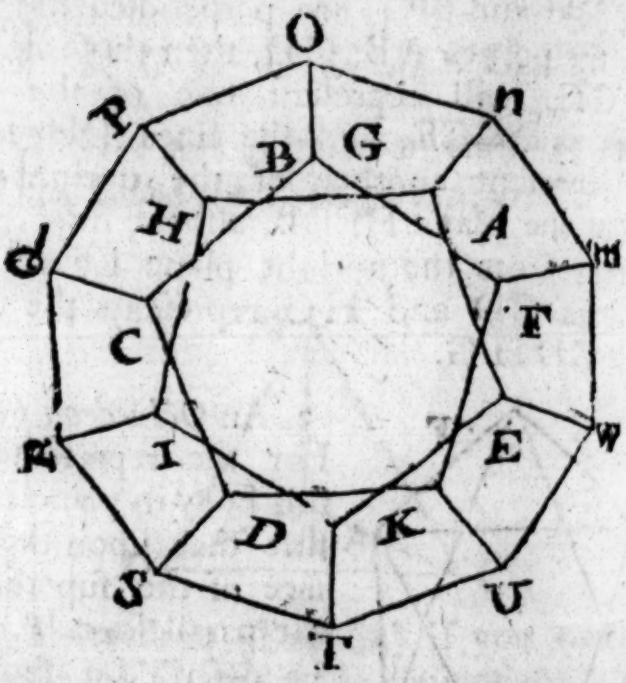
2. Hexaedron, (Def. 65) if you suppose ABCD, to be the base of the cube and EFGH, to be in the top over it, so that the lines AE, CG,

CG, DH and BF; are perpendiculars erected rom the points AB, CD, then thoſe lines AE, nd CG, will repreſent one of the upright lanes as AGCE, and the lines DH and BF ill repreſent another of the upright planes amely the plane HFDB, alſo the lines EF and B repreſents the upright plane EFAB, laſtly he lines CD and HG repreſents the upright lane CDHG.



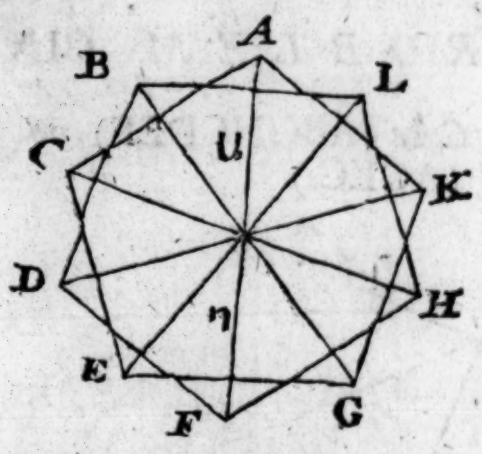
3. An *Octaedron* (def. 66). For the repreſentation of this body in plano imagine firſt that upon the upper face of the ſuperficies of the parallelogram ABCD, be deſcrib'd a ſquare pyramid having his four triangles AFB, AFE, CFD andDFB; equilateral and equiangular and concurring in F, alſo conceive hat on the lower face of the ſame parallelogram ABCD, be deſcribed, another pyramid aving its four triangles AEB, AEC, CED nd DEB; equilateral and equiangular and oncurring in E, and the deſcription may be onceiv'd.

4. *Dodecaedron* (Definition 67). Firſt imagine he pentagon ABCDE, to be the baſis of the ody, and then imagine the pentagon FGHIK, be oppoſite to the former, then between thoſe



two pentagons there will be ten pentagons pulled up, five from the five sides of the ground pentagon, as from AB the pentagon LMN and B will arise *&c.* and 5 from the five sides of the upper pentagon as from FG, the pentagon WMNGF, *&c.*

5. An *Icosaedron* (Def. 68) forasmuch as there are in this solid body twenty triangles and every triangle hath three angles the concourse of the triangles will be in twelve points as at ABGDEFGHKLM and N, where Note that in this plane figure the points M and N



re but one, but you muſt imagine one to be erected upwards and the other oppoſite to it downwards.

Now the five angles which concur in M and re imagined to be erected upwards

are,	BMD	and the 5 angles	A NC
	DMF	which concur in N	CNE
	FMH	and are conceiv'd	ENG
	HML	to be let fall down-	GNK
	LMD	wards are,	KNA

The other ten triangles which include this body are,

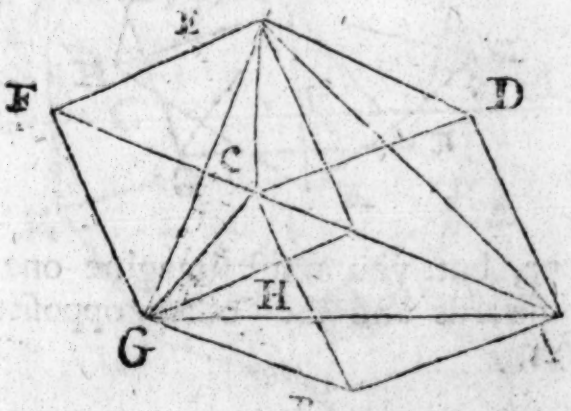
ABC		FGH
CDB		GHK
BDE	and	HKL
EFG		KLA
GEF		LAB

nd thus is the body compoſed.

PRO-

PROBLEM CLVII.

In a given Cube (ABGHCDEF) *to inſcribe a Pyramid* (AGEC.)



From the angle C, draw the diameters CA, CE, CG; and connect them with the diameters GA, GE, AC; and its done.

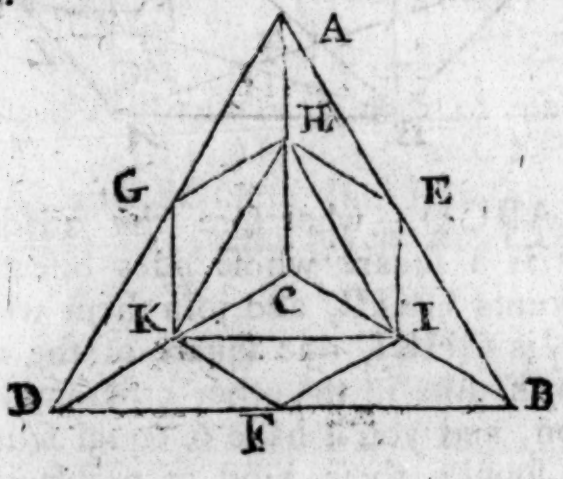
DEMONSTRATION.

For all of them are the diameters of equal ſquares, therefore the triangles CAG, CGE, CEA and EAG; are equilateral and equal, and conſequently AGEC, is a ſquare pyramid which inſiſts upon the angles of the cube, and therefore is inſcrib'd in the ſame as was required, 1 *E.* 15.

PRO-

PROBLEM CLVIII.

In a given Pyramid ABCD, *to describe an Octaedron.*



Bisect the sides of the given pyramid (or solid angle) in the points E, I, F, K, G, H; which join with the right-lines EI, IF, FK, &c.

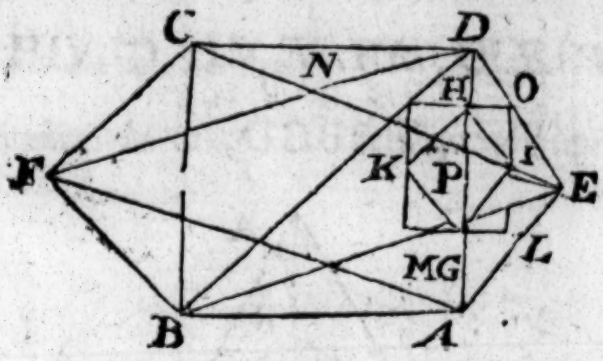
DEMONSTRATION.

All these being equal among themselves, do make three equal and equilateral triangles, and so make the octaedron EGKIFH, *inscrib'd in the square pyramid* ABCD, *as was required*; 2 E. 15.

PROBLEM CLIX.

In an Octaedron (ABCDEF) *to inscribe a Cube.*

Let the sides of the pyramid EABCD, whose base is the square ABCD, be equally bisected by the right-lines LM, MN, N*o* and *o*L, all these are equal and parallel to the sides of the square



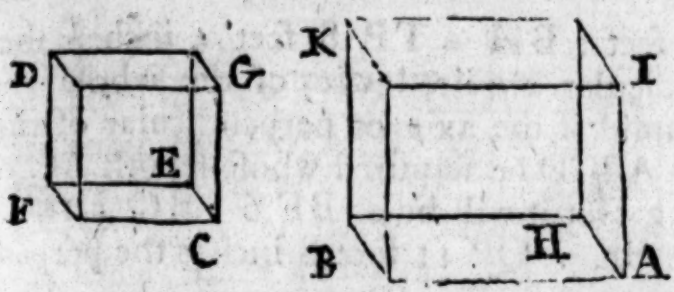
ſquare ABCD, wherefore the quadrilateral MNLo is a ſquare whoſe ſides biſect equally in the points GKHI, and join them with right lines, ſo is GHKI, one ſquare of the required cube, do the like in the other 5 pyramids of the octaedron, and you'll have 6 equal ſquares for the cube ſought, theſe kind of problems might be continued but as they are difficult to underſtand, and of no great uſe except for curioſity theſe ſhall ſuffice, a learner will beſt acquire the knowledge of theſe bodies as alſo of any other ſolid by making them with ſtiff paper as directed in their Definitions.

PROBLEM CLX.

Upon a given Right-line AB, *to make a Parallelopipedon* (AK) *like, and in like manner ſituate with a given one* (CD.)

Of the plane angles BAH, HAG and BAI; which are equal to the angles FCE, ECG and FCG; make the ſolid angle at A, equal the ſo-

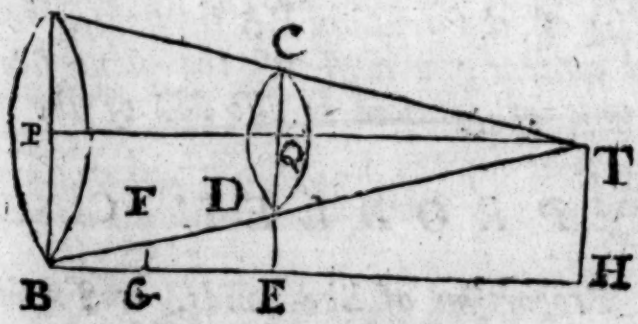
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olid angle at C, then (by Prob. 34) make it as CE : CF : : AB : AH, and as CE : CG : : AH : AI, and ſo finiſh the Parallelopipedon AK, which is like to the given one CD, as was equired; 27 *E.* 11.

PROBLEM CLXI,

To find the Axis of any Cone, Pyramid, Fruſtum, &c.



Faſten a chord or rule or cauſe ſome to hold t at B, the extremity of the baſe and ſtretch he other end any convenient length as to G, hen meaſure from G to the ſide BT, of the ſolid in a perpendicular line to BG. (which you may eaſily do by the help of a ſquare &c.) meaſure BF which ſuppoſe 6 feet and BG, 5 eet 10 inches, then ſay by the rule of three as BF, 6 : BG 5 : 10 : : BT, which ſuppoſe

24 feet : BH = TP 8 feet 4 inches the perpendicular required *viz.* of the whole cone or pyramid if the axis or perpendicular of the fruſtum ABCD be required whoſe length AC or BD is 12 foot it will be as BF 6 : BG 5,10 : : BD 12 : BE = QP 11 foot 8 inches the perpendicular required.

Alſo if you would find the diameter of the ſolids baſe ſuppoſe FG 7 inches ſay as BG 70 inches : FG 7 inches : : BH 276 inches : IH = PB 27,6 inches the ſemi-diameter which doubled is 55,2 inches the diameter required. All this is plain from Theo 1, for the angles BGF, BHT and BED; are ſimilar *&c.*

☞ *Always when you find the ſolid content of any ſuch like ſolid you ought to uſe the axis* TP, *or* PQ *of the fruſtum* BA *in finding the ſuperficial content you muſt uſe the ſlant ſide* BT *of the cone or pyramid as alſo* BD *of the fruſtum.*

PROBLEM CLXII.

The Proportion of like Solids, is Triplicate to that which the Homologous ſides have to each other.

That is as the ſolidity of any ſolid is to the cube of any ſide or part of it, ſo is the ſolidity of any other like ſolid to the cube of its like ſide or part.

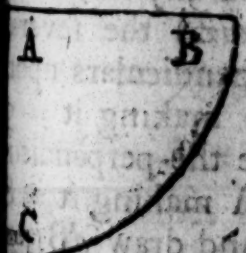
DEMONSTRATION.

Let H *equal the Height*, D *equal the depth* nd T *equal the thickneſs of any ſolid, and let* B *equal the rates of thoſe three quantities* HDF, *to three other quantities that is* B × H = BH *equal the height*, BD *equal the depth and* BT *equal the thickneſs of ſome other ſolid, now theſe two ſolids will be ſimilar becauſe their dimenſions are proportional, Def.* 41, *and therefore if* HDT *be the ſolidity of one*, R HDT *will be the ſolidity of the other, for if the figures are alike one rule muſt find both their contents, but as* HDT : T : : R³ HDT, R³ HDT = B³ T *&c.* of the reſt, *Q.E.D.* HDT :

As the increaſing or diminiſhing of ſurfaces depends on Theorem 3, ſo the like of ſolids depends on this Prob or rather Theo. 6 which might be geometrically applied in problems, but ſuch methods being prolix and tedious to learners I ſhall refer its uſe till we come to numbers.

PROBLEM CLXIII.

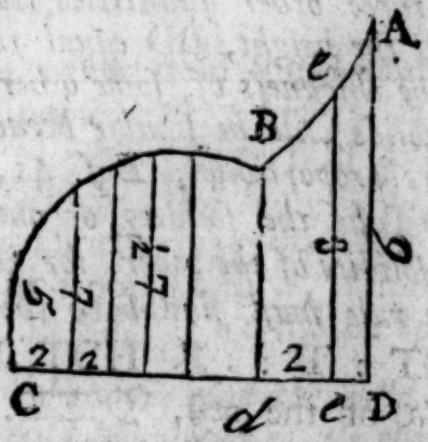
To make a Cone with Paſte Board, ſtiff Paper &c.



Upon the paſte board deſcribe a ſector as ABC which turn up and glue the ſides AB and AC together, ſo have you a cone whoſe vertex is A and circumference of its baſe is the arch BC, its ſlant height equal to AB or AC.

PROBLEM CLXIV.

To take the draught of any Curve or Arch ABC *and Protract the same.*



1. From the ends of the arch as from A and C draw two lines AC and CD *viz.* fasten two chords or rules, meeting each other at right-angles in D then from either AD, or ACD suppose CD, measure as many perpendicular distances as you please (the more the better) and suppose their lengths to be 5, 7, 7½ *&c.* as in the figure the line DC 12 D*e* is 1, *de* 2, *&c. per* fig.

2. To protract this curve first upon paper draw the line DC which from any scale of equal parts make equal to 12 (*viz.*) the several distances taken between the perpendiculars upon D raise the perpendicnlar DA making it = 9 make D*e* = 1 and upon *e* raise the perpendicular *ee* or draw *ee* parallel to DA making it = 8 also make D*d* = 3 or *ed* = 2 and draw *d*B parallel

rallel to D A making it = 6 and ſo do till all the ordinate parallels or perpendiculars be drawn, then over the ends A, *e*, B; *&c.* of thoſe lines draw the curve which muſt in reaſon be a true plan of the curve taken, thus may you have a true plan of any curve whether in a building or the hedge of a field, which with Prob. 16 or 17 you may have a plan or map of any ſurface whatever.

NOTE, *If in the curve there be any ſudden bend or turn as at* B, *you muſt be ſure to take a diſtance or perpendicular from that place otherwiſe when you plan it, ſuch a bend will not be in the map* &c.

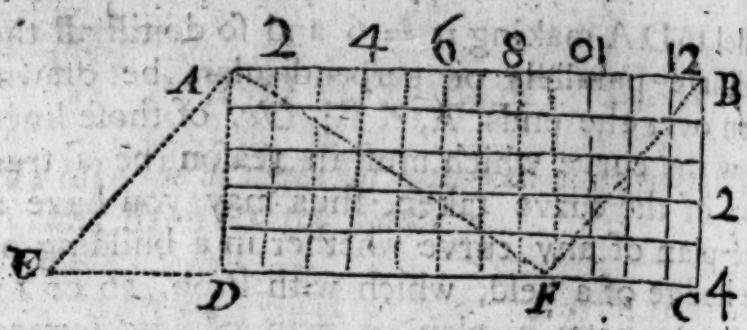
THEOREMS, *for* Meaſuring *&c.*

BEcauſe the general Theorem for menſurations *&c.* is only to be read by thoſe acquainted with fluxions for the ſatisfaction of thoſe that are unacquainted therewith I ſhall deduce from the foregoing Theorems ſome new ones, whereby the rules for meaſuring moſt figures in common practice are demonſtrated as follows.

PROBLEM CLXV.

To find the Area or Superficial content of Surfaces.

Let the ſides AB, and CD; of the rectangle ABCD, be divided into any number of equal parts



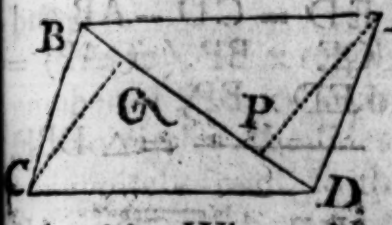
parts as 12, the ſame diſtance (in this figure) divides the ends or breadth AD and BC, into 5 equal parts, thro' which if you draw lines as in the figure they will form 60 equal ſquares as you'll ſee and number them, but thoſe ſquares conſtitute the area of the figure (*per* Def. 72) hence we have this Theo. 7, AB × BC, equal the area of any rectangle ABCD.

Alſo if DE = FC, then becauſe AD and BC, as alſo AB and DC are parallel, the ſides and angles in the triangles AED, and FBC are reſpectively equal, conſequently their areas will be equal, therefore it muſt be equal if from the end BC of the rectangle ABCD, you take the triangle FBC, and add it to the end AD thereof; that is becauſe triangle AED = triangle FBC, the parallelogram ABCD = parallelogram AEFB, which is another way of proof to Theorem 4. Prob. 84, hence we have Theo. 8, AB × BC = EF × AD equal area of the rhomboides &c, AEFB.

It is alſo provd at the latter end of Prob. 101, that every plane triangle AEF, is equal to half of its circumſcribing parallelogram EABF; therefore

herefore if EF × AD be = parallelogram EABF, consequently Theorem 9,

$$\frac{EF \times AD}{2} = \text{area triangle EAF.}$$



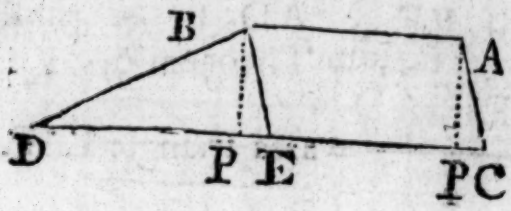
By this last Theorem you may measure any right-lin'd figure whatsoever, by dividing it into triangles &c. Where Note, that if two perpendiculars CQ aud AP fall upon one diagonal DB (as it doth in most 4 sided figures) the area of both triangles BDC and BDA may be had at one operation, for

$$\frac{BD \times CQ}{2} = \text{triangle BDC and}$$

$$\frac{BD \times AP}{2} = \text{triangle BDA,}$$

but it is manifest that these two triangles make up the area of the figure ABCD, that is, $\frac{1}{2}BD \times CQ + \frac{1}{2}BD \times AP =$ ABCD, therefore Theorem 10, $\frac{1}{2}BD \times \frac{1}{2}CQ + \frac{1}{2}BD \times AP = CQ + AP \times BD =$ the area of any Trapezia, &c.

If any two sides opposite as AB, and CD of a trapezia ABCD be parallel, the area thereof may be had without a diagonal: For, let EB, be parallel to AC let fall the perpendiculars BP and AP, which are equal; because AB and CD are



are parallel, now *per* fig. $ED = CD - AB$ and by the above Theorms $CE \times BP$ (or AP) = parallelogram ABEC, and $ED \times BP$,

$$\frac{\qquad}{2} = \triangle DBE,$$

but, $AB \times BP + \frac{1}{2} ED \times BP$, or $\frac{1}{2} \overline{CD - AB} \times BP = ABCD$, that is Theo. 11,

$$\frac{\overline{AB + CD} \times BP}{2} =$$

area ABCD.

Here we see the mistake of those who find the area of a 4 sided figure by multiplying half the sum of two opposite sides AB and CD, by half the sum of the other two opposite sides AC and BD, for if $\frac{AC \times BD}{2}$ be $= 2$ BP the area will be double what it ought to be, also,

if $\frac{1}{2} \overline{AC + BD}$ be equal 3 BP the area will be 3 times greater than it really is &c. so that unless the 4 angles ABC and D be each, near equal 90d. this method of adding sides &c. is vastly erronious, and always makes the area more than it is.

Every regular polygon is composed of some number of equal triangles it is therefore evident that if the area of any one of those triangles be multiply'd by the number of triangles or sides

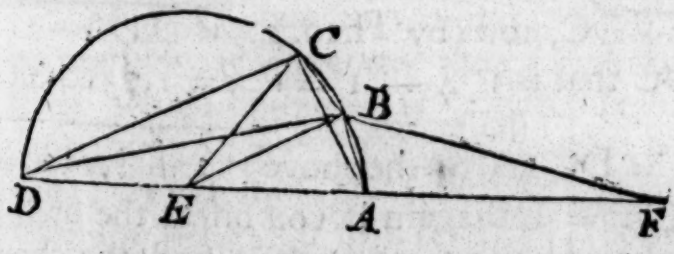
of

of the polygon the product will be the area of that polygon, let R equal the radius of the inſcrib'd circle S equal a ſide *n* equal the number of ſides of any regular polygon, then we have Theorem 12, $\frac{1}{2}RSN$ = the area of any regular polygon.

If we ſuppoſe a regular polygon of an infinite number of ſides to be circumſcrib'd, and inſcrib'd by two circles, the ſides of ſuch a polygon muſt be iufinitely ſmall, ſo that the peripheries of the two circles will become both one periphery whoſe radius will become R and its periphery = *n*, ſo the above Theo. $\frac{1}{2}RSn$, will become Theo. 13, $\frac{1}{2}Rn$ equal the area of a circle S being ſmall is rejected.

If $\frac{1}{2}Rn$, be equal the area of a circle whoſe radius is R, and circumference *n*, it will be eaſy to conceive Theo. 14 $\frac{1}{2}RA$, equal the area of any ſector whoſe arch is repreſented by *a*, and its radius by R.

To find the Periphery of a Circle Geometrically.



Let ABCD, be a ſemi-circle, and the line AC the chord of 60, ſo is DC the chord of the coarch DyC, now becauſe the ſum of the 3 angles of every plane triangle *is* = 180d. the arch ABC being

being the measure of the angle CEA, 60d. and the sides EA = EC *ergo* the ∠ EAC = ∠ ACE each = 60d (for 3 times 60 is 180) consequently the 3 sides EA, EC and AC are all equal; hence the side of any regular Hexagon is equal to the radius of its circumscribing circle, bisect the arch ABC, in B, make AF = DC and join DC, DB and BF; now because the sum of the two opposite angles of any trapezium inscrib'd in a circle is = 180 (that is in the trapezia, ABCD, the ∠ BAD + ∠ BCD = 180d) therefore 180° — ≺ BAD = ∠ DCB, and *per* fig. 180° — ≺ BAD = ≺ BAF, that is ≺ BAF = ≺ DCB also AF = DC and AB = BC by construction it is evident that FB = DB and consequently ≺ F = ≺ BDF = ≺ DBE therefore the triangles DEB and DFB are similar, so *per* Theo. 1, as DE (½ DA) : DB : : DB : DF (DC + DA) *Ergo* □ DB = ½ AD × DC + ½ □ AD, let AD = 2 than this equation will become □ DB = 1 × DC × 2, hence DB = $\sqrt{2 \text{ DC}}$, now (by Theo. 2,) $\sqrt{\square \text{DA} - \square \text{CA}}$ = DC that is $\sqrt{4 - 1} = \sqrt{3} = 1{,}7320508075$ &c. = DC and by the above $\sqrt{2 + 1{,}7320508075}$ &c. = DB again if you bisect the arch AB, you may by the same method find the chord of half that co-arch, and so on bisecting until you find the side of a Polygon of an infinite number of

of sides, which small side multiply'd by the number of sides must be nearly equal to the periphery of the circle.

$\sqrt{:2+1,7320508075:=1,9518516525}$
$\sqrt{:2+1,9518516525:=1,9828897227}$
$\sqrt{:2+1,9828897227:=1,9957178465}$
Thus $\sqrt{:2+1,9957178465:=1,9989291743}$
$\sqrt{:2+1,9989291743:=1,9997322757}$
$\sqrt{:2+1,9997322757:=1,9999330678}$
$\sqrt{:2+1,9999330678:=\sqrt{:3,9999330678}}$

The chord of the co-arch when the Polygon hath	
	12
	24
	48
	96
	192
	384
	768

} Sides.

Now by Theo. 2, the square of the last chord taken from 4 the square of the diameter, leaves the square of the side of a regular-polygon of 768 sides, whose square root multiply'd by 768 must be pretty near equal to the periphery of that circle whose diameter is 2, that is, $\sqrt{4-3,9999330678:=\sqrt{:0,0000669322:=0,00818121}}$, which multiply'd by 768, gives 6,28317 for the perimeter of a regular polygon of 768 sides inscrib'd in the circle; which, as the sides of the polygon very nearly coincide with the circumference of the circle, must also express the length of the said circumference very nearly. Now to find how near truth this is, let us sup-

poſe a circle to be circumſcrib'd and inſcrib'd by two regular polygons, each of 768 ſides, then by Theo. 1, it will be as 1, 999983264, the diameter of the inſcrib'd circle is to 6.28317 the perimeter of the inſcrib'd polygon, ſo is 2 the diameter of the circumſcrib'd to 6,28322 the perimeter of the circumſcrib'd polygon hence the periphery of the circle muſt be between 6,28317 and 6,28322, and therefore may be ſet at 6,28320 very near, for this number exceeds 6,28317 only by ,00003 and is only, 00002 leſs than 6,28322, the ſame with Theo. 27 following, and if you continue the biſection further you will ſtill have the periphery nearer.

If you ſtop at the polygon of 96 ſides, and work as you have done with that of 768 ſides (or by Prob. 172) you'll find that if the diameter of a circle be 7 the circumference will be leſs than 22 but greater than 21 $\frac{70}{71}$ or 21,985 &c. Between which limits it may be prov'd that as 9 is to 10 ſo is 4 times the ſide of a ſquare inſcrib'd in any circle to the periphery of that circle nearly, for if the the diameter of a circle be 7, then half of 7 is 3.5 which ſquar'd is 12,25, and doubled is 2,45, whoſe ſquare root is 4.949 the ſide of the inſcrib'd ſquare which multiply'd by 4 gives 19,796, then as 9 : 10 :: 19.796 : 21,995, which is more than 21,958, again, 4.95 is ſomething greater than the ſide of the inſcrib'd ſquare, ſo $4{,}95 \times 4 = 19.8$ then as 9 : 10 : : 19.8 : 22, as was to be prov'd.

Hence it follows, that as 9 is to 10 ſo is the chord of any arch (leſs than a Quadrant) to the length of the arch nearly.

PRO-

PROBLEM CLXVI.

To find the Solid, and ſuperficial content of Solids.

Theorem 15, *The ſolidity of any Parallelopipedon, Priſm, and Cylinder; is had by multiplying the Area at the Baſe into the Height.*

For if we ſuppoſe each ſide of each end, or baſe of a Parallelopipedon to be a foot or 12 inches, and its height to be 9 feet, the area at the baſe (*per* Theo. 7) will be 144 (*viz.* 12 x 12) inches or a ſquare foot, now if the ſolid be cut into 9 equal pieces (*viz.* at every foot in height) each of thoſe pieces will be a cube, every ſide whereof is a foot and the ſolid being 9 feet height will admit of 9 ſuch pieces or cubes conſequently its ſolidity is 9 ſolid or cubical feet (*viz.*) 9 foot the height multiply'd by 1 foot; the area at the baſe *Q. E. D.* The ſame muſt alſo hold good in priſms and cylinders, Theo. 16, the curve ſuperfices of the laſt mentioned ſolids is had by multiplying the circumference or ſum of the ſides at the baſe into the height, for if you imagine ſuch a ſolid to be cloſe ſewed in cloth, paper *&c.* except the baſes. That cloth rip'd open along the ſide of the ſolid from end to end, taken off and ſpread flat on the ground, will be a rectangle in form; whoſe length is equal to the height of the ſolid, and breadth equal to the perimeter of the ſolids baſe or end. *Q. E. D.*

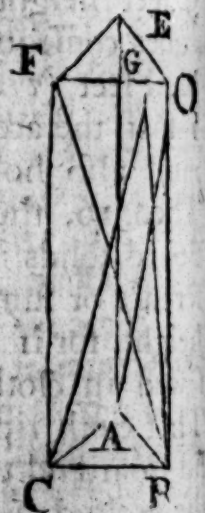
Theorem 17, All priſms cylinders *&c.* of equal heights are in proportion as their baſes.

This

This is plain from Theo. 15, for if A = the area of the baſe of any ſuch ſolid, and *a* = the area of the baſe of ſome other priſm, or cylinder P equal the height of each, then P A equal the ſolidity of one, and P *a* equal the ſolidity of the other ſolid (*per* Theo. 15) but as A : *a* : : AP : $\frac{aP}{A} = aP$, *Q. E. D.*

This Theorem alſo holds true in cones and pyramids of equal height.

THEOREM 18, *Every pyramid is a third part of a Priſm, which hath the ſame Baſe and height* (7 E. 12) *and ſo is every Cone a third part of its circumſcribing Cylinder.*

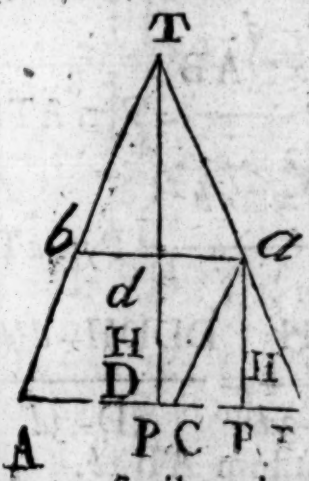


Let the, triangular pyramid BGAC, and priſm BACFEQ have the ſame height draw FB, FA, and OA; then the triangles BFC and BFO, are eqnal, therefore the pyramids BFCA, and BOFA are equal (for they are the ſame pyramids) therefore all the three BFCA, OEAF and BOFA are equal, and conſtitute the priſm BACFEO which priſm is therefore triple to the pyramid FBCA, that is (*per* Theo. 17) to BGAC, *Q. E. D.*

Hence (and *per* Theo. 17) multiply the area of any cones or pyramids baſe by $\frac{1}{3}$ of its axis and you have its ſolidity.

THEO-

THEOREM 19, the curve ſuperfices of any cone or pyramid, is equal to the circuit at the baſe multiply'd by half the ſlant height, this will appear by reaſoning as in Theo' 16, alſo ſee Prob. 163,



To find the ſolidity of a fruſtum, AB*a*B of a ſquare pyramid ATB, cut thro' by a plane *a b* parallel to its baſe AB, firſt put H = the perpendicular height *a*P, D = BA a ſide of the greater baſe, and *a* = *b a*, a ſide of the leſſer baſe of the fruſtum, let *a*C be parallel to TA then *per* ſimilar triangles,

Then *per* THEOREM 18,

we ſhall have,

1 $D - d : H :: d : \frac{dH}{D-d} = dT$

2 $\frac{dH}{D-d} + H = \frac{HD}{D-d} = dT + dP = PT$ [2 ÷ H]

3 $\frac{HDDD}{3D-3d} = \frac{PT \times \square AB}{3} = \text{Pyr. } ATB$ *per* Theo. 18.

4 $\frac{Hddd}{3D-3d} = \frac{dT \times \square ab}{3} = \text{Pyr. } aTb$

5 $\frac{DDDH}{3D-3d} - \frac{dddH}{3D-3d} = \frac{DDDH - dddH}{3D-3d} = \text{the Fruſtum } ABab.$ [6 − 4]

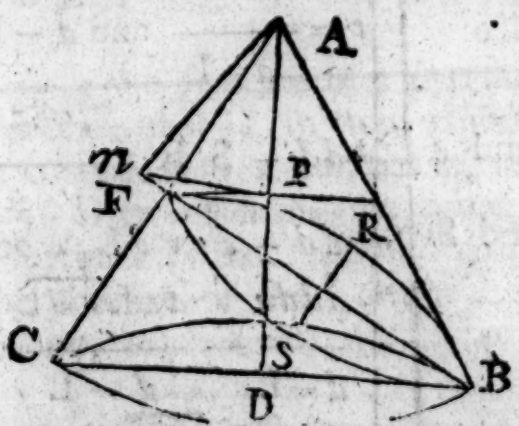
5 reduc'd 6 $DD + Dd + dd \times \frac{1}{3} H = \text{the Fruſt. } ABab$ Theorem 20.

Now becauſe DD equal the area of the greate and *dd* equal the area of the leſſer baſe of an ſquare fruſtum, it muſt follow that if DD be e qual the area of the greater and *dd* equal the a rea of the leſſer baſe of any fruſtum whatever That the ſame Theo. will alſo hold good *vi* $DD + Dd + dd : \times \frac{1}{3} H =$ any fruſtum, Theo rem 21:

Again

Again,
Suppose | 1 | $e = D - d$ and F = the Fruſtum,
Then, | 2 | $DD + Dd + dd = \frac{3F}{H}$ by the 6th
ſquar'd, | 3 | $ee = DD - 2Dd + dd$,
2 − 3, | 4 | $3Dd = \frac{3F}{H} = ee$
4 + ee, ÷ | 5 | $Dd + \frac{1}{3}ee = \frac{3F}{H}$
⅓ × H, | 6 | $Dd + \frac{1}{3}ee \times H = F$ Theorem 22.

To find the ſolidity of a conical ungula, or hoof BFCB (Def. 58) draw EF prrallel to BC cutting the axis of the cone in P, and the tranſ-



verſe diameter BF of the Ellipſis BRFS in F, produce BF, and draw a perpendicular thereto, then the part ABF, above the hoof BFCB, is an oblique eliptical cone let $d = BC$, $b = AD$ = AP, L = EF, and $a = DP$,

E e

per Theo. 18.

1 SR × FB × * 7854 × ⅓ *An* = the ſolidity of A B F, which

2 ,7854 *ddh* / 3 = the ſolidity of (call S, (the whole cone ABC

2 − 3

3 *ddh* − SR × FB, × *An* × 2618 = S the ſolidity of the hoof

per ſimilar △s

4 *An* × FB = BC (*d*) × AP (*t*) = *td*, or $An = \frac{dt}{FB}$

by the property of the ellipſis & cone

5 RS = √ *d*L, Note RS is the conjugate diameter of the ellip.

per ſimilar △s

6 $h = \frac{da}{d-L}$ for *d*−L: *a* : : *d* : *h* (*by* Theo. 1,

alſo

7 $t = \frac{aL}{d-L}$ and *d* − L : *a* : : (L : *t*,

from 3 and 4

8 $hdd - SR \times \frac{,7854}{3} dt = S$

from 5 and 8

9 $hdd - dt\sqrt{dL} \times 2618 = S$

6 7 9

10 $\frac{ddda}{d-L} - \frac{daL\sqrt{dL}}{d-L}$ (=S. × 2618,

So reduced

11 $\frac{dd-L\sqrt{dL}}{d-L} \times 2618\, da = S$

Theo. 23, greater hoof.

* (Note, *for* 0,7854 ſee Theorem 30)

Again,

Again, for the leſſer hoof, BFEB,	12	LLtc = the ſolidity of the top cone EAF,
	13	SR × FB × nA × c = AF.B as above, Note, here c = .2618,
13 — 12,	14	SR × FB × nA × c − LLtc = z, ſolidity of the leſſer hoof BFEB,
per 4, 5, 14;	15	$\frac{dt\sqrt{d}\mathrm{L} \times \mathrm{FB} \times c}{\mathrm{FB}}$ − LLtc, or $cdt\sqrt{d}\mathrm{L} - c\mathrm{LL}t = z$,
	16	$\frac{c\,a\mathrm{L}d\sqrt{d}\mathrm{L} - c\mathrm{LL}a\mathrm{L}}{d - \mathrm{L}}$ $= d\sqrt{d}\mathrm{L} - \mathrm{LL} \times \mathrm{L}ac$,
per 7, 15; Theo. 24,		$\frac{}{d - \mathrm{L}} = z$,

To reduce the fruſtum of a pyramid to a parallelopipedon, that is, in ſtep 1 above, Theorem 22, to find ſuch a number m, as that $em + d = d + \mathrm{D}m - dm$ may be equal the ſide of a parallelopipedon's baſe, whoſe ſolidity may be equal the ſolidity of the fruſtum ABab fig. to Theorem 20.

per ſtep 6 Theo. 22;	7	$D d + \frac{1}{3} e e$ = the ſide requir'd which call S;
per demand	8	$e m + d = S$
per 1 and 6	9	$dd + de + \frac{1}{3} e e = D d + \frac{1}{3} e e = S S$ for $D = d + e$
8 ſquared	10	$dd + 2 dem + e^2 m^2 = SS$;
6 = 10;	11	$dd + de + \frac{1}{3} ee = dd + 2 dem + e e m m$;
11 reduced	12	$3de + ee = 6 dem + 3 e e m m$ or $6md - 3d = e - 3 emm$
11 ÷ 1 − 3 m^2	13	$e = \frac{6m - 3}{1 + 3mm} \times d.$

Now if $m = \frac{5}{10}$ then $6md - 3d = e - 3 em^2$ will become $d \times 3\,0 - 3 = 0$, hence $e = 0$.

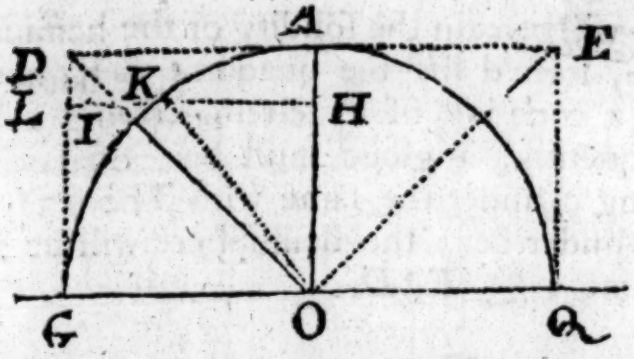
If $m = \frac{55}{100}$ then $3.30 - 3 = e - .9075e$, or $e = \frac{0.3d}{1 - ,9075}$

If $m = \sqrt{\frac{1}{3}}$, or 0,5774 &c. then $de = \frac{3,4644 - 3}{1 - 1,}$ here e muſt be infinite becauſe the diviſor is = 0, ſo that m can never be below 0.5, nor never above 0,5774 &c. Theo 25.

The ſame is alſo true if D, and d be the two diameter's of a cones fruſtum &c.

If a cone, a ſemi ſphere, and a cylinder have the ſame baſe and altitude, their ſolidities are as 1, 2 and 3.

If



If the right triangle DAO, the quadrant AIG and ſquare ADGQ; be all ſuppoſed to move round upon AO as an axis the triangle will deſcribe a cone, the quadrant a ſemi-ſphere, and the ſquare will deſcribe a cylinder; now if any line be taken perpendicular to the axis AO as the line HL, this HL muſt evidently deſcribe a circle of the cylinder, HI a circle of the cone, and HK one of the hemi-ſphere now becauſe AD = AO therefore HI = HO, but, (*per* Theo. 2,) □ HI = □ HL − □ HK (that is □ OH = □ OK − □ HK) now becauſe all circles are as the ſquares of their diameters (by Theo. 3) it is evident that the circle deſcribed by HI, is equal to the differenee of the circles deſcribed by HL and HK, that is equal to the ſection of the ſolid which remains when the ſphere is taken out of the cylinder, and becauſe this holds wherever the line HL be taken that is the ſections being every where equal, the ſolids themſelves muſt alſo be equal therefore if from the ſolidity of the cylinder EDGQ, form'd by the rotation of the ſquare ADGO, you take the ſolidity of the cone EOD form'd by the rotation of the triangle, AOD, there

there will remain the ſolidity of the hemi-ſphere QAG, form'd by the quadrant A K, now becauſe a cone is $\frac{1}{3}$ of its circumſcribing cylinder conſequently, a globe muſt be $\frac{2}{3}$ of its circumſcribing cylinder the ſame with Theo. 38, or if the cylinder be 3 the hemiſphere will be 2, and the cone 1 *Q, E, D.*

COROLLARY.

From hence it will be eaſy to find the ſuperficies of any globe for as an infinite number of triangles (Theo. 13) fill up a circle, ſo an infinite number of equal cones makes up a globe having their vertexes all in the center of the globe and their baſes in its ſurface, the ſemi-axis of the globe being the perpendicular height of each cone, and a portion of the globes ſurface equal the area of each cones baſe; now putting *a* equal the ſemi-axis of the globe, *c* equal its ſuperficies, we ſhall have,

per Theo. 18, 1 $\frac{1}{3}$ *ac* = the ſolidity of the ſphere (*viz.* of all the cones that compoſe it.

2 $\frac{4}{3}$ *paaa* = the ſolidity of the globe by the above, here $p = 3,1416$.

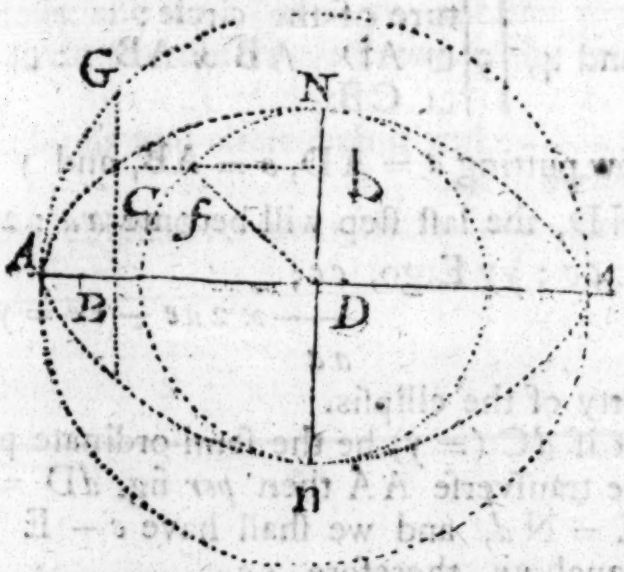
1, = 2, 3 $\frac{1}{3}$ *ac* = $\frac{4}{3}$ *paaa*, that is, $c = 4paa$ the ſame with Theo. 55,

But if $d = 2a$, then $pdd = c$,

Now becauſe 4 times 0,7854 is = 3,1416, it is the ſame thing whether you multiply the ſquare of the diameter by 0,7854, or multiply the ſquare

of

of half the diameter by 3,1416, this obſerve in Prob. 168, in which Prob. as alſo in other places where mention is made of the nature or property of the figure, ſignifies the ratio of the abſciſſa's to their ordinates, abſciſſa ſignifies a part cut off as AB is an abſciſſa cut off by CB, which CB, is called a ſemi-ordinate in the ellipſis, and CB a ſemi-ordinate in the circle &c.



Now to derive the properties of the circle and ellipſis, ſince an angle in a ſemi-circle is a right-angle, if GB be any where taken at right angles to the diameter AA, it always holds (by Theorem 1 and 2) AB × BA = □ GB put e = AB and a equal the ½ diameter AD, then we have $2ae - ee = yy$ (y being = GB) which is ſaid to expreſs the nature of the circle.

Again, for the Property of the Ellipſis.

Let the ellipſis be inſcrib'd and circumſcib'd by the prick'd circles, draw dC parallel to AA, from

from G draw GD, then becauſe CB is parallel, ND, the triangles DGB and fDd are ſimilar, therefore

per Theo. 1,	1	DG (= AD,) : GB : : Df (= DN) : dD (= CB);
that is,	2	AD : GB : : DN : CB;
2 in ſquares	3	□ AD : □ GB : : □ DN : □ CB,
But,	4	AB × BA = □ GB, by the nature of the circle;
3 and 4,	5	□ AD : AB × AB : : □ DN : □ CB.

Now putting a = AD, e = AB, and y = BC, c = ND, the laſt ſtep will become $aa : 2a - e \times e : : cc : yy$ Ergo, cc;

$\frac{}{aa} \times 2ae - ee = yy$, the property of the ellipſis.

Bnt if dC ($= y$) be the ſemi-ordinate parallel to the tranſverſe AA then *per* fig. dD = BC, put E = Nd, and we ſhall have $c - E^2$ = □ dD equal yy, therefore, cc,

$\frac{}{aa} \times 2ae - ee = yy = cc - 2cE + EE$, which reduc'd gives (by writing dC, y for e its equal, $yy = \frac{aa}{cc} \times 2cE - EE$,

ſo that if y = any ſemi : of an ellipſis, parallel to either the tranſverſe or conjugate and e = the abſciſſa

ſciſſa to that ſemi ordinate a = the diameter to the ſaid ſemi-ordinate and c = the other diameter, we have $\frac{cc}{aa} \times 2ae - ee = yy$ for the property of the ellipſis, or if $z = a - e$, then $\frac{cc}{aa} \times aa - zz = yy$ which is another property of the ellipſis, alſo $aa - zz = yy$, another property of the circle the difference between the property of the ellipſis and circle being only $\frac{cc}{aa} \times$, for when the ellipſis becomes a circle $\frac{cc}{aa}$ becomes unity.

PROB.

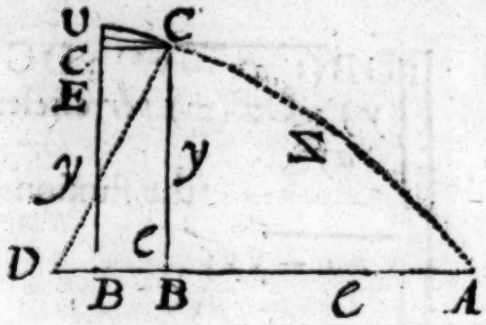
PROBLEM CLXVII.

To find a general Theorem for finding Areas, both for Right-lin'd and Curve-lin'd Figures.

LET AC, be either a right-line or a curve of any kind whose ordinates BC, are rightly apply'd, *viz.* perpendicular to the axis (AB,) Let the curve AC = z, AB = e, BC = y, space ABC = A, now if we suppose BC to move parallel to itself, and uniformly &c. BB = CE, will be = e · Ev = $\dot{y}$, Cv = $\dot{z}$, and the paralellogram BCEB = $\dot{A}$, (now *per* Theo. 7,) BC × BB, or $y\dot{e} = \dot{A}$, whose fluent A, is the area of the space ABC, by which general expression or Theorem the area of any such figure may be had, whose property is known and also the length of the curve, for *per* Theo. 2, $\sqrt{ee + yy} = z$ but with this I have little to do here, except in the circle; let therefore,

	1	$ce^n = y$ expres the nature of the figure ABC, c = the prar.
Then,	2	$y\dot{e}^n = \dot{A}$ the fluxion of its area by the above;
1, and 2,	3	$ce\dot{e}^n = \dot{A}$
	4	$\frac{ce^{n+1}}{n+1} = A$ the area sought, or fluent of the 3*d* Step,
1 and 4,	5	$\frac{ye}{n+1}$ equal A.

Now,



Now if $n = 1$ then the figure ABC, is a plane right-angled triangle, and ye, ——— will become $\frac{ye}{2}$ or $\frac{AB \times BC,}{2}$ $\frac{n+1,}{}$ = A the same with Theo. 9.

If $n = \frac{1}{2}$ then the figure ABC, will be half of a common parabola; and ye, ——— will become $\frac{2ye}{3} = A,$ $\frac{}{n+1}$

hence $\frac{2 : \times AB \times 2 BC :}{3}$ (Theo. 26)

equal area of a parabola.

If ABC, be the semi-segment of a circle, then let $a = DC$, the radius of that circle, and we shall have,

1 ∴ 1 $DB\,(\sqrt{a\cdot - y\cdot^2}) : DC\,(a) :: E$ $(y\cdot) : Cv\,(z\cdot)$ *per* circles property.

$\frac{a\,y\cdot}{\sqrt{a\cdot - y\cdot^2}} = z\cdot$ the fluxion of the arch

2 $\sqrt{aa - yy}$;

per Figure, 3 $AD - BD = AB$ or $a - \sqrt{aa - yy} = e$, $\frac{yy\cdot}{\sqrt{aa - yy}} = e\cdot$;

3 in Fluxions 4 $\frac{yyy\cdot}{\sqrt{aa - yy}}$ = the fluxion of the semi-

4 × y 5 segments area,

2 in Fluents, 6 $z = y + \frac{yyy\cdot}{6a\cdot^2} + \frac{3y\cdot^5}{40a\cdot^4} + \frac{15y\cdot^7}{336a\cdot^6}$, &c.

for the length of the arch AC;

5 in Fluents, 7 $A = \frac{y\cdot^3}{2a} + \frac{y^5}{12a^3} + \frac{3y^7}{80a^5}$ &c.

for the area of the semi-segmen ABC;

per 6, 8 $a + \frac{a}{6} + \frac{3a}{40} + \frac{15a}{336}$ &c. = z

when $y = a$, or ABC, is a quad

If $a = 1$ then 8, 9 $1 + \frac{1}{6} + \frac{3}{40} + \frac{15}{336}$ &c. = z

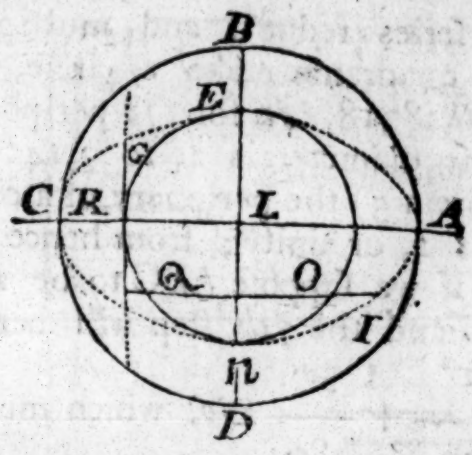
Whic

Which ſeries reduc'd and multiply'd by 4, becauſe 4 quadrants make a circle, there will come out 6,28318, &c. for the periphery of that circle whoſe diameter is 2, ſo 3,14159 &c, is (*per* Theorem 3) the periphery of a circle whoſe diameter is 1, or unity; from hence Theo. 27.

Again, if we ſuppoſe ABC to be a quadrant, then $y = a$ and the *7th* ſtep will become $A = \frac{a^2}{2} + \frac{a^2}{12} + \frac{3a^2}{80}$ &c, which multiply'd by 2, will be $2A = ca + \frac{aa}{6} + \frac{3aa}{40} + \frac{15aa}{336}$ &c; and divided by a, is $\frac{2A}{a} = z = a + \frac{a}{6} + \frac{3a}{40} + \frac{15a}{336}$ &c, the ſame with ſtep 8, by which it appears that ſtep 7, is agreeable to Theo. 14.

But according to the common way of finding an arch line, if B = the radius of a circle, d = the difference between the verſed ſine and radius and C = $\frac{1}{3}$ the chord of the ſegments baſe, then Theo. 28, $\frac{2\frac{1}{3}RR - \frac{1}{3}Rd - dd}{1\frac{1}{2}R + d} \times C$ equal the area of that ſegment, near enough for any ordinary practice, but by ſtep 7 before, you may continue the ſeries to what exactneſs you pleaſe.

Having



Having found the area of any circle ABCD, from that you may deduce the area of an ellipsis AEC*n*, for by the property of the ellipsis, as BD : E*n* : : EF : GH, therefore as □ BD : □ E*n* : : □ BD × ,7854 (*viz.* area circle ABCD)

$$: \frac{En \times \square BD \times ,7854}{BD} = 7854 \times En$$

equal the area of an ellipsis, Theo. 29.

Note, *If the diameter of a circle be equal* 1, *its periphery* (*by Theorem* 27,) *is* 3,1415.9 &c. *or* 3.1416 *may be used as a constant number, then* per *Theo.* 14, (*or by the above*) $\frac{1}{2}$ *of* 1, (*viz.* $\frac{1}{2}$

$$\times \frac{3,1416}{2} = 0,7854$$

the area of that circle whose diameter is unity or 1, *now since circles are like figures, and the square of* 1, *is* 1; *therefore* (per *Theo.* 3) *as* 1 : ,7854 : : *the square of any circles diameter to the area of that circle, hence we have Theo.* 30.

Also

Alſo as the ſquare of 3,1416 *the circumference, is to* 0,7854, *the area ſo is the ſquare of* 1, *the circumference,* (viz.) 1 *to* 0,795 *the area* &c. *the area of a circle whoſe circumference is* 1, *hence, Theo.* 31.

Again if R = BL, *or* LA *or* LC *the ſemi-tranſverſe of any ellipſis* g = Ln = LE *the ſemi-conjugate diameter thereof, and* d = LR *the difference between the verſed ſine* RC *and radius* LC, *then becauſe as* R : g : : ER : GR, *therefore as* R : g : : $\frac{2\frac{1}{3}RR - 1\frac{1}{2}Rd - dd}{1\frac{1}{2}R + d}$, $\times$ C, (*Theo.* 28) *to* $\frac{2\frac{1}{3}RR - 1\frac{1}{2}Rd - dd}{1\frac{1}{2}R + d} \times \frac{gC}{R}$ (*Theorem* 32) *the area of the elliptical ſegment* GHC, Note, *here* C = ER = BF.

Or becauſe R : g : : ER : GR, *take the two latter terms of the proportion, and the laſt Theo. will become* : $\frac{2\frac{1}{3}RR - 1\frac{1}{2}Rd - dd}{1\frac{1}{2}R + d} \times \frac{C \times GR}{ER}$ (*Theorem* 33) *the area of the ſame ſegment* GHC.

But if IH the chord of the baſe of IH *n* any elliptical ſegment be parallel to AC the tranſverſe diameter of the ellipſis.

Let R = AC, and *g* = E*n*, then as R : *g* : : 7854 RR : $\frac{,7854R^{\cdot 2}g}{R}$ = ,7854 R*g*, the area

of

of the whole ellipſis (*per* Theo. 29) alſo as RR : Rg : : RRa : $\frac{R \cdot ag}{RR} = Rga$, the ſame area ($a$ being = ,7854) but RR ; Rg : : Rg : $\frac{RR\ gg}{RR} = gg$, hence the area of any ellipſis, is a mean proportional between the areas of its circumſcribing and inſcribed circles, hence gga : A : : Rga : $\frac{RgaA}{gga} = \frac{RA}{g}$ the area of the ſegment I H n Theo. 34;

Note, A *is put for the area of the ſegment* O Q n.

PROBLEM CLXVIII.

To find a general Theorem that ſhall give both the Solid and Superficial content of any Solid, whoſe property is given;

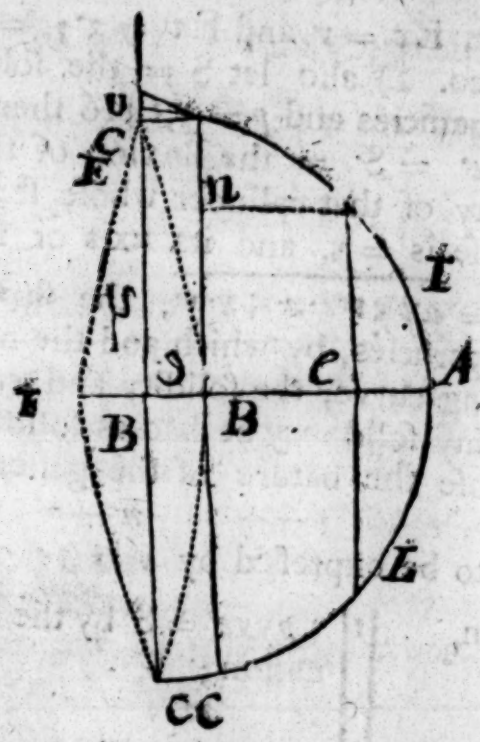
BUT for diſtinctions ſake the laſt problem might ſerve for this too, for let ABCC be a ſolid form'd by turning AC about the ordinate, or about the axis AB, then if CBC be the diameter of a circle moving uniformly from A towards F, it is evident the area of this moving circle will deſcribe the ſolidity, and its periphery the curve ſuperficies of the ſaid ſolid, therefore let AB = e, BC = y, then will

$BB = e$, $Ec = y$ and $Ev = z$; $= \sqrt{e \cdot e + y \cdot y}$ (*per* Theo. 2) alſo let S = the ſolidity, C = curve ſuperficies and $p = 3{,}1416$ then *per* Theo. 15, $pyye \cdot = S \cdot$ = the fluxion of the ſolidity, or ſolidity of that cylinder whoſe ſemi diameter of its baſe is $= y$, and its axis or length $= e \cdot$, and $C \cdot = 2py\sqrt{e \cdot e + y \cdot y}$, the fluxion of the curve ſuperficies, by which and the nature of the generating curve, the ſolidity and curve ſuperficies of any ſolid may be had as follows;

Suppoſe the nature of the generating curve ABC, to be expreſs'd by $y = ae^{\frac{n}{m}}$ or $y = ae^{\frac{m}{n}}$

Then,	1	$pyye \cdot = S \cdot$ by the above explanation;
per Curve,	2	$y = ae^{\frac{m}{2m}}$,
2 ſquared	3	$yy = aae^{\frac{em}{2m}} \cdot$
from 1 and 3,	4	$paae^{\frac{em}{2m}} e \cdot = S \cdot$
fluent of the 4	5	$\frac{paae}{2m+1} = S$,
per 3 and 5	6	$\frac{pyye,}{2m+1} = S$,

Now if the figure ABC, be a right-angled plane triangle, and be turn'd about upon AB, its baſe or perpendicular, it will form a cone, whoſe axis is AB, (= e) and diameter of its baſe



base CBC, (= 2y) in this case $m = 1$, and the last step will become $\frac{pyye}{3} = S$,

the same with Theo. 18,

Again, if the figure ABC, be a semi-parabola mov'd round its axis AB, then ABCC will be a parabolic conoid, and $m = \frac{1}{2}$ (if the curve ABC, be the common parabola) then $\frac{pyye}{2m+1}$ $= S$ will become $\frac{pyye}{2} = S$, Theo. 35;

Also if AC,BC be a parabolic spindle (y being the axis & e the greatest semi-ordinate of the generating parabola) then by examining the properties

perties of the common parabola you'll find that
m must be $= \frac{7}{8}$ so $\frac{pyyee}{16} = S$; will become $\frac{pyye}{2m+1}$
15 Theorem 36.

Likewise, if ACBC be the segment of any spheroid either right or oblate, let $a = \frac{1}{2}$ that diameter of the ellipsis upon which the semi-ellipsis is mov'd about when the solid is form'd, and e equal half of the other diameter of the said ellipsis, then by its property,

we have	1	$yy = \frac{cc}{aa} : \times 2ae - ee$
$1 \times pe$ —	2	$pe \cdot yy = \frac{pcc}{aa} : \times 2aee - eee$
		$: = S$ the fluxion of its solidity
Fluent 2d step	3	$\frac{pcc}{aa} : \times : aee - \frac{eee}{3} : = S$ the solidity of the segment required, Theo. 37;
if $e = a$, *then* $3 =$	4	$\frac{cc}{aa} p \times : \frac{2aaa}{3}$

Hence every globe, sphere or spheroid is $= \frac{2}{3}$ of its circumscribing cylinder Theorem 38, $S = \frac{2}{3} pcca$

if $c = a$ *then* $4 =$	5	$\frac{2paaa}{3} = S$ the solidity of any $\frac{1}{2}$

and 3 ſtep = 6 | globe or ½ ſphere Theo. 39. $paee - \frac{1}{2}peee$ = S the ſolidity of any ſegment of a globe or or ſphere, Theorem 40.

For when c becomes $= a$, the ellipſis becomes a circle, and then the ſpheriod muſt be a globe, from the Theorems laid down in this Problem it will be eaſy to deduce Theorems for finding the ſolidity's of the fruſtum of a parabolic connoid, and of the middle zones of a parabolic ſpindle, ſpheriod and globe or ſphere; thus,

Let ICCL, be the lower fruſtum of a parabolic conoid ACBC in the foregoing figure put y = BC the radius of the greater z = oL the radius of the leſſer baſe $h = o$B the height of the fruſtum, and q = Ao the height of the part or conoid LAIo to be cut off,

then *per* theo. 35 } 1 | $\frac{pyy \times h + q}{2}$ = the ſolidity of the whole conoid ACBC,

2 | $\frac{pzzq}{2}$ = the ſolidity of the conoid LAIo cut off,

1 − 2 | 3 | $\frac{p}{2} \times : yyh + yyq - zzq : =$ F the ſolidity of the fruſtum ICCL,

But, | 4 | $h + q : yy : : q : zz$ by the property of the parabola,

4 ∴ | 5 | $yyq = hzz + qzz$,

yyq

5 $-qzz$ | 6 $yyq - zzq = hzz$,

16 $\times \frac{2}{p}$ | 7 $yyh + yyq - zzq = \frac{}{p}$

7 — 6, | 8 $yyh = \frac{2F}{p} - hzz$

| $: yy + zz :$

8 reduc'd, | 9 $\frac{}{2} \times ph = F$ Theo. 41.

Alſo, if BC, be the axis of any common parabola, and AB a ſemi-ordinate thereof &c. then $: 2 \square BC + \square Io - \frac{2}{7} \square Cn : \times \frac{1}{7} Bo \times p$: equal the ſolidity of the half zone, ICCL Theo. 42, The ſolidity of any fruſtum, or zone may be found without ſuch ſubtraction, thus, if ICCL, be the half zone of a ſpheroid, put OB = e the length of the ſaid half zone, and a, c, y, the ſame as in Theo, 37, then by the property of the ellipſis.

we have, 1 | $yy = \frac{cc}{aa} : \times : aa - ee :$

1 $\times pe$ | 2 | $pyye = \frac{ccpe}{aa} : \times aa - ee = S$:

fluent of 2 ſtep | 3 | $\frac{pcc}{ra} : \times : a^2 e - \frac{1}{3} eee = S$ the ſolidity of the half zone requiſed, Theorem 43,

2 x

if c = a *then* 4 | $p \times : aae - \frac{1}{3} eee =$ S the ſolidity of the half zone of a globe, Theorem 44;

3d, = |

if e = a *then* 5 | $\frac{2pcc}{3aa} : \times aaa = \frac{2}{3} pcca$ the ſame with Theorem. 38.

But if $e =$ Bo the length of the half zone, $b =$ BC the radius of the greater baſe, $h =$ oL or oI, the radius of the leſſer baſe, and $p = 3{,}1416$ be given to find its ſolidity.

Then 1 *Step* = 6 $hh = bb - \frac{eeb^2}{aa}$, or $bb - hh = \frac{eeb^2}{aa}$, or $a^2 b^2 - h^2 a^2$ equal eeb

3 = 7 $pbbe - \frac{pbbeee}{3aa} =$ S, Note *here* b *is put for* c;

6 ÷ $bb - hh$ 8 $a \frac{eebb}{bb - hh}$

per 7 *and* 8, 9 $pbbe - \frac{pbbe \times : bb - hh}{3bb} =$ S Theorem 45; otherwiſe,

9 × 3 ÷ bb 10 S = $\frac{3pb^2e - pbbe + phhe}{3} =$ $pe \times : 2bb + hh$: Theo. 45

Which

Which holds for the middle half zone of any ſpheriod, globe or ſphere, and if *e* be equal the length of the whole zone, the ſame Theo. gives the ſolidity of the whole zone

If you would reduce any of the foremention'd fruſtums or zones into a cylinder, that is by multiplying the difference between the greater and leſſer diameters by ſuch a number, *m*; as that product added to the leſſer diameter may give a mean diameter D, equal to the diameter of a cylinder whoſe ſolidity is equal to that of the fruſtum or zone their heights being equal.

This is already done for the fruſtum of a cone in Theorem 25, and therefore for the fruſtum of a parabolic conoid, Theorem 41; put *d* equal the difference between the half diameters *y* and *z*,

per *Theo.* 41 | 1 $\frac{yy + zz}{2} \times ph = p\,DD\,h$, the ſolidity of the half fruſtum,

1 reduced | 3 $\frac{yy + zz}{2} = DD$, the ſquare of the mean diameter,

by what is ſaid before | 4 $z + d = y$ (for $y - z = d$) ergo $zz + 2zd + dd = yy$

| 5 $z + dm = D$, *Ergo*, $z + dm^2 = DD$,

3, 4, and 5, | 6 $zz + zd + \frac{dd}{2} = zz + 2z\,dm + d^2\,m^2 = z + dm^2$

z,

$6 - zz$ | 7 | $zd + \frac{dd}{2} = 2zdm + ddmm,$

$7 \pm, \div d -$ | 8 | $4zm - 2z = d - 2dmm$

$8 \div 1 - 2m^2$ | 9 | $d = \frac{:4m-2:\times z}{1-2mm}$ hence if $m = \frac{5}{13}$ then $d = \frac{4mz - 2z}{1 - 2m^2}$

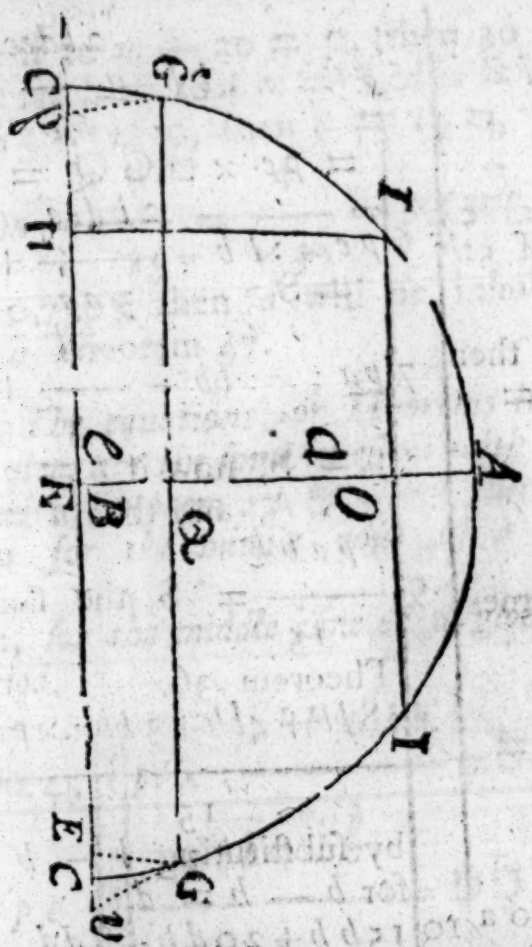
$= \frac{2,0-2}{1-5} = \frac{0}{5} = 0$

And if $1 - 2mm = 0$, then $1 = 2m$ and $m = \sqrt{\frac{1}{2}} =, 707$ &c. so when $m = 0,707$ &c. the denominator $1 - 2m^2$, will be $= 0$ and so d will be infinite in respect of z, Theo. 46.

Again for ICCI the half zone of a parabolic spindle.

Put $a =$ Bo half its length $b =$ BC the radius of the greater circle, or axis of the generating parabola) $h =$ Io the radius of the lesser base, D, m, and d the same as before; then,

per



per figure,	1	BC (b) − I o (h) = d = nC
per Prop. para.	2	□ nI (aa) : nC (d) : : □ Gg
		dee
		$(ee : \frac{dee}{aa}$ = Cg
		aa
per figure	3	BC − Cg = $b - \frac{dee}{aa}$ =
		GQ
		$2bdee$ $ddeeee,$
3 squared	4	$bb - \frac{2bdee}{aa} + \frac{ddeeee}{aaaa}$ = □
		GQ

4 × *pe*

$6 - zz$ | 7 | $zd + \frac{dd}{2} = 2zdm + ddmm,$

$7 \pm, \div d -$ | 8 | $4zm - 2z = d - 2dmm$

$8 \div 1 - 2m^2$ | 9 | $d = \frac{:4m-2:\times z}{1-2mm}$ hence if m

$= \frac{5}{10}$ then $d = \frac{4mz - 2z}{1 - 2m^2}$

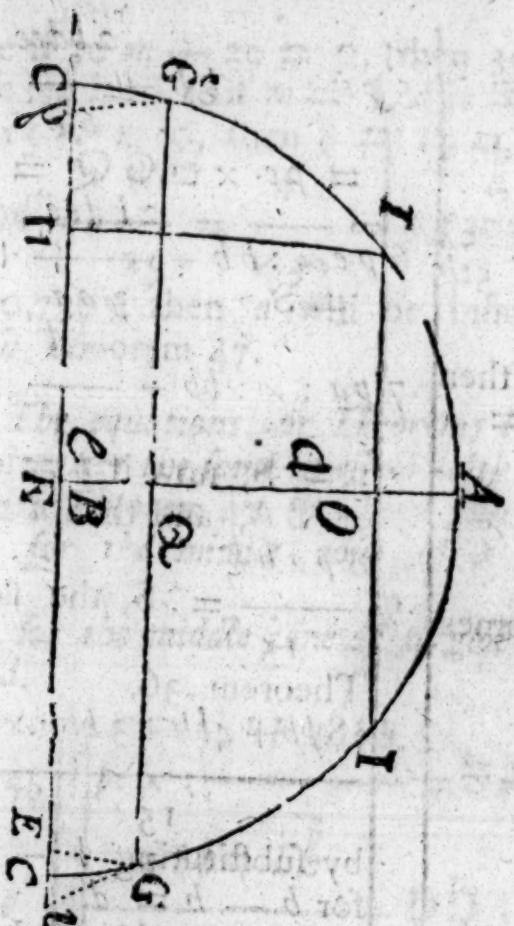
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And if $1 - 2mm = 0$, then $1 = 2m$ and $m = \sqrt{\frac{1}{2}} =, 707$ &c. so when $m = 0,707$ &c. the denominator $1 - 2m^2$, will be $= 0$ and so d will be infinite in respect of z, Theo. 46.

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per



per figure, 1 BC (b) − I o (b) = d = nC

per Prop. para. 2 □ nI (aa) : nC (d) : : □ Gg

$$(ee : \frac{dee}{aa} = Cg$$

per figure 3 $BC - Cg = b - \frac{dee}{aa} =$
GQ

3 squared 4 $bb - \frac{2bdee}{GQ} + \frac{ddeeee}{aaaa} = \square$
aa

4 × pe

$4 \times pe$ | 5 $pe : \times : bb - \frac{2bdee}{aa} + \frac{dde^4}{a^4}$
$= pe \times \square GQ = S$

Fluent of 5th | 6 $pe \times : bb - \frac{2bdee}{3aa} + \frac{dde^4}{5a^4}$
$= S$

If $e = a$ then 6th $=$ | 7 $pa : \times : bb - \frac{2bd}{3} + \frac{dd}{5} :$
$: = S$, now if $h = o$ then $o = BA$, and then $d = o$.

so 8 becomes | 9 $\frac{8\,pa\,bb}{15} = S$, the same with Theorem 36.

But 7 $=$ | 8 $\frac{: 8bb + 4bh + 3hh : \times pa :}{15} = S$
by substituting $b - h$ for d, for $b - h = d$,

7 reduc. to a com. denomi. | 10 $\frac{15hh + 20dh + 8dd}{15} = DD$
(by writing $h + d$, for b) $=$
$h + dm$

That is, | 11 $15hh + 20dh + 8dd = 15hh + 30hmd + 15d.^2 m.^2$

11 $- 15h^2 \div d$ | 12 $20h + 8d = 30hm + 15dm^2$
12 $- 15dm - 20h$ | 13 $30hm - 20h = 8d - 15dm^2$
13 $\div 8 - 15m^2$ | 14 $\frac{30mh - 20h}{8 - 15mm} = d$

Hence,

Hence if $30m - 20 = 0$, then $30m = 20$ and $m = \frac{20}{30} = \frac{2}{3}$, ſo if $m = \frac{2}{3}$, d is $= 0$, again if $8 - 15m^2 = 0$, then $8 = 15m$, and $m^2 = \frac{8}{15}$, ſo $m = \frac{\sqrt{8}}{15} = \frac{4}{\sqrt{30}} = 0{,}7303$, ſo that if $m = 0{,}7303$, then d will be infinite in reſpect of h Theorem 47.

Note, *The equations, or Theorems in the* 7th *and* 8th *ſteps are the ſame in effect with Theo.* 42 *ſo that with Theorem* 47, *you have four different Theorems for the middle zone of a parabolic ſpindle.*

Again, *for the middle zone of a globe, ſphere, or ſpheriod.*

per Theo. 45	1	$pe \times : \frac{2bb + hh}{3} = S = pe\,DD$;
$1 \div pe$	2	$\frac{2bb + hh}{3} \cdot = DD$
by what goes before,	3	$h + d = b$, Ergo $hh + 2hd + dd = bb$,
	4	$h + d\,m = D$, ſo, $hh + 2hd\,m + dd\,m^2 = DD$,
2 and 3	5	$h + \frac{4dh + 2dd}{3} = DD$,
$4 = 5$	6	$h.^2 + 2hd\,m + d.^2 m.^2 = h.^2 + \frac{4dh + 2dd}{3}$, which being

$4 \times pe$ | 5 $pe : \times : bb - \frac{2bdee}{aa} + \frac{dde^4}{a^4}$
$= pe \times \square GQ = S$

Fluent of 5th | 6 $pe \times : bb - \frac{2bdee}{3aa} + \frac{dde^4}{5a^4}$
$= S$

If $e = a$ then 6th $=$ | 7 $pa : \times : bb - \frac{2bd}{3} + \frac{dd}{5} :$
$: = S$, now if $h = o$ then $o = BA$, and then $d = o$.

so 8 becomes | 9 $\frac{8\,pabb}{15} = S$, the same with Theorem 36.

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by substituting $b - h$ for d, for $b - h = d$,

7 reduc. to a com. denomi. | 10 $\frac{15hh + 20dh + 8dd}{15} = DD$
(by writing $h + d$, for b) $=$
$h + dm$

That is, | 11 $15hh + 20dh + 8dd = 15hh + 30hmd + 15d.^2 m.^2$

11 $- 15h^2 \div d$ | 12 $20h + 8d = 30hm + 15dm^2$

12 $- 15dm - 20h$ | 13 $30hm - 20h = 8d - 15dm^2$
$30mh - 20h$

13 $\div 8 - 15m^2$ | 14 $\frac{30mh - 20h}{8 - 15mm} = d$

Hence,

Hence if $30\,m - 20 = 0$, then $30\,m = 20$ and $m = \frac{20}{30} = \frac{2}{3}$, ſo if $m = \frac{2}{3}$, d is $= 0$, again if $8 - 15\,m^2 = 0$, then $8 = 15\,m^2$ and $m^2 = \frac{8}{15}$, ſo $m = \frac{\sqrt{8}}{15} = \frac{4}{\sqrt{30}} = 0{,}7303$, ſo that if $m = 0{,}7303$, then d will be infinite in reſpect of h Theorem 47.

Note, *The equations, or Theorems in the* 7th *and* 8th *ſteps are the ſame in effect with Theo.* 42 *ſo that with Theorem* 47, *you have four different Theorems for the middle zone of a parabolic ſpindle.*

Again, *for the middle zone of a globe, ſphere, or ſpheriod.*

per Theo. 45	1	$pe \times : \frac{2bb + hh}{3} = S = pe\,DD$;
$1 \div pe$	2	$\frac{2bb + hh}{3} \cdot = DD$ —
by what goes before,	3	$h + d = b$, Ergo $hh + 2hd + dd = bb$,
	4	$h + d\,m = D$, ſo, $hh + 2hd\,m + dd\,m^2 = DD$,
2 and 3	5	$h + \frac{4dh + 2dd}{3} = DD$,
4 = 5	6	$h.^2 + 2hd\,m + d.^2 m.^2 = h.^2 + \frac{4dh + 2dd}{3}$, which being

wrought with as before we get $d = \frac{6\,m\,h - 4\,h}{2 - 3\,m.^2}$

Hence if $6\,m - 4 = 0$, then $6\,m = 4$ and $m = \frac{4}{6} = \frac{2}{3}$ so that if $m = \frac{2}{3}$ d is $= 0$. Again, if $3\,m^2 = 2$, then $m = \sqrt{\frac{2}{3}} =, 8166$, so if $m = 0{,}8166$ then d should be infinite in respect of h otherwise the multiplier will not be exact, Theo. 48,

From Theorems, 25, 46, 47, and 48; the limits of the multipliers, for all the common varieties of casks are found.

viz.	1	variety	,5		,5774
for	2	is	,5	and	,7072
the	3	between	,6666		,7303
	4		,6666		,8166

From which you may see that the difference of those multipliers, is not so much owing to different curvature of the sides, as to the greater and lesser difference between the head and bung diameters.

NOTE, *In the foregoing Theorems I have used the* $\frac{1}{2}$ *diameter of each base, but if you suppose those* $\frac{1}{2}$ *diameters to be whole ones the Theorems will be the very same, only* p *must be* $= 0{,}7854$ (viz. $= \frac{3{,}1416}{4}$,

Also, *Since finding the true content of a cask requires to know its true form, it may not be amiss to shew in this place how the form of a cask may be known in order to which, let* b = CC, h = II, a = oB, *half the casks length* m = GG,

a diameter taken in the middle between the head (II) *and bung* C C, *diameters in the forgoing figure, now if the cask be of the firſt form, viz. of two equal conical fruſtums, then its form is eaſy known, for the ſtaves are ſtreight from head to bung.*

for the 2d variety,	1	$ce = hh$, $cc + \frac{1}{2} ca = m^2$ and $ce + ca = bb$, ſee ſtep 1, Prob. 167
third variety	2	$aa : b - h :: \frac{1}{4} aa : \frac{1}{4} b - h =$ Cg *per* property of the curve
fourth variety	3	$tt = \frac{aabb}{bb - hh}$ and $tt = \frac{\frac{1}{4} aabb}{bb - m^2}$ Step 8 Theo. 48, $t = \frac{1}{2}$ diameter ellipſis.
from 1 ſtep	4	$m.^2 = \frac{b^2 + h^2}{2}$ Theorem 49.
2,	5	$m = \frac{3b + h}{4}$ Theorem 50.
3,	6	$m.^2 = \frac{3b.^2 + h.^2}{4}$ Theo. 51.

And becauſe the peripheries of circles are as their diameters you may as well take, b, h, and m; for the circles, at the Bung-head, and midway between the Bung and head, of the cask. And then by taking thoſe circles (*viz.*) by girding the cask at the head and bung you will have the values of h, and b; by which and the three laſt Theorems, you may find m, the circle in the middle between the head, and bung, and alſo

gird

gird the cask in the ſaid middle by which, you may ſoon ſee what form it is neareſt and ſo be able to gauge it exactly.

Alſo, by writing z for $\frac{1}{2}a$, the firſt 3 foregoing ſteps become;

7 $m.^2 = hh + \frac{bb - hh \times zz}{aa}$ Theo. 52.

8 $m = b - \frac{b + h}{aa} \times zz$ Theo. 53,

9 $m.^2 = bb - \frac{bb + hh \times zz}{aa}$ Theo. 54,

These three laſt Theorems are for ullaging casks that ſtand perpendicular to the horizon &c.

To find the curve ſurface of any ſpheriod or globe.

1 $y = \frac{c}{a} \sqrt{aa - ee}$ Step 1 foregoing Theorem 38,

1 in Fluxions, 2 $y\cdot = \frac{-cee}{a\sqrt{aa - ee}}$

2 ſquared 3 $y\cdot y\cdot = \frac{cc\,e\,e\,e\cdot e\cdot}{a^4 - a^2 e^2}$

3 + $e\,e\cdot \times 2p$ 4 $2py = \sqrt{e\,e\cdot} + \frac{cc\,e\,e\,e\cdot e\cdot}{a^4 - a^2 e}$

$= 2py\sqrt{e\,e\cdot} + y\cdot y\cdot = C\cdot$

$C =$

4 reduc'd & the Fluent taken

$$5\ C = 2pce \times 1 - \frac{bbee}{6a4} + \frac{b^4 e^4}{4a^8} - \frac{3b^6 e^6}{336a^{12}}\ \&c;$$

here, $b = a^2 - cc$,

For the curve ſuperficies of the middle zone of any ſpheriod, but when $a = c$ then $b = o$, then the ſpheriod becomes a globe, and the laſt ſtep will become, $C = 2pae$, the curve ſuperficies of the zone of any globe, and if $e = a$ then $2pae$ becomes $2paa$ Theo. 55,

For the curve ſurface of any globe or ſphere hence we have alſo another Theorem for the ſolidity of a globe, viz. $2paa\ (= C) \times \frac{1}{3}a = \frac{1}{3}paaa = S$ (as *per* Theo. 38) Theo. 56.

Note, *The periphery of the ellipſis curve of the parabola ſurface of the parabolic conoid* &c. *With the higher curves as the hyperbola* &c. *being ſeldom called in queſtion in common menſurations, I have declined them, hoping the following Problem will clear up all.*

PROB.

PROBLEM CLXIX.

To Meaſure any PLANE *or* SOLID *by* APPROXIMATION.

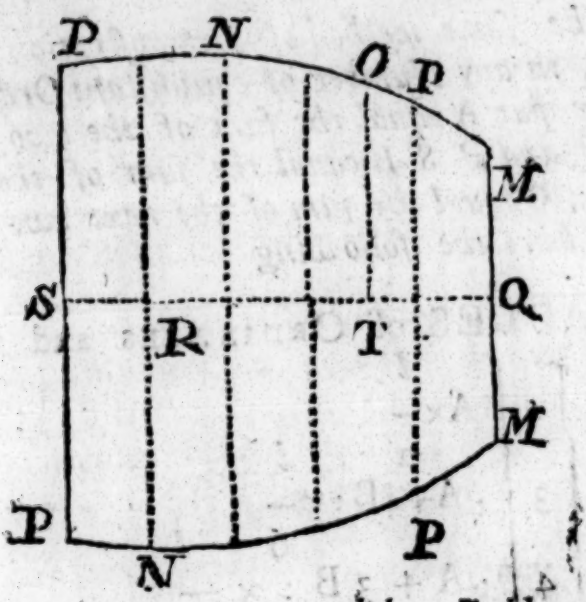
IN order to make this plain, Let us find the area of that curve, QMPS, whoſe equation is $y = a + eb + cee$ where e ſtands for any abſciiſs QT, and y its ordinate OT; then putting l = QS the diſtance of the extream ordinates, we ſhall have $ye^{\cdot} = ae^{\cdot} + ee^{\cdot} b + eee^{\cdot}c$ (*per* Prob. 167) whoſe fluent is $ye = A = ae + \frac{1}{2} be^{\cdot 2} + \frac{1}{3} ce^{\cdot 3}$ and when $l = e$, then $A = a + \frac{1}{2} bl + \frac{1}{3} cll \times l =: 6a + 3bl + 2cl^{\cdot 2} \times \frac{l}{6}$

But from the Equation of the curve we have theſe three following Equations:

y^1, y^2 y^3 being 3 equi-diſtant Ordinates

(viz.)
$$\left\{\begin{array}{l} 1\, y_1 = a = \mathrm{MQ}, \\ 2\, y^2 = a + \frac{1}{2} bl + \frac{cll}{4} = \mathrm{NR} \\ 3\, y^3 = a + bl + cll = \mathrm{PS} \end{array}\right.$$

4, y^2



2 − 1	4	$y^1 - a = \frac{bl}{2} + \frac{cll}{4}$
3 − 2	5	$y^3 - y^2 = \frac{bl}{2} + \frac{3cll}{4}$
5 − 4	6	$y^3 - 2y^2 + a = \frac{cll}{2}$
6 × 4	7	$4 : \times y^3 - 2y^2 + a : = 2cl^2$
3 − *a* − *cl*²	8	$y^3 - a - cl^2 = bl$
per 7 and 8	9	$3y^3 - 3a - 6 \times : y^3 - 2y^2 + a : = 3bl = -3y^3 + 12y^2 - 9a,$
per the above	10	$A = : 6a + 3bl + 2cl^2 : \times \frac{l}{6}$ = area QMPS
7, 9, 10, 11		$: y^4 + 4y^2 + y^3 : \times \frac{l}{6} =$ QM

PS, Theorem 57,

the

The same method of Demonstration also holds true, in any Number of equidistant Ordinates; so if we put A equal the sum of the two Ordinates M Q and P S B equal the sum of those next to them; C equal the sum of the next two &c. we shall have the following

TABLES of ORDINATES and AREAS

Table

2	$A \times \frac{l}{2}$
3	$: A+4B : \times \frac{l}{6}$
4	$: A+3B : \times \frac{l}{8}$
5	$: 7A+32B+2C : \times \frac{l}{90}$
6	$: 19A+75B+50C : \times \frac{l}{288}$
7	$: 41A+216B+27C+272D : \times \frac{l}{840}$
8	$: 751A+3577B+1323C+2989D : \times \frac{l}{17280}$ &c.

Now (in the figure to *Theorem* 11) if b=AB, B=CD and l=BP=AB: Then $\frac{B+b}{2} = 2A$, so $A \times \frac{l}{2}$ will become $B+b \times \frac{l}{2}$ the very same with

Theorem 11. If b=B then B+b=B+B=2B=A

So

So $A\times\frac{l}{2}=2B\times\frac{l}{2}$ the very ſame as Theorem 7

If b or $B=o$, then $B+o=B=A$; ſo $A\times\frac{l}{2}=\frac{1}{2}Bl$, the very ſame with Theorem 9.

But if you would meaſure a Curve-lin'd ſpace or figure by this method, and you do not know its property; Firſt draw a line QS which meaſure and perpendicular thereto, meaſure as many Ordinates as you pleaſe (the more the nearer you come to the true area): The two following Examples will in part ſhew the excellency of this method. See Mr: *Shirtcliffe's* GAUGING page 186

EXAMPLE I.

Let it be requir'd to Square *viz.* to find the Area of a Curve whoſe Equation is

$y=\frac{1}{1+e}$; That is $\frac{1}{1+e}$ is the Ordinate to every Abſciſſa e.

By Prob: 167. The area is found $e-\frac{1}{2}e^2+\frac{1}{3}e^3-\frac{1}{4}e^4+\frac{1}{5}e^5$ &c But by the foregoing Method let $e=1$, which divided into ſix equal parts, we will have the ſeven following equi-diſtant Ordinates,

$\frac{1}{1+0}$ $\frac{1}{1+\frac{1}{6}}$ $\frac{1}{1+\frac{2}{6}}$ $\frac{1}{1+\frac{3}{6}}$ $\frac{1}{1+\frac{4}{6}}$ $\frac{1}{1+\frac{5}{6}}$ $\frac{1}{1+\frac{6}{6}}$

$$\frac{6}{7}\quad\frac{6}{8}\quad\frac{6}{9}\quad\frac{6}{10}\quad\frac{6}{11}\quad\frac{6}{12}$$

So by the foregoing Tables

We have	1	$A = 1 + \frac{6}{12} = 1,5$
and	2	$B = \frac{6}{7} + \frac{6}{11} = 1,4025974$
alſo	3	$C = \frac{6}{3} + \frac{6}{10} = 1,35$
Likewiſe	4	$D = \frac{6}{9} + 0 = \frac{2}{3} = ,6666666$
1×41	5	$41\,A = 61,5$
2×216	6	$216\,B = 302,9610384$
3×27	7	$27\,C = 36,45$
4×272	8	$272\,D = 181,3333333$
$5+6+7+8$	9	$582,2443717$
$9 \times \frac{l}{840}$	10	0,6931479 the Area required (*Note* per Ex. $l = e = 1$)

To have got the area of this Curve by the above Series $e - \frac{1}{2}e^2 + \frac{1}{3}e^3$ &c. would require 1000000 Terms of the ſaid Series to have it in ſeven places of figures as it is turned out by this Method.

Note, This Area is the ſame with the Hyperbolic Logarithm of $1+e$, ſo if $e=t$ then $1+e=2$ hence the Hyperbolic Logarithm of 2 is equal ,6931479. In like manner if you would have the

the Hyperbolic Log. of 3, you muſt make $e=2$ and work as above &c. for any other number.

EXAMPLE II.

What is the Area of a Curve whoſe Equation is

$$t=\frac{1}{1+t^6}$$

Let t equal 1 and ſuppoſe it divided into ſix parts, and we ſhall have theſe ſeven equidiſtant Ordinates,

$$\frac{1}{1+0},\ \frac{1}{1+\frac{1}{36}},\ \frac{1}{1+\frac{8}{36}},\ \frac{1}{1+\frac{27}{36}},\ \frac{1}{1+\frac{64}{36}},\ \frac{1}{1+\frac{125}{36}},\ \frac{1}{1+\frac{216}{36}},\ \text{that is}$$

$$1,\ \frac{36}{37},\ \frac{36}{40},\ \frac{36}{45},\ \frac{36}{52},\ \frac{36}{61},\ \frac{36}{72}.$$

Hence we have

1 $A=1+\frac{36}{72}=1{,}5$

2 $B=\frac{36}{37}+\frac{36}{61}=1{,}563137$

3 $C=\frac{36}{40}+\frac{36}{52}=1{,}192308$

4 $D=0+\frac{36}{45}=0{,}8$

1	42 A = 61,5
2	216 B = 337,637592
3	27 C = 42,992316
4	272 D = 2176
	659,729908
5	785393 (erring but 5 in the last place) the Area ſought.

So *per* Table

$$5 + \frac{1+2+3+4}{840}\,l$$

NOTE $\frac{1}{1+tt}$ expreſſeth the length of a Circular arch whoſe radius is 1 and tangent t, ſo if $t = \frac{1}{1} = 1$, t is equal Tangent of 45^{0}; Conſequently 0,785393 is the length of a $\frac{1}{8}$ Quadrant whoſe diameter is 1, the ſame as of the Circle before *Theorem* 27.

The ſame Theo 57 extends to ſolids, for it is plain by the proceſs foregoing the ſaid Theo that there is no difference whether $y, {}^{1}y, {}^{2}y$, repreſent ordinates or ſpaces, therefore if the curve or ſpace QMPS revolve about QS, it will deſcribe a ſolid whoſe ſolidity is expreſs'd by $yy' + 4yy^{2} + yy^{1} \times \frac{lp}{6}$, The ſame alſo holds true in the tables by ſuppoſing A, B, C &c. to denote ſpaces or areas inſtead of lines &c.

Therefore if we put A = the area at the greater baſe a equal the area at the leſſer baſe and m = the area in the middle (parallel to each baſe of any conical, or pyramidical fruſtum or of any priſmoid &c. whether the baſes be ſimi-

la

lar or not provided they be parallel, and the solid streight lin'd from base to base; for its solidity we have this general Theoeem 58, *viz.*

$: A + a + 4m \times \frac{l}{6} = S$, l being equal the perpendicular length of the solid.

So if A = the area of the greater base, and a equal the area of the lesser base of the lower frustum of any cone or pyramid; then $\surd A$ = the side of a square at the greater, and $\surd a$ equal the side of a square at the lesser base, and by the property of the solid, $\frac{\surd A + \surd a}{2} = \surd m$ the side of a square in the middle between the two bases therefore $m = \frac{A + 2\surd A \times \surd a + a}{4}$, which being put in the *Theo.* 58 instead of m we have $2A + 2\surd Aa + 2a \times \frac{l}{6} = \frac{A + \surd Aa + a}{3} \times l$ the very same with Theorem 21.

If A equal the area of the base of any cone or pyramid and l equal its axis, then a will be equal 0; so the last equation will become $\frac{1}{3} Al$ the same with *Theorem* 18. Again, if there be such a prismoid as that $D \times B$ and $d \times b$ denotes the area of its two parallel bases, and D be parallel to d, also B parallel to b: Then will $\frac{B+b}{2} \times \frac{D+d}{2} = m$ denotes the area in the middle between and parallel to

to the two baſes, Hence DB = A, $db = a$ and $\frac{BD+Bd+bD+bd}{4} = m$, which ſubtituted in *Theorem* 58, for A, *a*, and *m* we ſhall have,

$2BD+Bd+bD+2bd : \frac{l}{6} = :BD+\frac{1}{3}Bd+\frac{1}{3}bD+bd : \times\frac{1}{3}l$

Theorem 59.

☞ *Alſo from Theo.* 58, *we have a* GENERAL *one for* Gauging *all ſorts of* Casks, *and alſo for* Ullaging *the ſame without any regard to their Forms* &c.

For let *l* = the length of the Cask, *h* = the Head diameter, *b* = the Bung diameter, *m* = a diameter taken in the middle betwixt Bung and Head, *p* = ,7854; Then we have *Theorem* 60, $bb+hh+4mm : \times\frac{lp}{6} = s$, nearly equal the Solidity of any Cask whatever. For by what goes before, this *Theorem* is prov'd to equal *Theorem* 21, and therefore holds true in the firſt form of Casks.

For the ſecond variety *per Theorem* 49, $m^2 = \frac{hh+bb}{2}$ therefore $: h^2+b^2+\frac{4h^2+4b^2}{2} : \times\frac{lp}{6}$ $= s = \frac{h^2+b^2}{2} : \times lp$ the very ſame with *Theo.* 41.

For the fourth Form *per Theorem* 51, $m^{\overline{2}} = \frac{3b^{\overline{2}}+h^{\overline{2}}}{4}$, which being put in *Theorem* 60, inſtead of $m^{\overline{2}}$ we ſhall have: $h^{\overline{2}}+b^{\overline{2}}+\frac{12b^{\overline{2}}+4h^{2}}{4} : \times \frac{lp}{6}$ $= s = \frac{1}{2} lp : \times : 2bb+hh$ the very ſame with *Theorem* 45.

Thus you ſee theſe three caſes agree exactly with truth, but the fourth form deviates a ſmall matter from truth, as appears *per* following tryal. *per* Theorem 50, $m = \frac{3b+h}{4}$ *Ergo* $m^{2} = \frac{3b+h|^{2}}{4}$ which put for $m.^{2}$ in *Theo.* 60, and we have $b^{2} + h^{2} + 4 \frac{3b+h|^{2}}{4} \times \frac{pl}{6} = S =$ $13bb + 6bh + 5h^{2} \times \frac{lp}{24}$ which is too much by $b - h^{\overline{2}} \times \frac{pl}{120}$, which can never amount to $\frac{1}{16}$ of a gallon in gauging any cask to be met with in practice. But this ſmall error will be rectified, if we ſuppoſe another fruſtum exactly equal to this and joined to it then we ſhall have the value of five equi-diſtant diameters of this ſolid, the extreme ones being $h\,h$, the next to theſe $m\,m$, and the middle one b; whence by the table for five ordinates we have: $7A + 32B +$

12

12 C. $\times \frac{pl}{6}$ wherein $A = ah^2 + ah^2 = 2ah^2$

$B = am^2 + am^2 = a \times \left| \frac{3b+h}{3} \right|^2$ and $C = ab$

which put for AB and C; we have : $14h^2 + \frac{32}{3} \times \left| 3b+h \right|^2 + 3h^2 : \times \frac{pl}{90} = : 2b^2 + h^2 - \left| b^2 - h \right|^2 \times \frac{pl}{3}$ exactly the same with *Theorem* 42.

By the same method you may also ullage any lying cask, for let h denote the area of the segment at the head, b that at the bung and m that at the middle between the head and bung, l the length of the cask, then we shall have : $b + h + 4m : \frac{1}{6} l$ equal the ullage *Theorem* 61.

This famous *Theorem* 58, still extends further as to Hoofs of all kinds, Segments of Globes Spheriods, Parabolic, Conoids, Spindles &c. however crosly cut.

For let d equal the Side of the lesser Base, D equal the Side of the greater Base of the greater Hoof of any square Pyramid; Then $DD = \frac{D+d}{2} \times \frac{DD+Dd}{4} = m$, (*Theo.* 58) and $\frac{1}{2}D \times \frac{D+d}{2} = \frac{DD+Dd}{4} = m$, Hence $A+m4+a : \times \frac{l}{6}$ becomes $DD + \frac{1}{2}Dd : \times \frac{1}{3} l$, *Theorem* 62. l being equal the Perpendicular height

of the Hoof, and $a=o$ becauſe the Area at the leſſer Baſe of the Hoof is equal o, but this Theorem does not hold in Conical Hoofs, becauſe $\frac{1}{2}D : x : \frac{D+d}{2}$ is not equal m or the Area in the Middle: And therefore Mr. WARD in his *Mathematical Guide* expreſſeth this *Theorem* 62. Thus, for Conical Hoofs, $:DD+\frac{1}{3}Dd+D-d : x\frac{1}{3}lp$ equal the Solidity of the greater Hoof *Theo.* 63, $:dd+\frac{1}{3}Dd+d-D : x\frac{1}{3}lp$ equal the Solidity of the leſſer Hoof *Theorem* 64. $D=\frac{1}{2}$ diameter of the greater Baſe, and $d=\frac{1}{2}$ diameter of the leſſer Baſe which Theorms are pretty near good, tho' not quite ſo exact as Theorems 23 and 24.

I ſhall go no farther to prove this UNIVERSAL *Theorem* 58, by the property of the Figure, ſince in Practice you can meet with no Figure to Meaſure, but you may take as many demenſions as you pleaſe; and ſo find the Content thereof by the foregoing TABLES of Ordinates and Areas, or Spaces and Solidities.

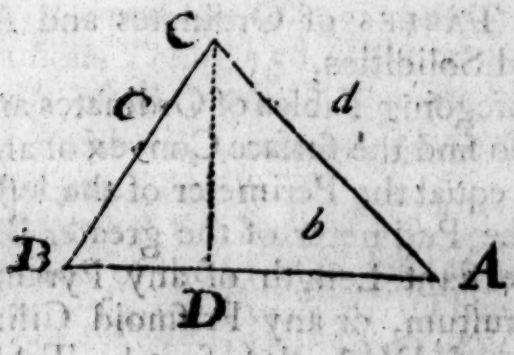
By the foregoing Tables of Ordinates and Areas we may alſo find the ſurface Convex of any Solid. For, let d equal the Perimeter of the leſſer Baſe, D equal the Perimeter of the greater Baſe, and l equal the ſlant Length of any Pyramidic, or Conical Fruſtum, or any Priſmoid Cilinderoid, &c. Then $d+D=A$, therefore by Table 2, $\frac{l}{6}A$ will become $:D+d : x\frac{1}{2}l$, *Theorem* 65, for the Curve ſuperficies of any ſuch Solid.

Again, ſuppoſe you meet with a Solid whoſe property you do not know, yet judge it to be ſomewhat like a Parabolic Conoid, and you are to find its Curve ſuperficies.

1. Take 3, 4, 5 *&c.* Girths as you think fit equally diſtant from each other, uppoſe 4; call the ſum of the two outermoſt A, and the ſum of the next two call B, and *l* the length of the Curve between the two outermoſt Girths: Then *per* Table 4th, we have $A+3B\times\frac{1}{8}l$ for the Curve Surface required.

PROBLEM CLXX.

Having given the three Sides b, c, *and* d, *of any Plane Triangle* ABC, *to find its Area* (a) *without the Perpendicular* CD.



Per 4. Axiom of plane Trigonometry

1. $b : d+c :: d-c : \frac{dd-cc}{b} = AD-DB$

2. $\frac{1}{2}b + \frac{dd-cc}{2b} = \frac{bb+dd+cc}{2b} = AD$

per Theorem 2. 3. $dd - \left|\frac{bb+dd-cc}{2b}\right|^2 = \square DC$

4. $4^{\overline{1}} \, 2 \times \frac{b}{2} \sqrt{:2b^{\overline{2}}d^{\overline{2}}+2b^{\overline{2}}c^{\overline{2}}+c^{\overline{2}}d^{\overline{2}}-b^{\overline{4}}-c^{\overline{4}}-d^{\overline{4}}} = a$ which is = the *Square-Root* of $\frac{-b+c+d}{2} \times \frac{b-c+d}{2} \times \frac{b+c-d}{2} \times \frac{b+c+d}{2}$ *Theo.* 66.

PROBLEM CLXXI.

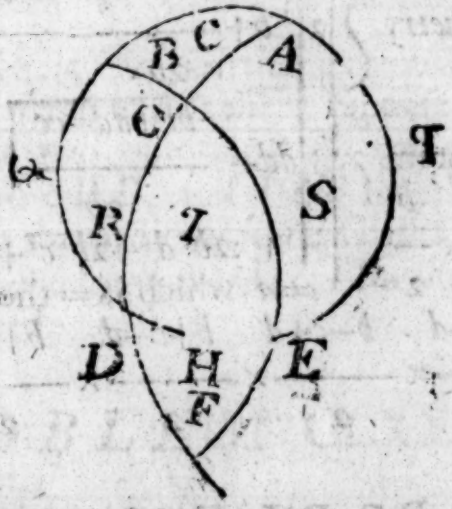
Given the three Angles A, B, *and* C, *of any Spherical Triangle* ABC, *to find its Area.*

There are ſeveral ways to do this, but that which plaineſt is the following one: And firſt *Note*,

If the Meridian Circle ARD be mov'd equally round the Globe, upon the Poles A and D, it will deſcribe equal ſpaces in equal Times, and therefore thoſe ſpaces muſt be as the Angles at the Poles; that is, putting D equal the Axis of the Sphere, and $p = 3,1416$ we have pDD equal the Surface of the whole Sphere (Theo. 55) Hence $360^\circ : p$DD :: ∠A or ∠D : ſpace AQD, put G = △ABC which is alſo equal Triangle EFD.

= H

=H, becauſe the Sides and Angles in ABC ar
reſpectively equal to thoſe in EFD, R equal ſpac
CBQD, F=CED, and S=ACEP, Then by th
fore oing



Note,	1	180 : $\frac{1}{2}p$DD :: A : G+R
	2	180 : $\frac{1}{2}p$DD :: B : G+S
	3	180 : $\frac{1}{2}p$DD :: C : G+T=H+T
1+2+3	4	3×180 : $\frac{1}{2}p$DD :: A+B+C : 3G+R +S+T
4÷3	5	180 : $\frac{1}{2}p$DD :: A+B+C : 3G+R +S+T
5−180	6	180 : $\frac{1}{2}p$DD :: A+B+C−180 : 3G +B+S+T−180
But *p*:Fig.	7	G+R+S+T=$\frac{1}{2}p$DD
Therefore	8	3G+R+S+T−$\frac{1}{2}p$DD=2G
	9	720 : A+B+C−180 :: 2*p*DD : 2G
And		:: *p*DD : G by quadrupling th

antecedent Terms; Hence we have.

$$G=\frac{A+B+C-180 : \times pDD}{720}$$ *Theorem* 67.

Now

Now becauſe all Polygons may be reduc'd into Triangles; Let *s* equal the Sum, and *n* equal the number of Angles of any ſpherical Polygon, of what ordinate or inordinate Figure ſoever it be, we ſhall have this Theorem for its Area,

(viz.) $\frac{360+s-180n}{720} \times p$ DD *Theorem* 68.

PROBLEM CLXXII.

Required the neareſt Ratio in Integer Terms, of 1 *to* 3,14159265358 98 viz. *of the Ratio of the Diameter of a Circle to its Circumference.*

The neareſt whoſe numbers in the leaſt Terms anſwering this Ratio, or 1 to 3, and 1 to 4, but 1 to 3 is too little by 0,14159 &c. and 1 to 4 is too much by 0,85840 &c. Therefore work as follows,

	1	1 ; 4—,8584073464102 +
	2	1 : 3+,14159265358 98—
		1ſt. *Caſe.*
	3	14159 &c.)85840 &c.(6
2×6	4	6 ; 18+,84955592153 88
4+1	5	7 : 22—,00885142487 14
		2d *C.*
	6	,00885&c.),14159&c.(15+
5×15	7	105 : 330—,13277137307 10
7+2	8	106 : 333+,00882128051 88
		3d. *C.*
	9	,00882 1&c),00885 1&c.(1+
3×1 &+5	10	113 : 355—,00003014435 26
		4th. *C.*
	11	,00003014&c.),00882 1&c(292+
10×292	12	32996 : 103660—,00880215095 92
12+8	13	33102 : 103993+,00001912955 96

Caſe 5th, &c. &c &c. From

From hence it appears that the Ratio of a Circle's diameter to its circumference can never be exactly expreſs'd in whole Numbers: And that the neareſt Ratio's in whole Numbers are, 1 to 4, or nearer as 1 to 3,

Or nearer, as 7 to 22, *Archimedes*'s Ratio,
Or nearer, as 106 to 333,
Or nearer, as 113 to 355, *Metius*'s Proportion,
Or nearer, as 33102 to 103993, *&c.*

NOTE, The Fraction in the ſecond Sep being nearer the truth than thoſe in the firſt Step, the ſaid ſecond Step is call'd the firſt Caſe, (viz.) the firſt continual increment; and ſo the Eraction in the firſt Step is divided by that in the ſecond Step, and then working as the Steps directs you have the ſecond Caſe in the fifth Step: Then divide the Fraction of the firſt Caſe by the Fraction of the ſecond Caſe *&c.* and you'll have the third Caſe in the 8th Step; then divide the Fraction of the ſecond Caſe by the Fraction of the third Caſe *&c.* ſtill dividing no longer than the Quotient is whole Numbers, ſee the foregoing work.

This PROB. *I have put in here that my Readers may want reaſon for nothing that I make uſe of in the next Part, ſee more of this in Mr.* Ronayne's *Algebra, from page* 428 *to* 433.

How

How to Compute LOGARITHMS.

PROBLEM CLXXIII.

SINCE the construction of the Sliding-Rule depends on a Table of Logarithms and such Tables being very common, I presume it will be as well taken if I shew the nature and construction of Logarithms, as to give a Table thereof which would take up so much room, that I might omit something more material.

To make this Problem plain it will be necessary to premise these Definitions.

I. That the sum of the Logarithms of any two Numbers is equal to the Log. of the Product of these two Numbers

II. The difference between the Logarithms of any two Numbers is equal to the Logarithm of the Quotient of one Number divided by the other.

III. If the Log. of any any Number be divided by the Index of any Root the Quotient will be the Log. of that Root.

IV. That is, what natural Numbers does by *Multiplication*, *Division* and *Extraction* of *Roots* Logarithms performs the same by *Addition Subtraction* and *Division*.

LEM-

LEMMA.

If from each of the indefinite Roots two Numbers an Unit be taken, the ſum of the two Remainders, will be equal the to the Product of the ſaid two indefinite Roots leſſen'd by 1; or which is the ſame in effect, if from the indefinite Root of any Number you ſubtract 1, I ſay the double of the remainder will be equal to the ſquare of the ſaid indefinite Root minus one: That is if $1+e$ repreſent any finite Number, and n an indefinite Number, then we are to prove that $\overline{1+e}\,|\,\overline{n-e\times 2}^{1}$ is equal to $\overline{1+e}\,|\,\overline{n}^{1}\times\overline{1+e}\,|\,\overline{n}^{1}-1$, That is, that $2:\overline{1+e}\,|\,\overline{n}^{1}-2=\overline{1+e}\,|\,\overline{n}^{2}-1$.

First by Evolution we ſhall have

$$1\,\Big|\,2:\overline{1+e}\,|\,\overline{n}^{1}=2+\frac{2}{n}e+\frac{2}{n}\times\tfrac{1}{2}\frac{2}{n}-1\,ee+\frac{2}{n}\times\tfrac{1}{2}\frac{2}{n}-1\times\tfrac{1}{3}\frac{2}{n}-2\,e^{3}\ \&c.$$

$$2\,\Big|\,\overline{1+e}\,|\,\overline{n}^{2}=1+\frac{2}{n}e+\frac{2}{n}\times\tfrac{1}{2}\frac{2}{n}-1\,ee+\frac{2}{n}\times\tfrac{1}{2}\frac{2}{n}-1\times\tfrac{1}{3}\frac{2}{n}-2\,e$$

$\overline{1-2}$

$$
\begin{array}{lll}
1-2 & 3 & 2 : 1+e \mid \frac{2}{n} - 2 = \frac{2}{n}e + \frac{2}{n} \times \frac{2}{n} - 1ee + \&c. \\
2-1 & 4 & 1+e \mid \frac{2}{n} - 1 = \frac{2}{n}e + \frac{2}{n} \times \frac{1}{2} \frac{2}{n} - 1\,ee + \&c. \\
3 \text{ or } 4 & 5 & \frac{2}{n}e \times \frac{2}{n} \times \frac{2}{n}ee + \frac{2}{n} \times \frac{2}{n} \times \frac{2}{3}eee \;\&c. \\
\textit{Reduced} & 6 & \frac{2}{n}e - \frac{2}{2n}ee + \frac{2}{3n}eee - \frac{2}{4n}eeee \;\&c. \\
\text{or } 5 = & 7 & \frac{2}{n} : \times : e - \frac{1}{2}ee + \frac{1}{3}eee - \frac{1}{4}e. \;\&c.
\end{array}
$$

Here the ſeries in the third and fourth ſteps are exactly the ſame as was to be prov'd, but becauſe n is ſuppoſed indefinetly great, it is manifeſt $\frac{2}{n}$ will be indefinetly ſmall and ſo $\frac{2}{n}$ being added to or taken from any number can make no ſenſible alteration, and therefore may be omitted in the Factors $\frac{2}{n} - 1$, $\frac{2}{n} - 2$ &c. and then the Series will be as in the 5*th* Step which contracted gives theſe in the 7*th* Step.

COROLARY I.

FROM hence it appears that if 1 be taken from $\overline{1+e}\Big|^{\frac{1}{n}}$ the indefinite root of $1+e$ any number the remainder $\frac{1}{n} \times e - \frac{1}{2}e^2 + \frac{1}{3}ee\frac{1}{4} - \frac{1}{1}e^4 + \frac{1}{5}e^5$ &c. would be the Logarithm of $1+e$ were it not indefinitely small (*viz.* multiply'd by $\frac{1}{n}$) but if we put $Q = \frac{10000\&c.}{n}$ (where 10000 &c. is supposed to consist of as many figures or places as n doth) the series will then become finite and we shall have $Q \times e - \frac{1}{2}e.^2 + \frac{1}{3}e.^3 - \frac{1}{4}e.^4 + \frac{1}{5}e^5$ &c. for the Logarithm of $1+e$.

COROLORY. II.

It is further evident, that there may be as many forms of Logarithms as n can be supposed different indefinite numbers, thus if $n = 1000$ &c. then Q will be $= 1$ aud the above series will become $e - \frac{1}{2}ee + \frac{1}{3}e^3 - \frac{1}{4}e^4 + \frac{1}{5}e^5$ &c. which series is the very same with that mentioned in *Example* I. Prob. 169, and is the Hyperbolical Logarithm of $1+e$: again if $n = 2,30251$ then

then Q will be = 0,434291 and then $Q \times e - \frac{1}{2}ee + \frac{1}{3}eee -$ &c. is the tabular Logarithm of $1+e$

But this Series tho' it be the moſt natural yet it converges ſo ſlowly, that it is of little uſe in the conſtruction of Logarithms, and therefore to find a Series that will converge ſwifter; Let us

Suppoſe $\frac{1}{1-e}$ to denote the Number whoſe Log. is required. Then by ordering

$$\frac{1}{1-e} \left| \begin{array}{l} \frac{1}{n} - 1 = \frac{1}{1+e} - 1 = \frac{1}{n} - 1 = 1 - e \\ \frac{1}{1-e} \end{array} \right| \frac{1}{n-1}$$ as you did $1+e \mid n-1$, (*viz.*) by taking the $n^{\overline{3}}$ Root of $1-e \mid \frac{1}{n}$ and ſubtracting 1 therefrom you will have $Q:\times:e+\frac{1}{2}ee+\frac{1}{3}e^{3}+\frac{1}{4}e^{\overline{4}}+\frac{1}{5}e^{\overline{5}}$ &c. for the Log. of $\frac{1}{1-e}$ to which add $Q:\times:e-\frac{1}{2}e^{\overline{2}}+\frac{1}{3}e^{\overline{3}}-\frac{1}{4}e^{\overline{4}}+\frac{1}{5}e^{\overline{5}}$ &c. the Log. of $1+e$ and you'll have $Q:\times 2e+\frac{2}{3}e^{\overline{3}}+\frac{2}{5}e^{\overline{5}}+\frac{2}{7}e^{\overline{7}}+\frac{2}{9}e^{\overline{9}}$ &c. for the Log of $\frac{1}{1-e} \times 1+e$, or $\frac{1+e}{1-e}$ that is, $2Q:\times e+\frac{1}{3}e^{\overline{3}}+\frac{1}{5}e^{\overline{5}}+\frac{1}{7}e^{\overline{7}}+\frac{1}{9}e^{\overline{9}}$ &c. is the Logarithm of $\frac{1+e}{1-e}$: And thus you have three ſeries (*viz.*) one

for

for the Log. of $1+e$, one for the Log. of $\frac{1}{1-e}$ (converges ſwifter) and one for the Logarithm of $\frac{1+e}{1-e}$ which converges ſwifteſt of all, and is the Series I ſhall uſe in the following Opperations, altho' by this method you may find ſeries at Pleaſure.

EXAMPLE I.

Required the Logarithm of any Number, N.

Becauſe $\frac{1+e}{1-e}$ denotes the given number therefore $\frac{1+e}{1-e}$ muſt be equal N, and if $\frac{1+e}{1-e} = N$ then $e = \frac{N-1}{N+1}$ which being ſubſtituted in the laſt ſeries inſtead of e we ſhall have the Logarithm of N required.

EXAMPLE II.

Required the Logarithm of the Fraction $\frac{N}{D}$

As before we muſt put $\frac{N}{D} = \frac{1+e}{1-e}$ and then

then we will find $e = \frac{N-D}{N+D}$ which write in the last series instead of e and you'll have the Logarithm of $\frac{N}{D}$,

What it the Hyporbolical or Nepers Logaritlm of 2 that is N = 2.

If N = 2 then $e = \frac{2-1}{2+1} = \frac{1}{3}$ so we shall have

$e = \frac{1}{3} = ,333333333333 +$
$\frac{1}{3} e.^3 = ,012345679912 +$
$\frac{1}{5} e.^5 = ,000823045268 -$
$\frac{1}{7} e.^7 = ,000065321c53$
$\frac{1}{9} e.^9 = ,000005645029 +$
$\frac{1}{11} e.^{11} = ,000000513185 -$
$\frac{1}{13} e.^{13} = ,000000048248$
$\frac{1}{15} e.^{15} = ,000000004646$
$\frac{1}{17} e.^{17} = ,000000000456 -$
$\frac{1}{19} e.^{19} = ,000000000045 +$
$\frac{1}{21} e.^{21} = ,000000000005 -$

$,346573590280 = e + \frac{1}{3} e.^3$ &c.
2 multiply for $2Q = 2$

$,693147180560 = 2Q < e + 3 e^{-}$ &c.

For the *Nepers* Logarithm of 2 true to the last place, and by the common series would require a multitude of terms, see this Logarithm found otherwise in *Example* I. Problem 169.

Again,

Again, let it be required to find the Tabular or Briggs *Log. of* 2.

1. If inſtead of multiplying 0,346573590280 by 2 Q = 2, you multiply it 2 Q = 0,8685889 63926 (Q being = 0,43429 + ſee *Corollary* 2,) you'll have the *Brigean*, or tabular Logarithm of 2, but this method being tedious, it will be better to find the *Tabular* Logarithm by the foregoing general ſeries, $R : \times e + \frac{eee}{3} + \frac{e^5}{5} + \frac{e^7}{7}$ &c. (R being = 2 Q = 0,86858896 &c.)

Thus e being before found $= \frac{1}{3}$ we ſhall have.

$$Re = \frac{0,8685889}{3} = ,2895296 +$$

$$\frac{Re\,3}{3} = \frac{0,8685889}{81} = ,0107233 +$$

$$\frac{Re\,5}{5} = \frac{0,8685889}{1215} = ,0007149 -$$

$$\frac{Re\,7}{7} = \frac{0,8685889}{15309} = ,0000568 -$$

$$\frac{Re\,9}{9} = \frac{0,8685889}{177147} = ,0000049 +$$

$$\frac{Re\,11}{11} = \frac{0,8685889}{1948617} = ,0000005$$

Sum = ,3010300 — for the *Tabular* Logarithm of 2. And

And thus having found the Logarithm of 2 you have alſo the Log. of 4, 8, 16, &c. for the Log. of 2 added to the Log. of 2, gives the Log. of 4, the Log. of 2 + Log. of 4 gives Log. of 8 &c. And by the above Series the Logarithms of the other Prime Numbers 3, 7, 11, 13, 17, &c. may be found But as we go to higher Numbers the Series will converge ſlower, and therefore it will be more expeditious to find the Logarithms of large Numbers by help of the Logarithms of ſmall Numbers already found: Thus,

Let a, b *and* c, *denote* 3 *Numbers in Arithmetical Progreſſion, we ſhall*

have { 1 $a=b-1$ } { 2 $c=b+1$ } *the common diff: being Unity*

1 × 2 | 3 | $ac-b^2-1$

3 + 1 | 4 | $ac+1=bb$. bb

4 ÷ ac | 5 | $\frac{bb}{ac}=\frac{ac}{ac}$ that is, 2 Log b—Log. a —Log. c = Log $\frac{bc+1}{ac}$

Put | 6 | $ac+1=N$, and $ac=D$, then $\frac{ac+1}{ac}=\frac{N}{D}$

p. Ex: 2 | 7 | $\frac{N-D}{N+D}=e=\frac{1}{2ac+1}$ hence Log. $\frac{ac+1}{ac}=Re+\frac{Re^3}{3}+\frac{Re^5}{5}$ &c. which put $=s$.

per 5 | 8 | 2 log. b—log a— log. $c=s$, or log $\frac{ac+1}{ac}$

from 8 | 9 | Log. $b=\frac{1}{2}$ log $a+\frac{1}{2}$ log $c+\frac{1}{2}s$, and log $a=2$ Log. $b-$ log $c-s$, alſo, Log. $c=2$ log $b-$log $a-s$.

Required

Required the Tabular Logarithm of 7.

Having found the Logarithm of 2 and 3, fro
them you may have the Logarithems of 8 and
(*viz.*) three times the Log. of 2 is the Log. of
and 3 times Log. 3 = Log. 9 Hence have $a=$
$b=8$ and $c=9$, and *per* 8th, Step Log. $7 = 2$ Lo
8—Log. 9—S. Now in order to find S we hav

per Step 7th. $e = \frac{1}{2ac+1} = \frac{1}{127}$ and ther

fore $s =$

$Re + \frac{Re^3}{3} + \frac{Re^5}{5} + \frac{Re^7}{7}$ &c. equal the tab

lar Log. of $\frac{ac+1}{ac}$ (*viz.*) Log. of $\frac{64}{63}$ *per* step

$$Re = ,\frac{868588963}{127} = ,006839283$$

$$\frac{Re^3}{3} = \frac{,868588963}{6145149} = 000000141$$

$$\frac{Re^5}{5} = \frac{,868588963}{165191847035} = ,000000000$$

Sum — ,006839424 = T

bular Logarithm of $\frac{64}{63}$,

Therefo

Therefore, 2 Log. 8 — Log. 9 — S = 0,845098040 the tabular Log. of 7, which Log. you see is had to 9 places of figures by 2 terms of the series only, and also turns out the Log. of the fraction $\frac{64}{63}$, But if we have greater numbers then fewer terms of the series will serve, thus if the Logarithms of 2, 3, and 5; be known, Then,

The Logarithms of 50 and 48 may be known from whence the Log. of 7 (= $\frac{1}{2}$ Log. 49) may be found, for put $a = 48$ $b = 49$ &c. $= 50$ then $e = \frac{1}{2ac+1} = \frac{1}{4801}$ and Re equal .868588896 &c. $\frac{1}{4801} = .0001809$ &c. $= S$ then *per* step 9 foregoing $\frac{\text{Log. } 48 + \text{Log. } 50 + S}{2} = \text{Log. } 49$: or $\frac{\text{Log. } 48 + \text{Log. } 50 + S}{4} =$ .845098040 the Log. of 7 as before, and takes in only the first term of the series, and would hold true to a place or two more if continued, and greater the numbers are the Log. by only the first term of the series, will still be truer to more and more places of figures, by which a table of Logarithms may be readily made.

PROBLEM CLXXVI.

Any Logarithm L *being given, to find the Natural Number* $1+e$ *thereunto belonging.*

If you revert the series $e - \frac{e^2}{2} + \frac{e^3}{3} - \frac{e^4}{4} + \frac{e^5}{5}$ &c. (found *per* Cor. 1. last Prob.) you will have the required value of $e + 1$, or otherwise thus, if $1 + e$ be the required number, then by the last Prob. $1 + e \mid \frac{1}{n} + 1 = L$ but since n being equal any indefinite number we may put $1, 1 + e \mid \frac{1}{n} = 1$; $\therefore 1 + e \mid \frac{1}{n} = 1$,

$$\text{That is } 1r : 1+e \Big| r : r^{\frac{1}{L}} = L$$

$$1-r \div r \;\; 2 \;\; 1+e \Big| r = 1 + \frac{1}{L}$$

$$\text{G. } r \;\; 3 \;\; 1+e = 1 + \frac{r}{1} + \frac{rL}{2} + \frac{r \times r-1\, L^2}{r^2} + \frac{r \times r-1 \times r-2\, L^3}{2} \times \frac{1}{3} + \frac{1}{rrr} + \&c.$$

$$3 = \;\; 4 \;\; 1+e = 1 + L + \frac{rrLL}{2rr} + \frac{r^3L^3}{6r^3} + \frac{r^4L^4}{24r^4} + \frac{r^5L^5}{120r^5} + \&c.$$

$$\text{that is } 4 \;\; 5 \;\; 1+e = 1 + L + \frac{LL}{2} + \frac{LLL}{2.3} + \frac{LLLL}{2.3.4} + \frac{LLLLL}{2.3.4.5} + \&c.$$

equal the number required.

In this work r being ſuppoſed indefinitely great, it can neither be ſenſibly increaſed or diminiſhed, and therefore the numbers — 1 — 2 — 3 *&c.* in the third ſtep may be left out from whence we get the fourth ſtep by which it appears that r and its powers vaniſhes ſo we get the ſeries in the fifth ſtep which are the ſeries required; L, being the Hyperbolic Logarithm of $1 + e$.

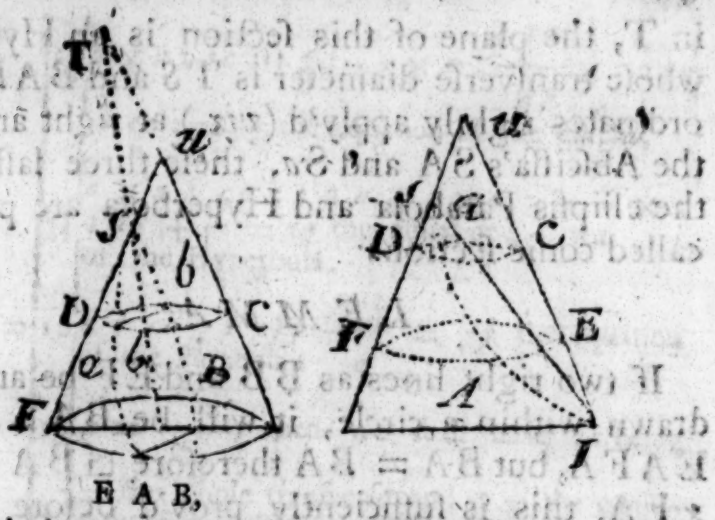
PROBLEM CLXXVII.

Any Conick being given, to find its Property.

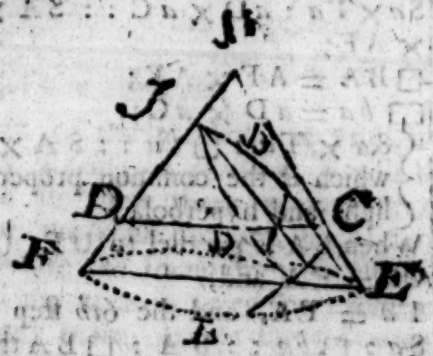
DEFINITIONS.

1. If a cone UEF (made by Prob. 163) be cut by a Plane paſſing thro' the vertex U, and center of its baſe dividing the cone into two equal parts or ſections the plane of each ſection will be an Iſcoſcles Triangle.

2. If it be cut any where by a plane CD parallel to its baſe EF the plane of that Section will be a circle.



3. If it be cut by a plane TS (*fig* 1.) from the extremity of the baſe to the oppoſite ſide UF the plane of that ſection will be an ellipſis whoſe tranſverſe diameter is TS *b a b*, and BAB ordinates at right angles to the Abſciſſa's S*a* and SB or T*a* and TB.



4. If the cutting plane SA (*fig* 3.) be parallel to the ſide CE of the cone the plane of this ſection is a common Parabola whoſe axis is ſet and *b a b*, an ordinate at right angles to the abſciſſa S*a*.

5. If the cutting plane SA (*fig.* 2,) paſs any how ſo that the Abſciſſa SA being produced will meet the other ſide EU of the cone produc'd as

in

in T, the plane of this section is an Hyperbola whose transverse diameter is TS and BAB, *bab* ordinates rightly apply'd (*viz.*) at right angles to the Abscissa's SA and S*a*, these three last (*viz.*) the ellipsis Parabola and Hyperbola are properly called conic sections.

LEMMA.

If two right lines as BB and EF be any how drawn within a circle, it will be BA × BA = EA FA, but BA = BA therefore □ BA = EA × FA, this is sufficiently prov'd before and is called the property of the circle, then in *fig* 1 and 2, it will be,

per Theorem 1;	1	S*a* : *a*D : : SA : AF, for the triangles S*a*D and SAF, are alike;
	2	T*a* : *a*C : : TA : AE for the triangles T*a*C and TAE are alike;
1 × 2	3	S*a* × T*a* : *a*D × *a*C : : SA × TA : AF × AE;
per Lemma	4	□BA = AF × AE;
	5	□ *ba* = *a*D × *a*C;
3, 4, 5;	6	S*a* × T*a* : □ *ba* : : SA × TA : □BA which is the common property of the ellipsis and hyperbola;
But	7	When SA is parallel to UE, (*fig.* 3,) then C*a* = EA and,
per step 2	8	I *a* = TA, and the 6*th* step will become,
	9	S*a* : □ *ba* : : SA : □BA the property of the Parabola.

Let *a* = ½ the transverse and *c* = ½ the conjugate diameters of the ellipsis and Hyperbola, but *a* = SA, the greatest Abscissa, or axis and *c* = BA the greatest semi-ordinate of the parabola (*fig.* 3,) also put *e* = S*a* any Abscissa, and *y* = *ba*, a semi-ordinate to that abscissa, then the 6th step will

will,

10 $a \times a : cc :: 2a - e \times e : \frac{cc}{aa} \times : 2ae - ee : = yy$ the equation of the ellipsis,

become

11 $a \times a : cc :: 2a + e \times e : \frac{cc}{aa} \times ; 2ae + ee = yy$ the equation of the Hyperbola.

and step 9 = 12 $a : cc :: e : \frac{ccc}{a} = yy$ the equation of the Parabola;

Put 13 $p = \frac{cc}{a}$, then we shall have (a being = the whole transverse and c = the conjugate of the ellipsis and Hyperbola)

10, 14 $a : p :: a - e \times e : yy = \frac{p}{a} \times \overline{ae - ee}$ equation of the ellipsis;

11, 15 $a : p :: a + e \times e : yy = \frac{p}{a} \times \overline{ae + ee}$ equation of the Hyperbola;

12, 16 $p : y :: y : e$ that is $pe = yy$ the equation of the Parabola.

By this equation of the parabola, in Prob. 15, projected being equal any abscissa at pleasure, y will be a semi-ordinate thereto in all these three sections or curves, from these properties of the conic sections, might be deduc'd many more for these are the foundation of all the rest, but as these have serv'd my turn in foregoing Problems, I think it needless to add any more, except the following one for determining the Focus &c.

PROBLEM CLXXVIII.

The Property of any Conick Section given to find its Latus Rectum or Parameter and Focus.

DEFINITION.

If we suppose an ordinate $= p$, then this ordinate is called the Parameter by which all the other ordinates are regulated as in the three last steps to the last Prob. now

that

that point in the axis or diameter cut by this ordinate p is called the Focus, of which there is one in the Parabola and Hyperbola, and two in the ellipsis, called its two Centers which are the two points where the ends of the thread is fastened when the ellipsis is described, see the Prob. for making an ellipsis, to find the distance of the Focus from the vertex S, of the Parabola *fig.* 3, put $\frac{1}{2}p = y$ then by the equation $pe = yy$ of the parabola, $pe = \frac{1}{4}pp$ therefore $e = \frac{1}{4}p$ then lastly for the ellipsis and Hyperbola, since the equation of these two curves differ only in the signs $-$ and $+$ that is in the ellipsis it is $a - e$, and in the Hyperbola $a + e$ these two equations may both be expressed at once by the sign $\ddagger$; put $z = \frac{1}{2}a - e =$ that part of the axis between the center or middle of the axis and any ordinate, then $ae + ee$ will become $\frac{1}{4}aa + zz$ and then $\frac{p}{a} \times : \frac{1}{4}aa + xz = yy$ another equation of the ellipsis and Hyperbola, suppose $y = \frac{1}{2}p$ as in the Parabola, then the last equation will become $\frac{p}{a} \times : \frac{1}{4}aa + zx = \frac{1}{4}pp$ whence $\mp\frac{1}{4}aa + zz = \frac{1}{4}pa$ therefore $\sqrt{\frac{1}{4}aa + \frac{1}{4}pa} = z$ the distance between the Focus and center of the Hyperbola and ellipsis, and writing $\frac{cc}{a}$ the value of p instead of p it will be $z = \sqrt{\frac{1}{4}aa - \frac{1}{4}cc}$ in the Ellipsis and $z = \sqrt{\frac{1}{4}aa + \frac{1}{4}cc}$ in the Hyperbola.

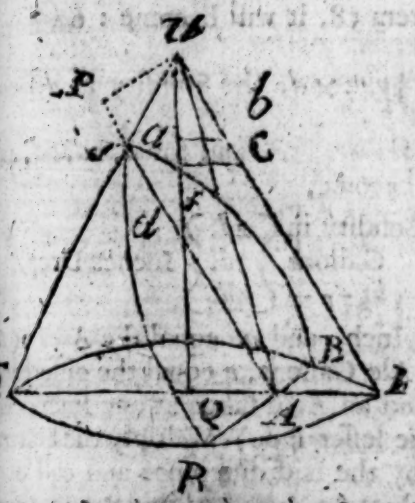
PROBLEM CLXXIX.

A Cone UEF, *being cut by any Plane* A, *into any two unequal Parts* UEAS *and* ASF, *to find the solidity of each Part.*

Let UQ be the Axis of the whole Cone, UEF the parallel to the Base EF, or Perpendicular to the Axis UQ draw Lines as

ab, *cd*, &c. and join AU cutting *ab*, *cd* &c. in *e f* &c. Then *per* ſimiliar Triangles.

will be $\begin{cases} 1 \\ 2 \\ 1 = 2 \end{cases}$ | U*e* : U*f* : : *be* : *cf*,
U*e* : U*f* : : U*a* : U*d*,
U*a* : U*d* : : *be* : *cf* &c.



Which lines *eb*, *cf*, EA &c. are verſed ſines of ſegments and bears the ſame proportion to the axis UQ that the radius *ab cd* EQ &c. do;

Hence it is evident, that the area of the ſegment EBABC, being multiplied by a third part of the axis UQ will give the ſolidity of the Part EUA, produce SA the axis of the plane upon which let fall the perpendicular UP, than after the ſame manner it may be proved that the ſolidity of the part UAS will be equal the area of the cutting plane SBABS multiply'd by $\frac{1}{3}$UP, which two ſolidites added together give the ſolidity of the part UEAS which taken from the whole cone leaves the other portion ASF as required.

By this Problem may the ſolidity of any Parabolic Hyperbolic, &c. hoofs be found, as of the elliptical hoofs, Theorem 23 and 24, but if ſuch hoofs be met with in practice before you can determine what kind of hoofs they are you may almoſt as ſoon by Prob. 169 find their ſolidities for which reaſon I decline the extent of this Problem.

PROBLEM CLXXX.

Any ſtreight Sided Tun &c. given, to find its Content at any given Depth d.

Let *aa* be equal the Area at the Bottom in Gallons, and *cc* equal the Area in Gallons at the Depth *d*: Now if any number of Areas be taken equally diſtant from, and parallel to one another

another, the Square Roots of those Areas will be in Arithmetical Progression whose common difference call e; then $a+e\,|^2 = cc$, or $a+e = e$, and $a+2e =$ the Square Root of the Area at 2d. Deep, $a+3e =$ the Square Root at the Area at 3d. Deep &c. Also $a+\frac{1}{2}e =$ the Square Root of the Area at $\frac{1}{2}$d. Deep &c. Now by writing aa for $A : a+e\,|^2 (cc)$ for a, and $2a+\frac{1}{2}e\,|^2$ for $4m$ in Theorem 58, it will become : $6aa+6ae+2ee : \times \frac{d}{6} = : aa+ae+\frac{1}{3}ee : \times d$, the Solidity at d Deep

Hence if for e we write $2e$, $3e$, $\frac{1}{2}e$, &c. and for d, $2d$, $3d$, $4d$, &c. the last expression will become,

$$\left.\begin{array}{l} : aa+2ae+\frac{4}{3}ee : \times 2d \\ : aa+3ae+\frac{9}{3}ee : \times 3d \\ aa+4ae+\frac{16}{3}ee : \times 4d \end{array}\right\} \text{Solidity in Gallons at} \left\{\begin{array}{l} 2d. \\ 3d. \\ 4d. \end{array}\right\} \text{Inches Deep.}$$

So that if d be equal one Inch, and aa equal the Area at the Bottom of the Vessel in Ale Gallons, e equal the difference between this bottom Area and an Area taken at one Inch deep (the Vessel standing upon the lesser Base) Multiply the Square Root of the bottom Area, by the said difference and call that the Product. Then to the bottom Area add this Product and $\frac{1}{3}$ of the Square of the difference and you'll have the content at one Inch deep; also, the bottom Area added to twice the said Product, and that Sum to $\frac{4}{3}$ of the Square of the said difference and doubled gives the Area at 2 Inches Deep. Again the sum of the bottom Area, 3 times the Product and $\frac{1}{2}$ or $\frac{3}{6}$ times the Square of the difference doubled, gives the Solidity at 3 Inches deep &c. By this Method any such Vessel as mentioned in Theorem 58, may readily and truly be Inch'd, half-Inch'd &c. according as you take $d = 1$ Inch or $\frac{1}{2}$ Inch &c.

OTHERWISE.

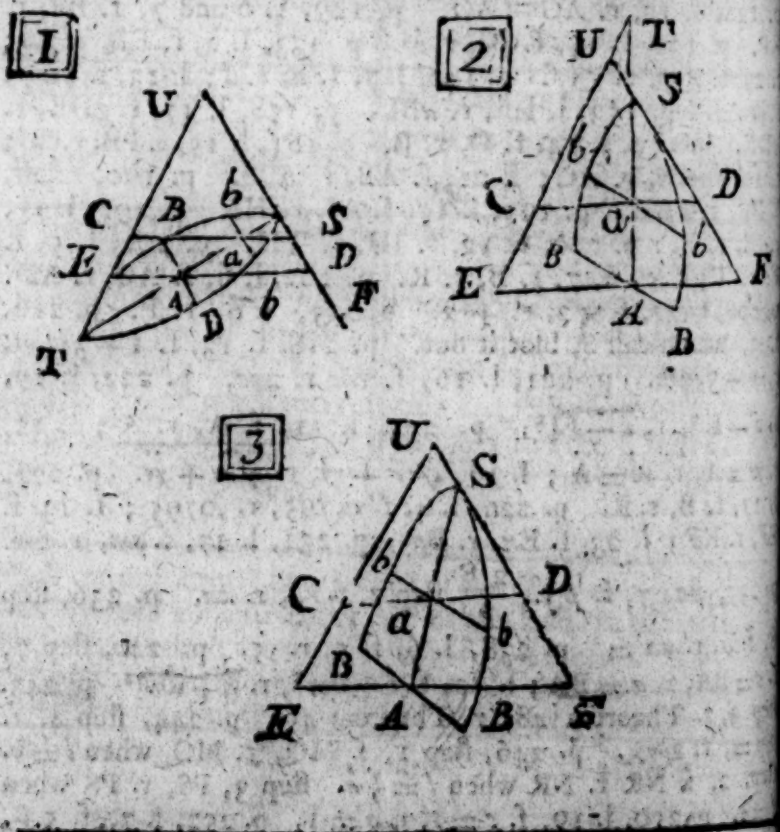
If you find three contiguous Contents, you may by this *Rule* find all the rest, (viz.) Divide each of the three contiguous Contents by their respective dry Inches and take the second Quotient from the third Quotient; then if to 3 times the remainder you add the first Quotient, and Multiply that Sum by the dry Inches belonging to the required Content, the Product will be that Content.

The END of the FIRST PART.

ERRATA.

PAGE 8 line 2 read 4 unequal. p. 49, l. 9, r. 7+8. p. 62, l. 16, f. 15 is 25^9, r. 15 is $,25^9$. p. 81, l. 4, r. FC. p. 82, l. 15, r. Theorem 1. p. 96, l. 17 and 18, f. $\angle$ r. Δ. p. 100, l. 12, f. C, r. Q; l. 18, f ABC, r. NBC, p. 103, l. 4, f. N*u*, r. M*u*; l. 6, r. 89; l. 14, f. WU or FM, r. WL or FT. p. 104, l. 4, f. AE r. AD; l. 7, f. DAE r. DC. p. 105, l. 10, f. *aebd* r. *ae*BD. p. 111, l. 4, 6, 8, 16, 18, f. C r. E. p. 112, l. 8, r. E*e* f. C*e*. p. 114, l. last, r. 83. p. 118, l. last, f. *za* r. A*a*. p. 119, l. 4, f. AC draw ED, r. AQ draw ZD; l. 9, r. HDZ. p. 128, l. 12, r. AG = AO, p. 129, l. 6 and 7, f. BP, r. RP. p. 132, l. 15, f. B*e*, r. *be*. p. 133, l. 2, f. EB, r. E*b*; l. 29, f. $\angle p$, r. $\angle f$. p. 140, l. 7, f. *a*, r. *d*; l. 13, f. 90—, r. 90=. p. 153, l. last, r. *a*BL. p. 158, l. 17, f. *g*DE, r. *a*DE. p. 159, l. 14, f. D, r. B. p. 181, l. 15, f. PB, r. SB; l. 19, f. AE, r. AC; l. 21, f. AB, r ASB. p. 190, l. last, f. BL, r. FBE. p. 191, l. 15, f. AI, r. HI. p. 192, l. 11, f. E, r. B. p. 200, l. 12, f. IH, r. TH. p. 201, l. 5, f. F, r. T; l. 5, 6, 7, f. B, r. R. p. 202, l. 6, f. AC, r. AD. p. 208, l, 22, f. $\times 2$, r. $+2$. p. 215, l. 6, r. F. p. 216, l. 3, which call S, blot it out. p. 218, l. 13, f. $1+3mm$, r. $1-3mm$. p. 221, l. 16, f. *zae* r. 2*ae*. p. 222, l. 19, f. $c-E^2$, r. $\overline{c-E}^2$. p. 224, l. 11, f. A, r. A; l. 12, f. $ye = A$, r. $ye = A$; l. 16, $\sqrt{ee+yy}$, r. $\sqrt{ee+yy}$. p. 227, l. 17, f. B, r. R. p. 229, l. 4, f. 0,795, r. ,0795; l. 13. f. BF, r. RF; l. 23, f. E*n*, r. B*n*. p. 231, l. 17, f. *em*, r. 2*m*. p. 235, step 7, f. p r. $\frac{2F}{p}$ step 3, f. *ra*, r. *aa*. p. 236, step 8, f. *a*, r. *aa* = p. 238, l. 6, f. 5, r. ,5. p. 240, step 7, f. $o = BA$, r. $a = BA$; l. 10, f. $b+dm^2$, r. $\overline{b+dm}^2$ p. 243. step 3, f. Theorem 48, r. Theorem 45. p. 244, step 4, f. $2py =$, r. $2py \times$. p. 246, step 1, f. MQ, r. MQ when $l = o$. step. 2, f. NR r. NR when $l = \frac{1}{2}e$. step 3, PS, r. PS when $l = e$. p. 250, l. 19, f. $e = 1$, r. $e = 1$, p. 252, l. 7, f. 5+, r. 5X. p.

p. 259, ſtep 4, f. $4^{\overline{2}}_{2}$, r. $\sqrt{3}$. p. 264, l. 3, f. two r. of $\frac{2}{1}$ l. 12, f. $n-e$, r. $n-1$. p. 265, ſtep 3, f. $1-2$, r. $1-\frac{2}{1}$ ſtep 4, f. $2-1$, r. $2-\overline{1}$. p. 266, l. 5, $\frac{y}{1}$ dele. p. 267, l. 11, f. $1+e$, r. $1-e$; l. 12, f. $n^{\overline{1}}$ r. nth. p. 271, ſtep 5, f. $be+1$, r. $ac+1$. p. 272, l. 4, f. 3 times, r. 2 times. p. 273, l. 24. f. $e\div 1$, r. $e+1$, l. 25, f. $\frac{1}{n}1$ $\frac{1}{n}-1$. p. 275, l. 6 and 7, SB and TB, r. SA and TA; l. 10, f. ſet, r. SA. p. 276, l. 10, f. EAFA, r. EA×FA. p. 278, l. 14, $\frac{1}{2}a-$, r. $\frac{1}{2}a \mp$; l. 17, f. xz, r. zz; l. 19, f. zx, r. zz; l. 20, f. $+$, r. $\pm$. p. 279, l. 15, f. EBABC, r. EBAB. p. 280, l. 4, f. $a+e=e$, r. $a+e=c$. In Prob. 144, for Weſt, r. Eaſt; and for Eaſt, r. Weſt. Problem 110 muſt be read thus, By a line drawn from a given $\angle$ A, in a $\triangle$ ABC, to cut off a part towards the $\angle$ B, that ſhall be $=$ to a given $\square$, whoſe ſide is $=$ to ab.-------The Figures to Problem 138 are theſe.



A TABLE OF THE Areas of Segments.

V.S.	S.A.	V.S.	S.A.	V.S.	S.A.	V.S.	S.A.
001	000042	031	007209	061	019716	091	035585
2	000119	32	007558	62	020196	92	036162
3	000219	33	007913	63	020689	93	036741
4	000337	34	008273	64	021168	94	037323
5	000470	35	008638	65	021659	95	038909
6	000618	36	009008	66	022154	96	038446
7	000779	37	009383	67	022652	97	036087
8	000951	38	009763	68	023154	98	039680
9	001135	39	010148	69	023659	99	040276
010	001329	40	010537	70	024168	100	040875
11	001533	41	010931	71	024680	101	041476
12	001746	42	011330	72	025195	102	042080
13	001968	43	011734	73	025714	103	042687
14	002199	44	012142	74	026236	104	043296
15	002438	45	012554	75	026761	105	043908
16	002685	46	012971	76	027289	106	044522
17	002940	47	013392	77	027821	107	045139
18	003202	48	013818	78	028356	108	045759
19	003471	49	014247	79	028894	109	046381
20	003748	50	014681	80	029435	110	047005
21	004031	51	015119	81	029979	111	047632
22	004322	52	015561	82	030526	112	048262
23	004618	58	016007	83	031076	113	048894
24	304921	54	016457	84	031629	114	049528
25	005230	55	016911	85	032186	115	050165
26	005546	56	017369	86	032745	116	050804
27	005867	57	017831	87	033307	117	051446
28	006194	58	018296	88	033872	118	052092
29	006527	59	018766	89	034441	119	052736
30	006865	60	019239	90	035011	120	053385

V.S.	S.A.	V.S.	S.A.	V.S.	S.A.	V.S.	S.A.
121	054036	141	067528	161	081846	181	09690
122	054689	142	068225	162	082524	182	09767
123	055345	143	068924	163	083320	183	09844
124	056003	144	069625	164	084059	184	09922
125	056663	145	070328	165	084801	185	09999
126	057326	146	071033	166	085544	186	10077
127	057991	147	071741	167	086289	187	10155
128	058658	148	072450	168	087036	188	10233
129	059327	149	073161	169	087785	189	10311
130	059999	150	073874	170	088535	190	10390
131	060672	151	074589	171	089287	191	10468
132	061348	152	075306	172	090041	192	10547
133	062026	153	076026	173	090797	193	10626
134	062707	154	076749	174	091554	194	10705
135	063389	155	077469	175	092313	195	10784
136	064074	156	078194	176	093074	196	10863
137	064760	157	078921	177	093836	197	10943
138	065449	158	079649	178	094601	198	11022
139	066140	159	080380	179	095366	199	11102
140	066833	160	081112	180	096134	200	11182

To find the TABULAR NUMBER *to any* Verſed Sine *in the Segment of a given Circle.*

See Page 96, 97, 98

1ſt. *To find the* Half-Chord *ZD.*

RULE.

From the Radius *CE* take the Verſed-ſine *ZE* there remains *CZ*. Then from the Square of the Radius *CD* take the Square of *CZ*, the Square Root of the remainder is the Half-Chord.

47. *E.* I

EXAMPLE

EXAMPLE.

The Diameter 1, *the Verſed-ſine* .01; *Required the Half-Chord?*

Semidiameter or Radius — .5
Verſed-ſine ſubtract — — .01

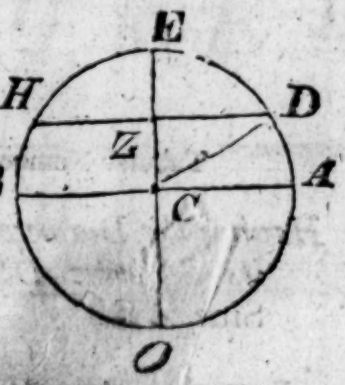
Co-ſine or difference — — .49
.49

441
196

Squar'd — — — — .2401
Radius Squared — — — .25

Remains the Square of *ZD* .0099

The Square Root — — .099 the Half-Chord requir'd.



2dly. *To find the* TABULAR NUMBER.

See *Rule* Page 97 PART II. Or, by this RULE.

From 7 times the Radius Squared, Subtract 4 times the Radius into the difference of Radius and Verſed-ſine, and 3 times the ſaid Difference Squar'd for a Dividend. ----- Then 4.5 times the Radius more 3 times the Difference for a Diviſor. ----- The Quotient Multiply'd by the Half-chord, gives the Area in Inches, or the ſame Meaſure with the Diameter.

EXAMPLE.

Given as above Diameter 1, *Verſed-ſine* .01; *Requir'd the Area?*

Radius Squar'd — — — — — — .25
Common Multiplier — — — — — 7

4 times Radius into the Diff.	.98	Sub.	1.75
3 times Square of the Diff.	.7203		1.7003
Sum.	1.7003		.0497 Dividend
			Radius

Rad. by 4.5 = 2.25
3 times the diff. 1,47

Divisor — — 3.72) .0497 (.01336
1250 .099 half Chord.
134
224 12024
12024

Tabular Number — — .00132264 to 8 places.

Having the Diameter of a Circle OE, and Chord of its Segment DH given, to find the Verſed-Sine *ZE.*

RULE.

From the Square of the Radius ſubtract the Square of half the Chord, the Square Root of the Remainder taken from Radius, gives the Verſed-Sine.

EXAMPLE.

Diameter as above 1, the Chord *DH* .98 : Required the Verſed-Sine ?

Radius Squar'd — — — .25
Square ½ Chord — — — .0099

Remainder — — — — .2401

Whoſe Square Root is — .49
From Radius — — — — .5

.01 Verſed-Sine.

Thus much I thought fit to add, inſtead of the long Table, when the Tabular Numbers are ſo eaſily found, the Rule may be of good uſe in the abſence of ſuch a Table.

The END of the SECOND PART.

THE UNIVERSAL *MEASURER.*

Part II.

CONTAINING,

I. Deſcription, Conſtruction, and Uſe, of *Coggleſhal*'s SLIDING-RULE.
II. Multiplication of Feet and Inches: Generally called Croſs Multiplication.
III. Decimal Arithmetic made Eaſy; with ſeveral New Contractions.
IV. Extraction of Roots, with an UNIVERSAL *Theorem*, for Extracting any Root out of a Simple Power.
V. Menſuration of Surfaces, every Caſe ſpoken particularly to.
VI. The ſeveral Artificer's Work, with an eaſy Method for thoſe that deal much to keep a regular Book, *&c.*
VII. Solid Meaſure; every particular Solid handled apart.
VIII. SURVEYING, Plotting and dividing of Land: alſo, how to make MAPS of the ſame.
IX. GAUGING, as before hinted at, in all its Parts.
X. Queſtions, with Solutions at large, which exerciſe the whole WORK.

By *A. FLETCHER, Philomath.*

WHITEHAVEN: Printed by W. MASHEDER, for the AUTHOR. MDCCLIII.

THE

UNIVERSAL

MENSURER.

THE

PREFACE.

AS the Deſign and Contents of the whole is mention'd in the *Preface* to the Firſt Part, there's leſs occaſion for a *Preface* to this Second Part; But as they may be ſold together or ſeperate, it may not be amiſs to inform the READER, this Part is an entire Treatiſe of Practical *Menſuration*.

THE former ſhews the Nature, Properties and delineation of Figures, with the invention of Rules to *Meaſure* the ſame.

THIS

THIS Second Part, is the application of thoſe Rules to all kinds of *Meaſuring*, whether *Lineal*, *Superficial*, or *Solid*.

IF the Firſt Part coſt more pains, the latter will afford more pleaſure.

AS no part of Mathematical Learning is more generally uſefull than the knowledge of *Numbers* & *Meaſuring*, ſo this Book is written to render theſe as eaſy as poſſible, and will ſupply the place of many Books.

THIS carefully Read you will need no other aſſiſtance, which is the intention and wiſh of the Author.

THE

THE *Universal Measurer.*

PART II.

Description and Use of the common SLIDING RULE.



THIS Rule consists of three parts *viz.* two Lines and a Slider, one of which Lines, number'd from the left hand beginning at one, and ending at nine about the middle of the Rule, where it again begins at 1, and goes on 1 2 3 4 5 6 7 8 9 10 to the end of the Rule, this is what is commonly call'd *Gunter's* Lines, the Slider is number'd the very same way in every respect.

The

The third Line, is of the ſame kind with theſe, but of a double Radius, the diſtance between 2 and 3 on this Line, is double to the diſtance between 2 and 3 on the Slider, and ſingle Line, *&c.* of any other; on ſome Sliding Rules this Line is mark'd with D at one end, but if it be not ſo mark'd it is eaſily known, by the marks *Girt. Line* or 12, for Timber meaſure, and *W. G.* for Wine Gallons, with *A. G.* for Ale Gallons call'd *Gage Point*.

Thoſe Lines are ſo divided, that if you were to multiply any two number thereupon, you muſt always extend with a pair of Compaſſes from 1 to the multiplier, that extent the ſame way reaches from the multiplicand to the Product, ſo that the diſtance between 1 and 2, added to the diſtance between 1 and 3, is equal to the diſtance between 1 and 6, for the Product of 2 and 3 is 6 *&c.*

Now if to the logarithm of any number, you add the log. of any other number, you will have the log. of the product of theſe two numbers, that is the log. of 2 added to the log. of 3, gives the log. of 6, *&c.*

From whence it appears that this Scale is made of a table of logarithms. Thus the log. of 1 being 0, of 2 is 301, of 3 is 477, of 4 is 602; Therefore from a Scale of equal parts take 301,

477,

477, 602, 699, 778, 845, 703, 954, 1000, respectively, and lay them down on a ſtreight Line, ſo have you the numbers 1 2 3 4 5 6 7 8 9 10, upon the Sliding Rule, and going on with the ſame diviſions, you have the number 2 3 4 5 6 7 8 9 10, now the diſtance between 1 and 2 being commonly divided into 25 parts, therefore from the ſame Scale of equal parts to the logs of 1 $\frac{1}{25}$ 1 $\frac{2}{25}$ 1 $\frac{3}{25}$ *&c.* reſpectively, and lay from 1 towards 2, ſo have you the diſtance between 1 and 2 exactly divided, and in the ſame manner is the diſtance between the other figures divided, which are commonly into fewer parts, becauſe the diviſions are nearer each another.

The Line D or gage point Line, is divided the ſame way, only its diſtances or diviſions are double to their reſpective diviſions of the other Lines.

SECT.

SECT. I.

The Use of the SLIDING RULE.

And first of Numeration; or to find any number thereupon.

THE figures 1 2 3 4 5 &c. beginning at the left Hand side of the Rule, may stand for unites as 1 2 3 &c. or tens as 10 20 30 &c. or 100 200 &c. and if these be unites, those in the following Line stands for tens, if the former Line be tens, the latter will be hundreds &c. that is the figures in the latter Line are always those in the former Line multiplied by 10; if you would find 25 on the Lines, in this case 2 must stand for 20, then 3 will denote 30, so 25 will stand 0,5 of the distance between 2 and 3; also if you would find 126 here 1 must denote 100, then 2 will signifie 200, so take 1 for 100, and 2 of the 10th. divisions for 20, and for 6, take 6 of the 10th. divisions (if the Rule be so divided) between 2 10ths and 3 10th, so have you 126 &c. that is if there be 10 divisions between the figures 1 and 2, then the first of those divsions is

is 110, the next 120, the next 130 *&c.* to the figure 2 which ſtands for 200, ſo having 120 at the 2d. diviſion you may eaſily perceive that 126 will be ſomewhat more than half way between the 2d. and 3d. diviſion. And thus may you find any other number, where Note, 1 that if you find 126 on the latter line, at that like point on the former Line you have 12. 6; if you have 12. 6 on the latter Line; at the like point on the former Line you have 1. 26 *&c.* for the latter Line being units, the former Line denotes tenths *&c.* as before.

Note. That Line which begins at 1 on the left Hand end of the Rule, and ends at 1 about the middle, I call the former Line, and the following Line, the latter Line; from the foregoing deſcription it is eaſy to conceive, that if four numbers be proportional, *viz* as the firſt is to the ſecond, ſo is the third to the fourth; that if you ſet the firſt number on the Slider, to the ſecond number on the Rule then againſt the third number on the Slider, ſtands the fourth number on the Rule, from which we have the following general rules for multiplication diviſion, and the rules of proportion.

RULE I.

Multiplication on the Sliding Rule.

As 1 *is to the multiplier, ſo is the multiplicand to the product, therefore always ſet* 1 *on the Slider to the multiplier on the Rule, than againſt the multiplicand on the Slider, is the Product on the Rule.*

RULE II.

Diviſion on the Sliding Rule.

As the diviſor is to the dividend, ſo is one to the quotient, therefore always ſet the diviſor on the Slider to the dividend on the Rule, and againſt 1 *on the Slider you'll have the quotient on the Rule.*

RULE III.

Rules of *Proportion* on the Sliding Rule.

This Rule being already laid down take the following examples.

EXAMPLE I.

If 1 *coſt* 7*s.* 6*d. what will* 3 *coſt at that rate?*

Set 1 on the Slider to 7.5 (becauſe 0.5 of a Shilling is equal to 6d.) on the Rule, then againſt 3 on the Slider, ſtands 22.5s. on the Rule for the anſwer.

EXAMPLE II.

If 1 *coſt* 7*s.* 6*d. what will* 200 *coſt at that rate?*

In this example ſome difficulty may ariſe to beginners for if you find 1 on the former Line upon the Slider, then the latter Line can only be tens; and therefore the figure 3 on the ſaid latter Line can be but 30, now in all ſuch caſes when the ſecond or third terms, are ſo big as to go off the Lines, divide the ſaid terms or term by 10, 100, 1000, *&c.* till they will come on the Line; ſo here 300 divided by 100 gives 3.00, therefore ſet 1 on the Slider to 7.5 on the Rule, then againſt 3 on the Slider ſtands 22.5 on the Rule, which being multiplied by 100, becauſe you divided the ſecond or third term by 100, give 2250s. for the anſwer.

EXAMPLE

EXAMPLE III.

If 16 *labourers do a piece of work in* 8 *days, in what time will* 256 *labourers do the ſame work ?*

Here 256 divided by 100 gives 2,56, but this proportion is inverſe, ſo 2,56 muſt be the diviſor or firſt term. therefore ſet 2,56 on the Slider to 8 on the Rule, then againſt 16 on the Slider ſtand 50 on the Rule, which 50 muſt be divided by 100, becauſe the firſt term was divided by 100, ſo you'll have 0,5 of a day for the anſwer.

3d. *Note.* That to divide any number by 10, is but to cutt off one figure (for a Decimal) towards the right Hand, to divide by 100, is but to cut off two figures *&c.* and to multiply by 10, 100, 1000, *&c.* is juſt contrary, ſo that the diviſion and multiplication uſed in theſe two examples, may be perform'd in the mind, which makes the uſe of this noble Inſtrument, both plain and eaſy.

This with what is ſaid in ſuperficial meaſure, ſufficiently ſhews the uſe of the Slider with Gunter's Lines. The 3 following rules belongs to the Slider, with the Gage Point Line, or Line mark'd D.

RULE

RULE IV.

To work PROB. 41, upon the Sliding Rule.

That is to find any fourth number, from three given ones, ſuch that as the firſt is to the ſquare of the ſecond, ſo is the third to the ſquare of the fourth.

EXAMPLE IV.

Let the three given numbers be 3, 60, *and* 7, *ſuch that as the ſquare of* 3 *is to the ſquare of* 60, *ſo is* 7 *to the fourth number ſought?*

Set 3 on the Line D to 7 on the Slider, than 60 being off the Line D divide it by 10, ſo againſt 6 on the Line D ſtands 28 on the Slider, which 28 multiplied by the ſquare of 10, gives 2800 the number required, for as 9 : 3600 : : 7 : 2800.

RULE V.

Between two given numbers, to find a Geometrical mean Proportional. (as *per* Prob. 36.)

EXAMPLE V.

Between 4 *and* 9 *to find a mean Proportional?*

Set

Set the leſſer number 4 on the Slider to the ſaid leſſer number 4 on the Line D, then againſt the greater number 9 on the Slider, ſtands the mean proportional on the Line D, *viz.* 6.

RULE VI.

Extraction of the *Square Root* by the Sliding Rule *Prob.* 37.

This is the ſame thing as to find a mean proportional between 1 *and the number given to be extracted.*

EXAMPLE VI.

What is the Square Root of 15129 *?*

Set 1 on the Slider to 1 on the Line D, then 15129 goes off the Rule, ſo I divide it by the ſquare of 10 or ſquare of 100 *&c.* till it come on the Rule, ſo 1529 divided by the ſquare of 100 gives 1.5129, againſt which on the Slider, ſtands 1,23 on the Line D, which being multiplied by 100 (becauſe I divided by the ſquare of 100,) gives 123 the root required.

The reaſon of theſe three rules is plain, if we conſider that the Sliding Rule is logarithmical, and that the Line D is of a double radius to the other Lines *&c.* Thoſe that would ſee more examples in theſe three rules, are deſired to trace ſolid meaſure and Gauging.

SECT.

SECT. II.

Multiplication of Feet and Inches.

THERE are ſeveral ways of working multiplication of feet and inches, commonly call'd croſs multiplication, but the eaſieſt for a Learner is that in *Harvney's* Compleat Meaſurer, which is as follows :

12 *Inches*	make	1 *Foot*,
12 *Parts*		1 *Inch*,
12 *Seconds*		1 *Part*,
12 *Thirds*		1 *Second*,
12 *Fourths*		1 *Third*,

Now if each ſide of a ſquare, be 1 foot or 12 inches, then its area (*per* Theorem 7) will be 1 foot, *viz.* 1 x 1 = 1, or 1 foot multiplied by 12 inches is equal to 12, which here are called inches, becauſe being divided by 12 makes the area 1 foot as before, alſo 12 inches multiplied by 12 inches, gives 144 ſquare inches which divided by 12 gives the abovenamed inches &c.

From

From hence we have the following rules;

Feet multiplied by *Feet* give *Feet.*

Feet by Inches	give	*Inches.*
Feet by Parts		*Parts.*
Inches by Inches		*Parts.*
Inches by Parts		*Seconds.*
Parts by Parts		*Thirds.*

So that if you begin to multiply parts by feet in the firſt row, parts by inches in the ſecond row, and parts by parts in the third row, the firſt figure in every row will ſtand a place more towards the right hand, ſee the following Examples,

EXAMPLE I.

	F.	I.	P.
Multiply	8	9	
By	3	6	
	26	3	
	4	4	6
Anſwer	30	7	6

EXAMPLE II.

	F.	I.	P.
	24	10	
	8	9	
	198	8	
	18	7	6
	217	3	6

In EXAMPLE I,

Multiply 9 inches by 3 feet ſaying, 3 times 9 is 27 quotes 2 feet and 3 remains, which 3 ſet down under inches and carry 2, ſaying 3 times 8 is 24 and 2 that I carry makes 26 which ſet down under feet.

In

2. In the ſecond multiplier which is 6 Inches, I begin thus, 6 times 9 is 54 which is 4 inches and 6 parts, (for 54 divided by 12 quotes 4 and 6 remains,) ſo ſet 6 down under parts and carry 4, ſaying 6 times 8 is 48 and 4 that I carry make 52, which divided by 12 gives 4 feet and 4 inches remains which ſet down as you ſee in the operation, and add to 26 feet and 3 inches, and you'll have 30 feet 7 inches and 6 parts for the anſwer, and the ſecond example is wrought the ſame way.

EXAMPLE III.

Multiply 3 *Feet* 4 ½ *Inches by* 2 *Feet* 8 ¼ *Inches?*

Becauſe 12 parts is an inch, ½ an inch will be 6 parts, and ¼ of an inch 3 parts, ſo this Example will ſtand thus,

F.	I.	P.	
3	4	06	
2	8	03	
6	19	: 00	S. T.
2	3	: 00 : 0	: 0
0	: 0	: 10 : 1	: 6
Anſwer	9	: 0 : 10 : 1	: 6

Here I begin to multiply 6 parts by 2 feet and ſo get the firſt row then I begin to multiply 6 parts by 8 inches and ſo get the ſecond row, laſtly multiplying with the 3 parts, to get the third row &c, as before.

But if there are more figures in the feet than one it will be easier to work by aliquot parts thus, multiply all the feet inches parts &c. in the multiplicand by the feet in the multiplier as you did before, and then divide the multiplicand by such alliquot parts, as you find contain'd in the inches parts &c. of the multiplier, which added together give the answer.

So the first and second Examples stands thus,

EXAMPLE I.

F.	I.	P.	
8	9		
3	6		
26	3		
4	4	6	½ of 8 feet 9 inches
Answer 30	7	6	

EXAMPLE II.

F.	I.	P.	
24	10		
8	9		
198	8		
12	5		½ 24 feet 10 inches
6	2	6	¼ of 24 feet 10 inches
Answer 217	3	6	

In EXAMPLE I.

Six Inches being half a Foot I take half of the multiplicand, *viz.* half of 8 feet 9 inches, which is 4 feet 4 inches, and ½ an inch or 6 parts, becauſe 12 parts is an inch, ſo you have the anſwer as before.

In EXAMPLE II.

The aliquot parts of the inches, 9 in the multiplier are ½ and ¼ *viz.* 6 inches is ½ a foot and 3 inches is ¼ of a foot, (and 6 + 3 = 9,) therefore I take ½ the multiplicand which is 12 feet 5 inches and ¼ thereof, which is 6 feet 2 inches, and ¾ of ½ remains which is 6 parts, then adding the two laſt rows together you have the anſwer as before, and thus by theſe rules may you work any example in feet and inches, to aſſiſt wherein, take the following Table.

A TABLE of *Aliquot Parts.*

1 *Inch*	is	1 *Twelfth*	of a Foot.
2 *Inches*		1 *Sixth*	
3 *Inches*		1 *Fourth*	
4 *Inches*		1 *Third*	
5 *Inches*		1 *Third* & 1 *Twelfth*	
6 *Inches*		1 *Second*	
7 *Inches*		1 *Second* & 1 *Twelfth*	
8 *Inches*		1 *Second* & 1 *Sixth*	
9 *Inches*		1 *Second* & 1 *Fourth*	
10 *Inches*		1 *Second* & 1 *Third*	
11 *Inches*		2 *Thirds* & 1 *Fourth*	

EXAMPLE IV.

	F.	I.
	97	8
	8	9
Here 97 feet 8 inches × 8 is —	781	4
¼ of 97 feet 8 inches is ——	48	10
¼ of 97 feet 8 inches is ——	24	5
Answer	854	7

EXAMPLE V.

	F.	I.	
	75	9	
	17	7	
	525		
	75		
½ of 75 feet 9 inches is —	25	3	P.
¾ of 75 feet 9 inches is —	18	11	3
½ of 17 feet is ——	8	6	0
¼ of 17 feet ——	4	3	0
Answer	1331	11	3

When the feet in the multiplier are above 12 (*as in Example* 5 *they are* 17,) it will be easier to multiply the feet 75 and 17 together letting the 9 inches stand, till you have wrought for the 7 inches, then because the alliquot parts of 9 inches are ½ and ¼; I take the ½ of 17 feet, and ¼ of 17 feet

feet, (*not meddling with the 7 inches becauſe they were noted before*) and add all together, as you may ſee by the operation; This method alſo holds true in feet inches parts *&c.* but as one part being one twelfth of an inch, is near enough to take dimenſions, I ſhall go no lower only add the following Examples for Practice.

EXAMPLE VI.

	F.	I.	P.		
	37	7	5		
	4	8	6		
	150	5	8	S.	
⅓	12	6	5	8	
⅓	12	6	5	8	T.
½	1	6	9	8	6
	177	1	5	0	6 Anſwer.

Here becauſe 6 parts is ½ an inch, and feet multiplied or divided by inches give inches, therefore thus ⅓ of 37 feet, muſt be 18 ½ inches or 18 inches and 6 parts, which is 1 foot 6 inches and 6 parts, ſo ſet 1 under feet and 6 under inches, then this ⅓ 7 inches muſt be 3 ½ parts, or 3 parts 6 ſeconds, ſo 3 parts added to the above 6 parts gives 9 parts, which ſet down under parts alſo ⅓ of 5 parts in this caſe muſt be 2 ½ ſeconds or 2 ſeconds 6 thirds, ſo 2 ſeconds added to the laſt 6 ſeconds gives 8 ſeconds, which ſet down under

under 1 or ſeconds, and for the $\frac{1}{T}$ ſecond or 6 thirds, ſet 6 down under thirds or T which added toge-the gives 177 feet 1 inch 5 parts 0 ſeconds and 6 thirds as before.

EXAMPLE VII.

Let 8 feet, 4 inches, 3 parts, 5 ſeconds, 6 thirds, be multiplied by 3 feet, 3 inches, 7 parts, 8 ſeconds, 2 thirds ?

F.	I.	P.	S.	T.				
8	4	3	5	6				
3	3	7	8	2				
25	0	10	4	6				
2	1	0	10	4	6			
	4	10	6	0	2	6		
		5	6	10	3	8	0	
			1	4	8	6	11	0
Prod. 27	7	3	5	1	8	8	11	0

In this laſt Example there is no difficulty, if you do but obſerve the former directions, and ſet every row a place more to the right Hand.

SECT.

SECT, III.

Notation of DECIMALS.

IF the denominator of any Fraction be 1 and a certain number of cyphers, then ſuch a Fraction is call'd a Decimal Fraction, $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{10}$ $\frac{5}{0}$ &c. are all Decimal Fractions, now ſuch denominators as 10, 100, 1000 &c. being always 1 and a number of cyphers, the numerators may be expreſs'd without thoſe denominators thus, 0,5 may ſtand for $\frac{5}{10}$, and ,05 for $\frac{5}{100}$ &c. where the Comma, or Period . ſtands in the place of unites, the firſt figure in the place of tens, the ſecond figurein the place of hundreds, the third in the place of thouſands &c. beginning at the left hand and numbering towards the right hand juſt contrary to whole numbers, as you may ſee, in the following Table.

TABLE.

TABLE.

C. Thousands	X. Thousands	Thousands	Hundreds	Tens	Units	Tenth Parts	Hundredth Parts	Thousandth Parts	X. Thousandth Parts	C. Thousandth Parts
-	6	5	4	3	2	1.	2	3	4	5	6

Therefore if you were to write down any whole number, suppose 4 thousand, you write it thus 4000, but if you write it down 4 thousand parts to Decimals, you must see it down thus ,004 the comma always denoting unites.

COROLLARY.

From hence it appears that the true value of all Decimal parts, are known by their distance from the units place, and therefore cyphers on the left hand side of Decimals, must be carefully observ'd, but cyphers on the right hand side of Decimals, effects their value nothing, and so need not be set down, unless for illustrations sake, as for Example, $\frac{5}{10}$ *viz.* 0,5 or .5 is equal to .50 or $\frac{50}{100}$, because both the numerator 5 and denominator 10, are multiplied by 10, so the value is not altered, consequently ,5 = ,500 = ,5000 &c.

NOTE

NOTE I.

☞ Some define a Fraction thus, if the integer or whole part, be divided into any number of equal parts ſuppoſe 36, then theſe 36 equal parts may be term'd inches, and the integer a yard, now any number of theſe parts leſs than the whole ſuppoſe 7, may be expreſs'd thus, 7 inches or $\frac{7}{36}$ of a yard, that is 7 of the 36 parts of a yard *&c.* Alſo in diviſion of whole numbers; if after diviſion is perform'd there be a remainder, this remainder is call'd the numerator, and the diviſor the denominator and ſo we have a fraction or broken number; Example, if 80 be divided by 3 the quotient will be 26, and the fraction $\frac{2}{3}$ ſo the full quotient is $26\frac{2}{3}$ *&c.* now to reduce theſe Vulgar Fractions to Decimal Fractions, obſerve the following Rules.

Reduction of Decimals.

To reduce a Vulgar Fraction to a Decimal.

RULE.

ANNEX or bring down as many cyphers to the numerator, as being divided by the denominator may leave no remainder, (if it can be ſo) *and the Quotient is the Decimal Fraction requir'd.*

EXAMPLES.

EXAMPLE I.

Ruduce $\frac{1}{2}$ and $\frac{1}{25}$ to Decimals?

2) 1,0 (,5 = $\frac{1}{2}$
1 0
0

EXAMPLE II.

20) 1,00 (,05 = $\frac{1}{25}$
100
0.

EXAMPLE III.

Reduce $\frac{1}{200}$ and $\frac{1}{8}$ to Decimals?

200) 1,000 = $\frac{1}{200}$
,005

EXAMPLE IV.

8) 1,000 = $\frac{1}{8}$
,125

NOTE II.

☞ To determine the value of these Decimals I shall explain Example third thus, first I say how often the whole denominator 200, in the whole numerator 1, it goes no times, so I write down the comma, in the quotient, then I say how often the denominator 200, in the numerator 1 and one cypher, and it goes 0, so I write 0 in the quotient after the comma, again how often 200 in 1 and 2 cyphers, it goes no times again, so I write a second cypher in the quotient, again I try with 200 in 1 and 3 cyphers, and find it goes just 5 times so I set 5 in the quotient after the second 0, so I have the Decimal ,005 equal $\frac{1}{200}$ as was requir'd, this observe in every Example of this kind, and you'll easily get the Decimal.

The

The reaſon of this note and the foregoing rule is evident from the operation, as in Example 2, if you multiply both the numerator and denominator ot the given Fraction $\frac{1}{20}$ by 5 it will become $\frac{5}{100}$ whieh is equal to ,05, by the foregoing notation, and ,05 = $\frac{1}{20}$ by the foregoing corollary &c.

EXAMPLE V.

Reduce $\frac{1}{13}$ to a Decimal?

13) 1,0000 (,076923076 &c. = $\frac{1}{13}$ nearly.

91

90
78

120
117

30
26

40
39

10

NOTE III.

☞ Here I annex 4 cyphers to the numerator 1 and bring down 3 more, but I ſee there will always ſomething remain, for the laſt remainder is

is 1 which is equal to the laſt dividend, and therefore the figures in the quotient will return again, and circulate in the ſame order as before, ſo that in all ſuch caſes, you may continue the quotient as far as you pleaſe without diviſion, by only repeating the ſame figures again as you ſee here where obſerve, that more figures you have in the quotient the nearer the truth you come, tho' in all ſuch caſes your Decimal Fraction, will never be exactly equal to the given Vulgar Fraction, yet 4 or 5 places of figures in the Decimal, are ſufficiently exact in moſt buſineſs, for the fourth place of Decimals ſtands in the place of thouſands, and therefore what it differs from truth can only be ſome parts of ten thouſand, which is but ſmall in reſpect of the whole &c.

EXAMPLES.

Reduce $\frac{1}{3}$ $\frac{2}{3}$ and $\frac{11}{3}$ to Decimals?

Example 5.

3) 1 000 } &c.
,333 } equal $\frac{1}{3}$ nearly.

Example 6

3) 2,00 } &c.
,66 } equal $\frac{2}{3}$

Example 7.

3) 11,000 } &c.
3,666 } equal $\frac{11}{3}$ nearly.

NOTE IV.

Becauſe 4 places of Decimals are prov'd to be ſufficient, I have only continued theſe 3 Examples to 4 places where obſerve, that if the fifth place

place be 5 or more, you may venture to make the fourth place one more than it is as in Example 6 and 7, the fifth place would be 6, so I call the fourth place *viz.* 6 one more than it is *viz.* 7 as *per* opperation; but when the said fifth place is under 5 as in Example 5, this increase is too much to be made *&c.* this Note will be of great use in Addition and Multiplication of Decimals, for it is easy to understand that if one number be a small matter too much, and another a small matter too little, that in Adding or Multiplying they may nearly ballance; but in Substraction or Division, the error may be more considerable, and therefore in Subtraction and Division, the Decimals should be wrought with as the turn out; in the improper Fraction Example 7, the denominator 3 is contain'd 3 times in the numerator 11, so this 3 in the quotient must be a whole number.

EXAMPLES

Reduce 5 inches to the Decimal parts of a foot, and also of a yard?

Example 8.

12) 5,00 (,4167 = $\frac{5}{12}$ parts of
48 a Foot nearly.

Here 12 inches are a foot and 36 inches one yard, so we only have $\frac{5}{12}$ and $\frac{5}{36}$ to reduce to Decimals.

20
12
80
72
8 &c.

Example

Example 9.

36(5,00(,1389 = $\frac{5}{32}$ Parts of a Yard nearly.
36

140
108

320
288

32 *&c.*

Example 10.

Reduce 2 $\frac{1}{2}$ or 2,5 inches, to the Decimal of a Foot?

24) 5,000 (,2083 = $\frac{5}{24}$ nearly,
48

200
192

80
72

8 *&c.*

For 2 $\frac{1}{2}$ inches are = 5 half inches, & 12 inches are = 24 half inches *&c.*

Otherwise.

12) 2,5000 } = $\frac{25}{12}$ nearly.
,2083 }

After the same method you may reduce Coin, Weight, Measure *&c.*

Example

Example 11.

Reduce 17 s. 6 d. to the Decimal of a Pound Sterling?

In 1 Pound are 40 Sixpences, and in 17 s. 6 d. are 35 Sixpences, ſo we have $\frac{35}{40}$ to reduce to a Decimal.

40) 35,000 } &c. &c. }
,875 } = 17 s. 6 d. } the Integer 1 £.

Example 12.

Reduce 8 d. $\frac{1}{2}$ to the Decimal of a Shilling Sterling?

In 1 Shilling are 24 half Pence, and in 8 d $\frac{1}{2}$ are 17 half Pence, ſo we have $\frac{17}{24}$ to reduce to a Decimal.

24) 17,00 (,7083 = $\frac{17}{24}$ = 8 d. $\frac{1}{2}$ nearly.
168

200
192

80
72

8 &c.

All or any of the Examples may be wrought by the Sliding Rule (per *Rule* 2 *Section* 1,) thus

thus in Example 11, ſet 40 on the Slider to 35 on the Rule, then againſt 1 on the Slider ſtands ,875 on the Rule *&c.* for any other whatever.

On the out edge of moſt common Sliding Rules, againſt the Line of Inches, ſtands a Line of 12 Inches being Decimally divided *viz.* into 100 equal parts number'd 10, 20, 30 *&c.* to 100, by which inches may be had in Decimal parts of a foot by inſpection.

As for Example,

Let it be required to find the Decimals of 6 inches, and of 1 $\frac{1}{2}$ inches, the integer being a Foot by the Sliding Rule.

Againſt 6 on the inches, ſtands 50 that is $\frac{50}{100}$ or 0,5 on the Decimal Line; alſo againſt 1 $\frac{1}{2}$ that is 0,125 on the Decimal Line, ſo that the Decimal of 6 inches is ,5 and of $\frac{1}{12}$ inches is ,125 *&c.* for any ſuch like. But if you practice what is taught at the beginning of Superficial Meaſure, all this Inſpection, Reduction *&c.* need never be us'd in Menſuration Gauging *&c.*

ADDITION

ADDITION and SUBTRACTION of *Decimal Fractions.*

THESE are perform'd in all respects like Whole Numbers, regard being had to place units under units, tens under tens &c. as well in the Integers as in the Decimal parts.

Examples in Addition.

Example 13.	*Example* 14.	*Example* 15.
,0057	,785623	25,687
,0002	,02	1032,02
,06	,5	4,5
,083	,079	4232,0689
,1489	1,384623	5294,2759

Examples in Subtraction.

	Example 16.	*Example* 17.	*Example* 18
From	,3123	25,02	270
Take	,0102	3,98	,028
Remains	,3021	21,04	269,972

MULTIPLICATION of *Decimals.*

IF you multiply 3,000 by 7,00, the Product will be 21,000,00 from whence we have this general Rule.

Rule { *There must always be as many places of Decimals in the Product, as are in both Factors,* viz. *in the Multiplier and Multiplicand.*

EXAMPLES.

	Example 19.	*Example* 20.	*Example* 21.
Multiplicand	3,024	32,12	,0347
Multiplier	2,23	24,3	,0236
	9072	96 36	2082
	6048	12848	1041
	6048	6424	694
Product	6,74352	780,516	,00081892

NOTE V.

If there be not so many figures in the Product, as there are places of Decimals in both Factors, you must prefix Cyphers to the Product, as in Example 21.

NOTE VI.

When the Factors together contain more places of Decimals, than they would chose to have in the Product, they may be contracted to any

any desired number, and yet the Product be as true as far as it goes, as if the Factors had been multiplied at large, for which observe this Rule,

Under that place of the multiplicand which you would reserve, set the unites place of the multiplier, and write the rest of the figures in an inverse order; then begin to multiply each figure in the multiplicand, by that figure in multiplier which stand exactly underneath it, (having due regard to the increase that would be brought thither, from the foregoing figures,) and set every first figure under the units place of the multiplier, and the rest towards the left hand as in common multiplication, and the sum of these Products give that sought.

Let it be required to multiply 3,141592 with 52,7438, and to have but four place in the Product. And 257,356 by 76,48 and no Decimals in the Product. Also 0,248264 by 0,725234, and to have five places of Decimals in the Product.

Example 22.	*Example* 23.	*Example* 24.
3,141 592	257,356	,248264
8 347,25	84,67	432527
1 5707 96	18015	17378
628 32	1544	497
219 91	103	124
12 57	20	5
94		
25	19682	,18004
165,6995		
4 Places.	Whole Numbers.	5 Places.

Tha

These 3 Examples with the above Rule, and a little practice will make Contracted Multiplication easy, and that this method is true, and to exercise Multiplication further, observe these 3 Examples wrought at large thus.

EXAMPLE XXII.

3,1414592
52,738

25,132736
94,24776
1256,6368
21991,144
628,3184
17,5079680

165,6995,001296

EXAMPLE XXIII.

257,356
76,48

20,58848
102,9424
1544,136
18014,92

19682,58688

EXAMPLE XXIV.

,248264
,725234

993056
744792
4 96528
124 1320
496 528
17378 48

,180004 493776

By these 3 Operations at large, the common way you may see the reason of inverting the *Multiplier*, for which I have drawn a Line thro' each Operation, to illustrate this method which is exact and short, by the common way.

Or by *Note* 4, you may have a round guess of any *Decimal* Product, thus in *Example* 24, I take the 3 first figures in the *Multiplicand*, and the two first in the *Multiplier*, only calling the figure 2 a 3, and you see the Product ,18104 differs only ,001 from truth, and it would not have erred so much, but the figures which follows 8 and 5, in the Factors are but small *viz.* each but 2 *&c.*

Ex. 24. *Otherwise.*

,248
,73
744
1736
,18104

To find the value of any Decimal Fraction?

This is commonly done in *Reduction*, but because it is perform'd by multiplying the given *Decimal*, by the number of parts in the *Integer &c.* I have done it here.

EXAMPLES.

What is the value of ,875 *parts of a Pound Sterling; and of* ,70833 *&c. of a Shilling?*

EXAMPLE XXV.

,875 ℔

20 *Shilling in a Pound.*

17,500

12 *Pence in a Shilling.*

6,000 *Answer* 17 *s.* 6 *d.*

EXAMPLE

EXAMPLE XXVI.

,708333
12 *Pence in a Shilling.*

8,49999
4 *Farthings in* 1 *Penny.*

1,99999 *Answer* 8 £.

What is the Value of ,75 *of a Lineal Foot?*

EXAMPLE XXVII.

,75 *Parts of a Foot in Length.*
12 *Inches in* 1 *Foot.*

150
75

9,00 *Answer* 9 *Inches.*

From hence you may learn, that this is no more but just Proof to *Reduction*, as appears by comparing *Example* 25 and 26, with *Example* 11 and 12.

Division

DIVISION of DECIMALS.

AS in *Whole Numbers*, *Diviſion* proves *Multiplication*, ſo in *Decimals* it doth the the ſame, therefore if there are more figures in the *Dividend* than in the *Diviſor*, Divide them as *Whole Numbers*, but if there be more figures in the *Diviſor* than in the *Dividend*, you muſt annex cyphers to the *Dividend*, till *Diviſion* can be made, and then the *Decimals* in the Quotient are had by this general *Rule*.

RULE.

The number of Decimals *in the Quotient, muſt always be equal to the difference between the number of* Decimals *in the* Dividend, *and the* Diviſor, *which is juſt contrary to the foregoing* Rnle *for* Multiplication.

EXAMPLES:

Divide 780,516, *by* 24,3 ?

EXAMPLE XXIX.

24,3) 780,516 (32,12

515

291

486

(0)

EXAMPLE

EXAMPLE 30.

Divide ,00081892 *by* ,0236.

,0236),00081892(,0347
708

1109
944

1652
1652
---
(0)

EXAMPLE 31. *Divide* 1 *by* ,7854.

,7854)1,0000(1,2732 &c.
7854

21460
15708

57520
54978

25420
23562

18580
15708

2272 &c.

NOTE VI.

NOTE VII.

Having once determin'd the first Decimal Figure in the Quotient, the following Figures will be Decimals, if you bring down ever so many Cyphers, as Ex: 31. *So that when Division is ended if there be a Remainder, you may bring down Cyphers and go on as far as you please, as in* Reduction.

NOTE VIII.

Since any Dividend Divided by the Quotient will produce the Divisor; so in Ex: 31. if we Divide 1. by 1,2732 the Quotient will be ,7854. But 1 Multiplied by ,7854 is equal to ,7854; so that 1. Divided by 1,2732 or Multiplied by ,7854 is the same thing: Consequently any Number Multiplied by ,7854 is equal to the same Number Divided by 1,2732. And thus you may turn any Multiplier into a Divisor, or Divisor into a Multiplier by only Dividing 1 by the given Number.

Thus if 282,231 *and* 2150,42 *be each Divisors Let it be required to Convert them into Multipliers.*

Example 32.

282) 1,000 (,003546
846

1540
1410

1300
1128

1720
1692

28

Example 33.

231) 1,000 (,004329
924

760
693

670
462

2080
2079

1

Example 34.

2150,4) 1,000000 (,000465
860168

1398320
1290252

1080680
1075210

(5470)0

Because Multiplication is shorter and easier to Work than Division I have in Mensurations *&c.* made Use of Multiplication as much as possible by the Pen, and since Divisors are used by the sliding Rule I have here given the above Note, with its 3 Examples; whereby any of that kind may be done.

NOTE

NOTE IX.

The foregoing Rule conſiders the Diviſor, Dividend and Quotient, all Decimals; but if any of them (viz.) *the Diviſor and Dividend be mixt Numbers, its alſo true, if you conſider the Integeral parts by themſelves as for* Example *if there be* 3 *places of Decimals in the Diviſor and* 2 *places of Decimals in the Dividend (let the whole Numbers be what they will) you muſt annex a Cypher to the Dividend and then the Decimals in the Diviſor and Dividend will be equal* (viz.) *each equal to* 3 *places in this* Example, *and ſo the Quotient will be whole Numbers while that Diviſion continues*, &c. *of any other. As Multiplication may be contracted, ſo there is alſo a Method for contracting Diviſion which will be of great Uſe, when there are many Decimals in the Diviſor and it is as follows*;

Having determined how many places of whole Numbers there muſt be in the Quotient, if there be any at all or if there be none at all it is the ſame, for every figure you place in the Quotient leave out one in the Diviſor towards the right Hand, having due regard to the increaſe that would ariſe from the figure ſo left out, and ſo Divide on, as uſually.

EX. 35.

EXAMPLE 35.

Divide 70,23 *by* 7,9863,

Contracted.	The same at Large.
7,9863) 70,2300 (8,7938	7,9863) 70,2300 (8,7938
638904	638904
63396	63396 0
55904	55904 1
7492	7491 90
7187	7187 67
305	304 230
239	239 589
66	64 6410
63	63 8904
3	07506

Thus I have given the grounds of Decimal Arithmetic, which being well understood may be applied to all the Rules in common Arithmetic, But since such Rules are of little Service in this Book, I decline them here, And so come to Extracting Roots.

SEC.

SECTION IV.

Extraction *of the* SQUARE ROOT *the common Way* &c.

DEFINITION I.

IF you Multiply any Number by itſelf, the Product is called the Square of that Number, and the Number itſelf is called the Square Root of that Product, ſo 2 multiplied by 2 gives 4, therefore 4 is the ſquare of 2, and 2 is the Square Root of 4, alſo 25, is the Square of 5, and 5 is the Square Root of 25 *&c.* In like Manner the Cube of any Number is had by Multiplying the Number 3 times, the *Biquadrate* of any Number by Multiplying it 4 times into its ſelf, the *Surſolid* 5 Times (continually) *&c.* ſo the Square *Cube*, *Biquadrate*, *Surſolid* and the ſixth Power of C, is 262144, and the *Square Root*, the *Biquadrate Root*, the *Cube Root* the *Surſolid Root*, and the Root of the 6*th* Power of 262144 is C, *&c.* After which Method the following Table is calculated.

A Table of *Powers.*

Root, or 1ſt. Power.	Square, or 2d. Power.	Cube, or 3d. Power.	Biquadrat, or 4th. P.	Surfolid, or 5th. P.	Square Cubed or Cube Squared; 6th. P.	2d. Surfolid, or the 7th. Power.	Biquadrat Squared, or 8th. Power	Cube Cubed, or 9th. Power.
	Index (2)	*Index* (3)	*Index* (4)	*Index* (5)	*Index* (6)	*Index* (7)	*Index* (8)	*Index* (9)
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683
4	16	64	256	1024	4096	16384	65536	262144
5	25	125	625	3125	15625	78125	390625	1953125
6	36	216	1296	7776	46656	279936	1679616	10077696
7	49	343	2401	16807	117649	823543	5764801	40353607
8	64	512	4096	32768	262144	2097152	16777216	134217728
9	81	729	6561	59049	531441	4782969	43046721	387420489

By

By this Table, you may find the Power of any Number under 10, to any Power under 10 and ſo take the neareſt Root of any Square *Cube*, *Biquadrate* &c. of any Number whoſe Root is a ſingle Figure.

DEFINITION II.

The Square of 10, is 100 (by the 1 DEF.) *viz.* 1 and two Cyphers, the *Cube* of 10 is 1000 (*viz.*) 1 and 3 Cyphers, the *Biquadrate* of 10 is 1 and 4 Cyphers *&c.* the Square of 100 is 10000, its Cube 1,000,000, its *Biquadrat* 100,000,000, *&c.* that is the Number of Cyphers multiplied by 2 gives the Square, by 3 gives the Cube by 4, the *Biquadrate* by 5, the *Surſolid &c.* hence the Index of the Square is 2 of the Cube 3 *&c.* As in the Table.

DEFINITION III.

When any Number is given to be extracted the firſt Thing to be done, is to divide it into Periods by Points ſet over the Figures, as the Index of the Power directs, and from thence we ſee how many places of Figures will be in the Root, and in pointing the Figures you muſt always begin at the Units Place, and make a Point over every ſecond Figure for the Square Root over every third Figure, for the Cube Root

Root *&c.* both in whole Numbers and Decimals, as for *Example*, if you were to Extract any Root out of 756403872 46 it must be pointed thus,

viz. For the { Root of the Square 75640387246 / Cube Root 75640387246 / Biquadrat Root 75640387246

Or if the Number were the Decimal, 0,6740359 82.

Then for the { Square Root, 0,6740359820 / Cube Root, 0,674035982 / Biquadrat Root 0,6740359820 00 / Sursolid Root 0,6740359820 *&c.*

So that it appears the Square Root of 756403 87246, will consist of 6 places of Figures, its *Cube Root* of 4 places, its *Biquadrat Root* of 3 places *&c.* also the *Square Root* of the Decimal ,674035982, will be 6 places of Decimal Figures, its *Cube Root* 4 places *&c.*

DEFINITION IV.

If any Decimal Fraction be given to be extracted, you must by annexing Cyphers, make the Number of Places or Figures therein equal to some Multiple of the Index of the Root which you

you are to Extract, as in the above Decimal, there is one Cypher annex'd for the Square Root, and then the Number of places are even, or diviſible by 2, the Index of the Square Root: Alſo, for the *Biquadrat Root*, there is 3 Cyphers annexed and then the Number of places is diviſible by 4, the Index of the *Biquadrat* &c. The Reaſon of which is evident from the ſecond Definition, for the Denominator of every Decimal Fraction, being 1, and a Number of Cyphers if the Number of the ſaid Cyphers be not diviſible by the Index of the Root, the exact Root of the ſaid Denominator cannot be had or when you have taken your required Root, and attempt to prove it, it will not produce the Decimal firſt taken, as for *Example*; if you were to take the *Cube Root* of 0,8, and call it 0,2 (becauſe by the Table, 2 is the *Cube Root* of 8) Then 0,2 multiplied by 0,2 and that Product by 0,2 gives ,008 which ought to have been 0,8; ſo ,2 is not the Cube Root of ,8, but the *Cube Root* of 0,8000 will be the *Cube Root* of 0,8 &c. for any other.

Thus I ſuppoſe I have fully defin'd the Extraction of *Roots*, I ſhall now ſhew how to particularize the ſame, and firſt of the *Square Root* after the common Method.

1. Having pointed the given Number as before directed into Periods of 2 Figures each take the neareſt leſs Square Root of the firſt Period towards the left hand, (which you may eaſily find by the foregoing Table) and place it like a Quotient Figure in common Diviſion,

then Subtract its Square from the said first Period, and to the remainder bring down the next Period for a Dividend, and double the Root or Quotient Figure for a Divisor inquiring how oft it may be had in that Dividend, so as when the Quotient Figure from thence arising, is annexed to the Divisor and multiplied by this Quotient Figure, the Product thus had may be the greatest that can be taken out of that Dividend, which Subtract out of the Dividend and to the remainder bring down the next Period, or two Figures for a new Dividend, and double the whole Quotieut for a new Divisor and so proceed as before, and thus go on untill all the Periods be brought down, and if nothing remain you have the true Root required, but if there be a remainder, as mostly happens, you may bring down Cyphers two at a Time, and so run the Root into Decimals as far as you please: A few *Examples* will make this Plain.

EXAMPLE I.

What is the Square Root *of* 321489.

This being prepared by Points as *per* DEFINITION 3d stands Thus,

```
                . . .
          321489 ( 567 Root reqd;
            25
          ------
1 Divisor 106) 714
               636
               ---
2 Divisor 1127) 7889
                7889
                ----
                  (0)
```

Here

Here the neareſt leſs Root to 32 the firſt Period is 5 whoſe Square is 25 which taken from 32 leaves 7 to which bring down the next Period 14 and you have 714 for the firſt Dividend and the double of the Root 5 is 10 for the firſt Diviſor, then ſay how oft 10 is 71 (leaving the Figure 4 out becauſe the Figure ariſing is to be annexed to 10) and it goes 6 times, ſo ſet 6 after 10 and it becomes 106, which multiplied by 6 gives 636 and taken from 714 leaves 78 to which bringing down the next Period 89 you have 7889 for a ſecond Dividend and 56 doubled is 112 for a ſecond Diviſor, and you'll find 112 to be contained 7 times in 788, ſo 7 is annexed to 112 and all multiply'd by 7 gives 7889 which is exactly equal to the Dividend 7889, and therefore 567 is the juſt *Square Root* of 321489.

EXAMPLE II.

Required the Square Root *of* 32,1489,

```
32,1489 (5,67 Root ſought,
25
---
106)714
    636
---
1127) 7889
      7889
---
      (0)
```

EXAMPLES

EXAMPLE III.

Required the Square Root *of* 321,489;

```
. . . .
321,4890) 17,9301 &c. for the Root re-
1                      [quired, viz. of
                       [321,489.
27) 221
    189
349) 3248
     3141
3583) 10790
      10749
358601) 410000
        358601
        51399,) Remainder.
```

EXAMPLE IV.

Required the Square Root *of* ,0321489;

```
. . . .
,0321489 (0,1793 for the Square Root
1                [of ,0321489
27) 221
    189
349) 3248
     3141
3583) 10790
      10749
      (41) Remainder.
```

EXAMPLE

EXAMPLE V.

Required the Square Root *of* $\frac{4}{9}$

The Decimal of $\frac{4}{9}$ is = ,6666 (,81649 *&c.*

64 { *for the* Square Root *of* $\frac{4}{9}$

161) 266
161

1626) 10566
9756

16324) 81066
65296

163289) 1577066
1469601

Remains (107465)

In *Example* 2d, the Decimal places are juſt Periods themſelves, but in *Example* 3*d* and 4*th* there is a Cypher annexed to make them ſo, alſo in theſe *Examples* 3 and 4, there is a remainder ſo Cyphers may be brought down (two at a Time) as long as you pleaſe, in *Example* 3*d* I have brought down Cyphers twice, but becauſe the Decimal of $\frac{4}{9}$ is always 6's, in *Example* 5, I bring down 66 at a Time *&c.* ſee the Work.

EXTRACTION *of* ROOTS *out of any Single Power by one* General METHOD, *deduced from the* CONVERGING SERIES.

In order to perform this, Observe carefully the following General Rules.

RULE 1. POint the given Number as directed in the 3d. DEF. foregoing.

RULE 2. Find the neareſt Root to the 1ſt. Period towards the Left Hand, and to this Root annex a Cypher for every Period, except one, in the given Number *vide* 3d DEFINITION, and raiſe it to the Height of the given Power.

RULE 3. Take the difference between this involv'd Root and this given Number for a Dividend, which *Note* with the ſign + if the given Number be greater, but with — if the given Number was leſs than the involv'd Root.

RULE 4. Raiſe the ſaid Root and Cyphers to a Power whoſe Index is one leſs than the Index of the given Power, and Multiply this Number by the Index of the given Power for a Diviſor, by which Divide the Dividend, taking but one Figure into the Quotient.

RULE 5. If the foreſaid difference was noted with + this Quotient Figure muſt be annexed to the firſt taken Root, but if noted

noted with – it must be taken from the said Root, and the Sum or difference must be wrought with in every Respect as with the first taken Root, and the next Quotient you get, you take two Figures and the next Quotient after that you may take 3 or 4 Figures *&c.* every time nearly, doubling the Number of Figures, by which Means you may have the Root of any single Power to a great many Places of Figures, which compared with the old Method is vastly easy: We shall examine this Method by an *Example* or two of the *Square Root*, and then apply it to higher Powers.

Ex. VI. *What is the Square Root of* 321489?

OPERATION

360000 = *Square of* 600

321489 *given Number Subtract.*

1200)38511 (600 *first Root*
36000 30 *Quotient Subtracted*

Remainder 2511 560 *second Root*

321489 *given Number*
313600 *Square of* 560 *Subtract*

1120)+7889(560 *second Root*
7840 7,0 *Quotient Add*

Remainder 49,0 567,0 *third Root*

Here

Here the neareſt Root to the firſt Period 32 is 6 and there are two Periods more in the given Number, ſo I put two Cyphers to 6 and it makes 600 which multiplied by its ſelf, gives 360000 which being greater then 321489, I Subtract 321489 therefrom and ſet the ſign — before the Remainder or Dividend 38511 now the Index of the Square being 2 and the next leſs Power to the Square being 1 or the firſt Power, (*Rule* 4.) the double of 600 *viz.* 1200, is the firſt Diviſor and the Quotient comes out 32, but we are to take but the firſt Figure 3, ſo I put a Cypher in the Room of 2, and here *Note*, the Reaſon of taking but 1 Figure in the firſt Diviſor, 2 Figures in the ſecond Diviſor *&c.* is, becauſe if we take more, the Root will be too much. hence, when the Quotient is to be ſubtracted you may carry 1 to the firſt Figure towards the Right Hand, ſo in this *Example*, inſtead of taking 30 from 600 I take 40 therefrom for it is to be ſubtracted becauſe 38511 is noted with —, the carrying of this 1, would ariſe from taking the 2 out of 0, (*viz*,) 32 from 600, but when the Quotient is to be added, ſuch increaſe muſt not be regarded.

But 40 taken from 600 leaves 560 for the ſecond Root, which gives 313600 when ſquared and being leſs than the given Number, is ſubtracted therefrom and + 7889 remains for a ſecond Dividend, which divided by twice 560 *viz.* 1120 gives 7,0 for here we are to take two Figures into the Quotient, and becauſe the Dividend is *noted* with + this 7,0 is added to 560 which

which gives 567 for the third Root, and being squared is equal to the given Number, so 567 is the true *Square Root* of 321489.

EXAMPLE VII.

What is the Square Root of 7.

OPERATION

9 the nearest Square to 7,
7 the given Number,

6) — 2,0 { 3 the first Root,
18 { 0,3 the Quotient Subtract;

Remains (2) 2,6 second Root,

7 given Number,
6,76 Square of the second Root;

5,2) + ,240 (2,6 second Root
208 ,046 Quotient add.

320 2,646 third Root,
312

(8) *Remainder.*

7,0001316 Square of 2,646.
7 — — — given Number Subtract
6,292) — ,0013160 { 2,646 third Root,
10584 { ,0002486 Quo. Subtd

25760 2,6457513 fourth,
21168

45920 { Root true to the
42336 { last Figure,

35840
31752

Remains (4088)

Here the neareſt Squares to the given Number 7, are 4 and 9, but 9 is nearer to 7 than 4 ſo I take 9 whoſe *Square Root* is 3, and twice 3 is 6 the firſt Diviſor, alſo becauſe the Quotient 0,3 is to be ſubtracted from the firſt *Root* 3, I Subtract 0,4 as directed in the laſt *Example*, for the ſame Reaſon I Subtract ,0002487, inſtead of ,0002486 from the third *Root* 2,646, and ſo get 2,6457513 for the *Square Root* of 7, and if you Square this laſt *Root* and proceed as before, you may rely on 7 or 8 Figures in the Quotient, and ſo have the *Root* to 14 or 15 Places of Decimals.

This Method of Extracting *Roots* proves itſelf as it goes on, for here 3 is more than the *Square Root* of 7, but 2,6 is ſomewhat leſs, alſo 2,646 is ſomewhat more, & again 2,6457513 is ſomewhat too little ſo that every one of theſe *Roots* come nearer the Truth than the preceeding one, that is 2,6 is nearer than 3, but 2,646 is nearer than either 3, or 2,6 *&c.*

Ex. VIII. *What is the Cube Root of* 1728?

OPERATION.

1728		the given Number,
1000		the *Cube* of 10 Subtract
300)+728	(10	firſt *Root*,
600	2	Quotient add,
Remains 128	12	ſecond *Root*.

Here

Here the *Cube Root* of 1, the first Period is 1 to which annex a Cypher because there is another Period in the proposed Number 1728, and it becomes 10, whose Cube 1000 taken from 1728 leaves 728 for a Dividend, now the next Power below the Cube is the *Square*, and the Index of the Cube is 3, therefore by the fourth Rule, the Square of 10 is 100, which multiply'd by 3, gives 300 for the first Divisor, and the Quotient 2 added to 10 gives 12 for the second Root, which cubed gives 1728, hence 12 is the exact *Cube Root* of 1728.

EXAMPLE IX.

Let it be required to Extract the Cube Root *out of*

,067507824239 *given Number,*

64 *Cube of* 4,

Three times the Square of ,4 = 48) + ,003507324239 } ,4 *first Root,*
336 } ,007 *Quo. add,*

Remains 14 ,407 *second Root*

,067507824239 *given Number,*
,067419143 *Sub.* C. *of* ,407 2d R,

Three times the Square of ,407 *is* ,496947) + ,000088681239 } ,407 2d. *Root*
496947 } ,000178 *Q. Add*

3898653 ,407178(3d *Root*
3478629 *to the last Figure.*

4200249
3975576

Remains, 224673

NOTE,

NOTE, *Because the first Figure that arises in the first Quotient is* 0, *you may in such Cases take two Figures into the Quotient, so here I take in* 07, *and because two Figures are taken into this Quotient, you may take* 3 *or* 4 *Figures into the next Quotient, as* per *Work,*

So that the Cube Root of ,067507824239 *is* ,4 *or nearer* ,407, *or nearer* ,407178.

EXAMPLE X.

What is the Cube Root of 282? *And of* 231?

OPERATION I.

	343	*Cube of* 7	
	282	*given Number*	
147)	—61,0	7	*1st. Root*
	58,8	0,4	*Quo. Sub.*
Rem.	22	6,5	*2d. Root.*
	282	*given Number*	
	274,625	*Cube of* 6,5	
126,75)	+7,3750	6,5	*2d. Root*
	6 3375	,058	*Q. A.*
	103750	6,558	*3 Root*
	101400		
Remains	2350		

OPERATION II.

	216		
	231		
108)	+150	6	
	108	0,1	
Rem.	42	6,1	
	231		
	226,981		
111,63)	+4,0190	6,1	
	3 3489	,036	
	67010	6,136	
	66978		
Remains	32		

So the Cube Root of 282 *is* 6.558, *And the Cube Root of* 231 *is* 61,36 *nearly.*

EXAMPLE

EXAMPLE XII.

What is the Cube Root of 2150,42 ?

OPERATION.

2150,42 *given Number,*
1000 *Cube of* 10,

300) + 1150,42 } 10 *first Root.*
900 } 3, *Quotient add*

Remains 250,42 13 2d *Root.*

2197 *Cube of* 13,
2150,42 *given Number,*

1521) — 46,58 { 13 *second Root,*
4563 { 0,0306 *Quo. Subt.*

9500 12,9693 *third Root.*
9126

Remains (374)

EXAMPLE

EXAMPLE XIII.

What is the Cube Root of 1 $\frac{1}{3}$?

OPERATION.

The Decimal of 1 $\frac{1}{3}$ *is* 1, 3333 *given Number,*
1 *Cube of* 1,

3) 0,3333 { 1 *first Root,*
,3 { 0,1 *Quo. add*

Remains ,033 *&c.* 1,1 2d. *Root,*

1,3333 *given Number,*
1,331 *Cube of* 1,1

3,63) +,002333 { 1,1 *second Root*
2178 { ,0006427

1553 1,1006427
1452

1013
726

2873
2541

Remains (332)

In *Example* 12, Becauſe the firſt Figure in the ſecond Quotient is a Cypher, I take 4 Figures into that Quotient and ſo get the Cube Root of 2150,42 to be 12,9693, but in *Example*

13

13, the two first Figures in the second Quotient are Cyphers *viz.* the places of Hundreds and Thousands are each a Cypher; and therefore in this Case, I take Figures into the said Quotient (you may take 7 or 8 Figures if you please) and so get 1,1006327 for the Cube Root of $1\frac{1}{3}$, or 1,333333333 *&c.*

EXAMPLE XIV.

What is the Biquadrat Root of 4857532416?

OPERATION.

8100000000 *Biquadrat of* 300,

4857532416 *given Number,*

1080000000)−3242467584 { 300 *first Root,*
324000000 { 30 *Quo. Subtract*

Remains 2467584 260 2d. *Root,*

4857532416 *given Number*
4569760000 *Biq. of* 260 *Subt.*

70304000)+287772416 { 260 *Secona Root,*
281216000 { 4,0 *Quo. Add,*

Remains (6556416,0) 264 3d. *Root,*

Having pointed the given Number according to the third *Definition*, I find that the *Biquadrat* of 2 is 16, (*per* Table) and the *Biquadrat* of 3 is 81; now 16, the *Biquadrat* of 2 is nearer to 48 the first Period, than 81, the Biquadrat of 3

by

by a small Matter, but I take 81, and then the first Divisor is greater than if I had taken 16, and so the given Number Converges swifter, yet if you take 16 it will be the same at last, only it will afford an Operation more.

But taking 81 whose *Biquadrat Root* is 3 and there being two Periods more in the proposed Number, I put two Cyphers to 3, and it becomes 300, which cubed gives 27000000, and (*per* RULE 4) multiply'd by 4 the Index of the fourth Power, gives 108000000 for the first Divisor, also the second Root 260 cubed and multiply'd by 4 gives 70304000 for the second Divisor &c. see the above Work.

NOTE, *In Extracting* Roots *out of higher Powers, the Work may be oft made easier by using more Extractions than* 1. *Thus, take any two, three, four* &c. *Numbers which being multiply'd together may produce the Index of the required* Root, *and then Extract such* Roots *out of the given Number, as those Numbers so multiply'd denotes the Index of, and the last* Root *will be that required, so in Extracting the* Biquadrat Root, *whose Index is* 4; *here* 2 *multiplied by* 2 *gives* 4, *but* 2 *is the Index of the Square, so if you take the* Square Root *of the given Number* 4857532416, *it will be found* 69696, *and the* Square Root *of* 69696 *will be found* 264 *which is the* Biquadrat Root *of* 4857532416 *the same as before, and is the true* Root *required, for* 264, *raised to the fourth Power is equal to the given Number.*

EX. 15.

EXAMPLE XV.

What is the Surſolid Root *of* 12309502009375?

OPERATION.

12309502009375 *given Num.*
10240000000000 *Sur. of* 4000

128000000000)+2069502009375 { 400 *1ſt.* R.
1280000000000 { 10 *2d. add*

Remains 789502009375 410 S. R.

12309502009375 *given Num.*
11585620100000 *Sur. of* 410

1412880500000)+713881909375 { 410 *S.* R.
706440250000 { 5,0 *2d.*

Remains (7441659375,0) 415,3*dr*

This third Root 415 raiſed to the fifth Power is juſt equal to the given Number, conſequently is the true Surſolid Root thereof.

☞ "That as no Numbers whole Numbers "being multiply'd together can produce 5, "the Index of the Surſolid, its Root muſt "be extracted as above where the firſt Di- "viſor is the Biquadrat of 400 multiplied "by 5, (ſee Rule 4.) and the ſecond Divi- "ſor is the Biquadrat of the ſecond Root "410 multiplied by 5."

EXAMPLE XVI.

Let it be required to Extract the Square *cubed* Root, *or* Root *of the* 6th *Power out of* $\frac{1}{2}$ *or* ,5

OPERATION.

FIRST, for the Square Root of $\frac{1}{2}$

,500000	*given Number*
,49	
1,4)+010000	{ 0,7 *first Root.*
98	{ ,0071 *Quo. Add.*
20	,7071 2d. *Root,*
14	
Remains (6)	

,500000	*given Number,*
4999941	*Square of* ,7071
1,4142)+,00000059000	{ ,7071 2d. *Root,*
56568	{ ,000004171970 *Q.* (*A.*
24320	,707104171970
14142	(*third Root,*
101780 *&c.*	

In this *Example*, becauſe 2 Times 3 is 6, therefore (*per* Note *to* Ex. 14) I firſt take the Square Root of $\frac{1}{2}$, which becauſe Cyphers turn ſo out in the firſt and ſecond Quotient, I have at two Dividings got the Square Root of $\frac{1}{2}$ or ,5 to

to 12 Places of Figures (see the *Note* to *Example* 9) Now the Cube Root of this Square Root will be the Root sought, which take as follows.

	.729 — — — — —		*Cube of* .9
	.707104171970 — — —		*given Number,*
2,43)—	021895828030	,9	*first Root,*
	2187	,009	*Quotient Subtract.*
	Remains 2	.890	*second Root,*
	.707104171970 — — —		*given Number,*
	704969 — — — — —		*Cube of* .89
2.3763) +	.00213517	.890 — — —	*second Root,*
	190104	.0008985 —	*Quotient Add,*
	23413	.8908985	*third Root for the Cu.*
	21386	*Root of* .707104171970	*or the Squ.*
See Example 35, SECTION 3.	2026 / 1901	*Cub'd Root of* 0,5	*as was required,*
	125		
	118		
	Remains (7)		

EX. XVII.

EXAMPLE XVII.

Let it be required to Extract the Root of the 18*th Power out of* $\frac{1}{2}$,

FIRST, Because 2 times 3 is 6, and 3 times 6 is 18, therefore (by the Note to *Example* 14) you may first find the Cube Root of $\frac{1}{2}$ and then the Cube Root of this Cube Root will be the Cube, Cubed Root of $\frac{1}{2}$, the Square Root of which will be the Root sought (for twice 9 is 18) but from the last *Example* we have 0,8908985 for the Cube squared Root of $\frac{1}{2}$, so because 3 times 6 is 18, if we take the cubed Root out of 0,8908985 it will give the Root required.

OPERATION.

1, ——— *Cube of* 1,
0,890898500 — *given Number;*

3) —. 1091015 ——{ 1 *first Root,*
9 { 0,03 *Quotient Subtract.*

Remasns 1 .96 *second Root,*

.8908985 ——— *given Number,*
.884736— — —*Cube of* .96

3,7648) + .0061625 {.96 ——— *second Root,*
55295 {.0022 —— *Quotient Add,*

5329 .9622 *for the Root required, viz.*
5529
——— *the Cube cubed squar'd Root of*
Remains (799)

I think it needleſs to add any more Examples being perſwaded that if the foregoing Rules be conſidered the Method of Extracting Roots muſt be eaſy, but here you may obſerve that the Roots chiefly uſed in Books of this kind are only the Square and cubed Roots which I have not ſpared with Examples.

Since mean Proportionals are of Uſe in Working by the ſliding Rule, I ſhall ſhew here how to do it by the Pen, that the Learner may be able to prove the Rule by the Pen.

EXAMPLE XVIII.

Between any two Numbers (ſuppoſe 90 *and* 40*) to find a mean Proportional.*

RULE, } The Square Root of the Product of the two given Numbers is the mean Proportional required.

See Problem 36, *and* RULE 5 *Section* 1.

40 }
90 } *The two given Numbers.*

3600 *their Product,*
60 *The Square Root of that Product or mean Proportional ſought.*

PROOF.

PROOF.

If 40 *give* 60, *what will* 60 *give? or, as* 90 : 60 :: 60 : 40.

	60		60
40)	3600	90)	3600
Answer	90	*Answer*	40

After the same Manner you may find a mean Proportional, between any two Numbers whatever.

SECTION V.

SUPERFICIAL Meaſure.

THE Area and Dimenſions are commonly kept in one Name: That is, if the Area be required in Inches, the Dimenſions are taken in Inches; If the Area be required in Feet, the Dimenſions are taken in Feet and Inches; If the Area be required in Yards the Dimenſions are taken in Yards, Feet and Inches *&c.* Then thoſe odd Parts being reduced to Decimal Parts of the Integer (*vide Reduction*) the required Area is had by Working as the Nature of the Figure requires, or without ſuch *Reduction* of odd Parts the Area may be had by Feet and Inches: But eaſieſt of all, by taking the Dimenſions without Inches or thoſe odd Parts; thus, if you are to give the Area in Feet take the Dimenſions in Feet and Decimal Parts of a Foot, and ſo you have the Dimenſions ready reduc'd to your Hand and

and is also exacter than to take Feet and Inches (if you Work decimally) because in *Reduction* of Decimals there often happens to be a Remainder *&c.* now for this purpose, on the Edge of *Coglesbels* Sliding-Rule, there is a Line 12 Inch long Decimally divided (*viz.*) into 100 equal Parts, which may be taken instead of Inches, as for *Example* if you find any Line to be 13 Feet 1 $\frac{1}{2}$ Inches long, instead of observing the Line of Inches look this Decimal Line, and for 1 $\frac{1}{2}$ Inches you'll find 12 $\frac{1}{2}$, that is 12 hundred Parts of a Foot, or 0,125 Decima Parts of a Foot, so the foresaid Line is 13,125 Feet in Length, also any Line or side of 7 Foot 6 Inches Length is found 7,50 Feet, or 7,5 Feet *&c.* There are also Rules a Yard or 36 Inches long which are divided into 100, or 1000 equal Parts, for taking Dimensions in Yards and Decimal Parts of a Yard *&c.* which Methods being once practised by any that can Multiply Decimally, I am perswaded they will be easy about any other Method by the Pen: But that this Book may not fall short, I shall Work many of the following *Examples* both by Multipllcation of Feet and Inches, and by Decimals: But before I begin with the *Examples*, it may not be amiss to lay down the two following Tables (*viz.*) one of long and the other of Square Measure, which shew the Parts used in Surveying, as well as those in other Superficial Measure.

A TABLE of Square MEASURE.

	Inches.	Links.	Feet.	Yards.	Perches.	Chains.	Acres.	Mile.
Inches.	1							
Links.	62,764	1						
Feet.	144	2,295	1					
Yards.	1296	20,755	9	1				
Perches	39204	625	272,25	30,25	1			
Chains.	627246	10000	4356	484	16	1		
Acres.	6272640	100000	43560	4840	160	10	1	
Miles.	4014489600	6400000	2787 8400	3097600	102400	6400	940	1

That is, 1296 Square Inches or 20,755 Square Links, or 9 Square Feet make 1 Square Yard &c.

A TABLE *of* Long Measure.

Length in Inches

Link.	Foot	Yard.	Perch	Chain	Miles.
7,92	12	36	198	792	63360
Links.	1,515	4,56	25	100	8000
	Feet.	3	16,5	66	5280
		Yards	5,5	22	1760
			Perch:	4	320
				Chains	80

That is, 36 *Inches or* 4,56 *Links or* 3 *Feet in Length, make* 1 *Yard in Length* &c.

How to Reduce one kind of Measure *to another.*

RULE, } When you Reduce a leſs Number to a greater, uſe Multiplication; but a great Number is reduc'd to a leſs by Diviſion.

EXAMPLE I.

(*In* Square Meaſure) *In* 54,5 *Square Yards how many Square Feet and Square Inches*

OPERATION

OPERATION.

54,5 *Square Yards,*
9 *Square Feet in a Square Yard,*

490,5 *Square Feet in* 54 ½ *Square Yards,*
144 *Square Inches in* 1 *Square Foot,*

19620
19620
4905

70632,0 *Square Inches in* 490,5 *Square Feet, or* 54,5 *Square Yards.*

EXAMPLE II.

(*In* Long MEASURE) *In* 54½ *Yards, how many Feet and Inches?*

OPERATION.

54,5 *Yards,*
3 *Feet in one Yard,*

163,5 *Feet in* 54½ *Yards,*
12 *Inches in one Foot,*

3270
1635

1962.0 *Inches in* 163,5 *Feet, or* 54½ *Yards.*

And

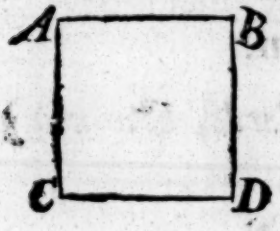
And if you are to Reduce Inches into Feet and Yards, it is but Dividing instead of Multiplying.

How to Measure a Square;

RULE 2, Multiply any Side of the given Square by itself and the Product is the Area, or Superficial Content by THEOREM 7, *Problem* 164.

EXAMPLE III.

If each Side of a Square be 7 Feet 6 Inch, What is its Area?



By Feet and Inches.			*Decimally*
F.	I.		F.
7	6	= AB =	7.5
7	6	= CD =	7.5
52	6		375
3	9	0	525
Answer 56	3	*Answer*	56.25

By the SLIDING RULE.

Set 1 on the *Slider*, to 7,5 on the RULE, then against 7,5 on the *Slider* stands 56¼ on the RULE the Area required.

How to Measure a right-angled Parallelogram.

RULE, 3. Multiply the Length by the Breadth and the Product is the Area, (by THEO. 7, *Problem* 164).

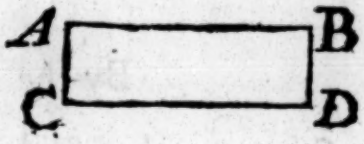
EX. IV.

EXAMPLE IV.

If the Length AB *be* 10 F. 9 I. *and the Breadth* AC 2 F. *What is the Area in Feet and Inches?*

By F. *and* I.

F.	I.		F.
10	9	= AB =	10 .75
2	—	= AC =	2
Anſ. 21	6	*Anſwer*	21 .50.



By the SLIDING RULE,

SET 1 on the SLIDER, to 10.75 on the *Rule*, then againſt 2 on the *Slider*, ſtands 21.5 on the RULE, the Area required.

How to Meaſure an Oblique-angled Parallelogram as a Rhombus or Rhomboides.

RULE 4. Multiply the Perpendicular let fall from 1 of the Obtuſe Angles upon the oppoſite Side, by the Side oppoſite and the Product is the Area. *By* THEOREM 8.

EXAMPLE V.

If the Length DE *be* 2 F. 10 $\frac{1}{2}$ I. *and the Perpendicular* DP 9 *Inch, What is the Area?*

By



	By Feet and Inches,			*Decimally.*
	F.	I.	P.	F.
DE =	2	10	6	2.875
DP = S		9.		.75
Answer	2	1	10 6	14375 20125
Answer				2.15625

By the *Sliding Rule*,

Set 1 on the *Slider* to 2,875 on the *Rule* then againſt 7,5 on the *Slider*, ſtands 21,5625 on the *Rule*.

But becauſe the ſecond Term ,75 is multiplied by 10, (*viz.*) made 7,5 the above fourth Number 21,5625 muſt be divided by 10 and then it becomes 2,15625. The Area required by SECTION I.

How to Meaſure any Plane Triangle.

RULE 5. Multiply the Baſe by the Perpendicular, half that Product is the Area, or Multiply the Baſe by half the Perpendicular, or half the Baſe by the Perpendicular, any of theſe ways gives the Area, *By Theorem* 9.

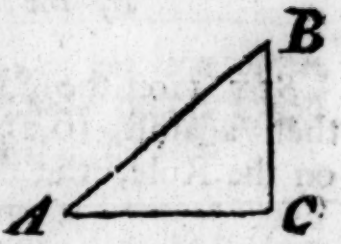
EXAMPLE VI.

In the right-angled Plane Triangle ABC, *if the Baſe* AC, *be* 7 F. 10 I. *and the Perpendicular* BC 2 F. 3 I. *What is the Area?*

By

By Feet and Inches. *Decimally.*

F.	I.			F.
7	10	=	AC	7.8333
2	3	=	BC	= 2.25
15	8		P.	391665
1	11		6	156666
				156666
2 | 17	7		6	
				2)17.624925
Anſr. 8	9		9	
			Anſwer	8.8124675



By the SLIDING RULE,

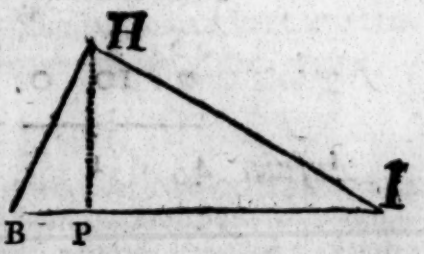
SET 2 on the *Slider* to 7,83 on the *Rule*, then againſt 2,25 on the SLIDER, ſtands 8,81 on the *Rule* the Area required.

EXAMPLE VII.

In the Scalene *Plane Triangle* IBH *if the Baſe* IB *be* 10 *Feet* 3 *Inches and the Perpendicular* HP 6 *Feet* 10 *Inches, What is the Area of that Triangle?*

By Feet and Inches. *Decimally.*

F.	I.			F.
10	3	=	IB	3.4167 = ½ PH
3	5	= ½	HP	10,25 = BI
30	9		P	170835
4	3		3	68334
				341670
(35	0		3)	35,021175 *Anſwer.*



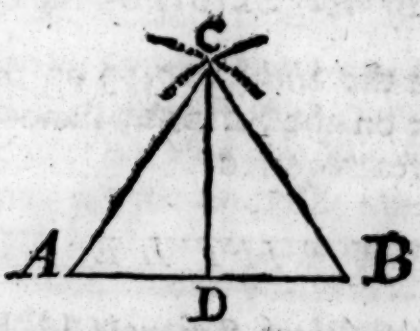
By

By the SLIDING RULE

SET 1 on the *Slider* to 3,416 on the Rule, then against 10,25 on the Slider ſtands 35,02 on the Rule, the Area required.

EXAMPLE VIII.

In the Iſoſcles *Triangle* ABC *if the Baſe* AB *be* 7 Feet 4 Inches *and the Perpendicular* CD 11 Feet 9 Inches, *What is the Area?*



By Feet and Inches.				*Decimally.*
F.	I.			F.
11	9	= CD	=	11,75
3	8	= ½ AB	=	3,667
35	3			8225
7	10	0		7050
				7050
Anſwer 43	1			3525

Anſwer 43,08725

By the SLIDING RULE.

Set 2 on the *Slider* to 7,333 on the *Rule*, then against 11,75 on the *Slider* ſtands 43,08 on the *Rule*, the Area required.

NOTE, *The three laſt* Examples *are wrought three different ways according to the* Rule 5, *but in any Plane Triangle whatſoever, the Learner may take any of thoſe ways which he likes beſt. Alſo in the right-angled Triangle*, Example 6: *If a Perpendicular be taken from the Angle* A, *to the Hypothenuſe* BC, *half the Product of this Perpendicular and Hypothenuſe will give the Area of the Triangle, the ſame as before.*

" Since the Method of Feet and Inches is " fully explained in *Section* 2, I think it needleſs " to Work any more *Examples* thereby; " and ſo in the following *Examples*, I ſhall ſuppoſe the Dimenſions to be taken Decimally as " is taught at the beginning of this *Section*, by " which means the Area of any Figure is ſoon " diſcovered as in the following *Examples*"

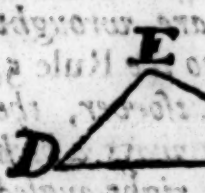
How to Meaſure a Plane Triangle without a Perpendicular by having the three ſides given.

RULE VI.

From half the Sum of the three Sides, Subtract each Side ſeverally, and Multiply the ſaid half Sum and three Remainders together, according to continual Multiplication, the *Square Root* of the laſt Product is the Area by *Theo.* 66, PROB. 170.

EXAMPLE IX.

In the Oblique angled Plane Triangle DEF, *if the Sides* DE *be* 29 EF 41 *and* FD 56 *What is the Area thereof?*



29 = DE, 63 63 63
41 = EF, 56 41 29
56 = FD, ——
7 22 34
126 *Sum Sides*,
63 *half Sum Sides*,
7 *first Difference*,
441
22 *second Difference*,
882
882
9702
34 *third Difference*,
329868
291066

Last Product 329868 (574,341 *Area required*
107) 798
1144) 4968
11483) 39200
114864) 475100

Remains, 15644

This

This Method being ſomewhat tedious need only be uſed when great exactneſs is required, *Otherwiſe*, you may from the three given Sides lay down the Triangle (as in Prob. 7 or Problem 128) and ſo Meaſure out a Perpendicular and Work by the fifth *Rule*; *Note*, that *Rule* 5 is inferior to *Rule* 6 in Truth (for they are the ſame by *Prob.* 170) But the Sides of the Triangle being large Numbers you muſt Uſe true Scales and great Accuracy in drawing the Figure if you get the Perpendicular ſo true as to Perform the ſame exactneſs with the above Rule 6, this Method is of great Uſe in Surveying, when by Reaſon of ſome Obſtacle a Perpendicular cannot be taken *&c.*

By SCALE *and* COMPASSES.

Having found the ½ Sum 63, and the three Differences, 7, 22, 34; as before, ſet 1, to 6,3 then againſt 7 ſtands 44,1, ſet 1 to 44,1, then againſt 2,2 ſtands 97,02; again ſet 1 to 97, 02 then againſt 34 ſtands 329,868 ſo the Product is 329868, for whoſe Square Root, ſet 1 on the *Slider* to 1 on the *Line* D, then againſt 3300, on the *Slider* ſtands 575, that is 575 for the Area required, See *Section* 1.

☞ "Becauſe 329868 is near 330000 I take the "Square Root of 3300 which being 57,5 "and multiplied by 10 (becauſe 330000 "was divided by the Square of 10) gives "575, for the Square Root of 330000 the "Area required nearly."

How

How to Measure a TRAPEZIA.

RULE 7.

Multiply half the Sum of the two Perpendiculars by the Diagonal, and the Product is the Area: *Theorem* 10, or the Sum of the Perpendiculars by ½ the Diagonal.

EXAMPLE X.

In the Trapezia ACBE, *let the Diagonal* AB *be* 28,2 *the Perpendicular* CD, 10½, *and the Perpendicular* FE, *What is the Area?*



10,5 = CD,
8, = EF,

18,5 *Sum Perpen.*
14,1 = ½ AB,

18,5
740
185

260,85 *Area sought.*

By the SLIDING RULE.

SET 2 on the SLIDER, to 18,5 on the *Rule* then against 28,2 on the *Slider* stands 260,85 on the *Rule*, the Area required.

How

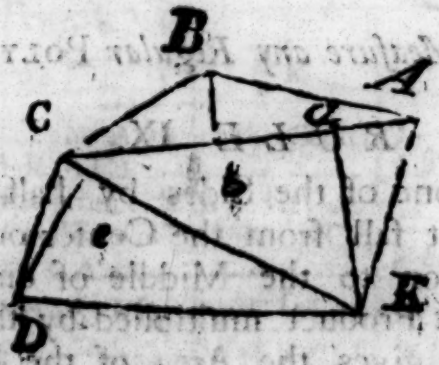
How to Measure an Irregular FIGURE.

RULE 8.

Divide the given irregular Figure into Triangles and Trarpezias by drawing Diagonals and Perpendiculars then the Sum of all the Areas of those Triangles and Trapezias gives the Area of the proposed Figure.

EXAMPLE XI.

Let it be required to Measure the irregular Pentagon ABCDE, *by Measuring the Diagonals* AC 20,75 *and* EC 24, *the Figure is Divided into the Triangle* CDE, *and Trapezia* CEAB *whose Perpendiculars are* De 8,3 Bg 6 *and* Ea 10,4.



8.3 = De
12 = ½ CE.

166
83

99.6 *Area* CDE. 6 =

6 = Bg
10,4 = E*a*,

2) 16,4 *Sum Perpendiculars*,

8,2 = *Sum Perpendiculars*,
20,75 = AC,

4150
16600

to 170,150 *Area Trapezia* ABCE,
add 99,6 *Area Triangle*, CDE,

269,75 *Area of* ABCDE *requirtd*.

The Measuring of this by the SLIDING RULE *is easy from* Example 8 *and* 10.

How to Measure any Regular POLYGON.

RULE IX.

Multiply one of the Sides by half the Perpendicular let fall from the Center or Middle of the Polygon to the Middle of one of the Sides and that Product multiplied by the Number of Sides gives the Area of the Polygon.

EXAMPLE XII.

If each Side of a Regular Hexagon *be* 30 *and the Perpendicular or Radius of its inscrib'd Circle* 26, *What is its Area?*

OPERATION

OPERATION.

13 = ½ Ce *the Perpendicular,*
30 = AB, *a Side of the Polygon,*

390 = *Area Triangle* ABC,
6 = *Number Sides in the Polygon,*

2340 = *Area required.*



But in Measuing Regular Polygons it often happens that there is nothing given but the Sides and then the Perpendicular may be found by Plain TRIGONOMETRY.

Thus,

As Sine half the Angle at the Center viz. Sine Angle ACP	} 30 d.	9.698970
Is to half the given Side AB viz. Ae	15,	1,176091
So is Sine Complement Angle ACc, viz. Sine Angle CAc	} 60 d.	9.937531
To the required Perpendicular Ce	25,99	1,414652

OR, If we suppose the Side AB = 1, (the Angles being still the same) we shall find the Perpendicular Ce to be, 866 which multiplied by 3 (half the Sum of the Sides of the given *Hexagon* see *Theorem* 12) gives 2,598 for the Area of a Hexagon whose Side is Unity or 1, Therefore by Theo. 3, Prob. 69, if you Multiply

tiply the Square of the Side of any Regular Hexagon by 2,598, the Product will be the Area of that Hexagon; so if we repeat the last *Example* wherein the Sides of the given *Hexagon* are each 30.

Then, 30 = BB,
30 = AB,

900 *Square of* 1 *Side*,
Multiply, 2,598 *Area of a Regular* HEXAGON, (*whose Side is Unity*,
2331,200 *Area near as before.*

A TABLE *for Measuring* POLYGONS, Regular.

The Names of the POLYGONS.	*Number of Sides.*	*Half the Angle at the Center.*	*Area, the Sides being Unity.*	*These Multipliers turn'd into Divisors.*
Trigon	3	60° 00′	,433013	2,309401
Square	4	45 00	1,000000	1,000000
Pentagon	5	36 00	1,720477	,581234
Hexagon	6	30 00	2,598076	,384900
Heptagon	7	25 42$\frac{6}{7}$	3,633959	,275181
Octagon	8	22 30	4,828427	,207107
Nonagon	9	20 00	6,181824	,161769
Decagon	10	18 00	7,694209	,129968

Becau

Becauſe every Circle contains 360° and a ſem-Circle 180°, therefore, if you divide 180 by the Number of Sides in the Polygon, the Quotient will ſhew the half Angle at the Center, ſo 180° divided by 3 gives 60° for the Equilateral Triangle or Trigon &c. of the reſt as in the *Table*.

Likewiſe if the Side of a Regular Polygon be 1 its Perpendicular may be found by plain Trigonometry as before, and alſo its Area: So if the Side of a Trigon be 1, its Area will be 0,433013, alſo if the Side of any Regular Hexagon be 1 its Area will be 2,598076 &c. as in the foregoing TABLE, And thus you may find the Area of any regular Polygon, whoſe ſide is Unity or 1, (the foregoing *Table* only extends to one of 10 Sides) and then, by *Theo*. 3, we have this *General*

RULE 10. Multiply the Square of the Side of any regular Polygon by its Tabular Number and the Product is the Area of that Polygon whoſe Side you ſquar'd, or the Square of the Side divided by the Diviſor gives the Area.

EXAMPLE XIII.

If the Side of a Regular Pentagon be 50, *What is its* AREA?

OPERATION.

50	1,720477	*Tabular Number for a Pentagon,*
50	2500	*Multiply,*
2500	860238500	*Square of a Side*
	3440954	
	4301,192500	*Area required.*

By the SLIDING RULE.

SET 1 on the Line D, to (the Tabular Number 1,720477, or) 1.72 (will ſerve) on the *Slider* then againſt 5 on the *Line* D, (for 50 is off the RULE) ſtands 43,01 on the SLIDER which multiplied by the Square of 10 (becauſe 50 was divided by 10) gives 4301 for the Area of a required Pentagon whoſe ſide is 50, whether Inches, Feet or Yards *&c.*

Again, for the Hexagon EXAMPLE XII.

SET 1 on the *Line* D to 2,598 on the *Slider* then againſt 3 on the Line D (for 30 is off the *Rule*) ſtands 23,38 on the *Slider* which multiplied by 100, (becauſe 30 was divided by 10) gives 2338 for the Area as before, ſee Rule *Section* 1.

How to Measure a CIRCLE

RULE XI.

Multiply the Diameter by the Circumference, that Product divided by 4, gives the Area: Or, Multiply half the Circumference by half the Diameter, and the Product is the Area. by Theo 13,

EXAMPLE XIV.

If the Diameter AB *be* 100 *and the Circumference* AB, AB; 314,16, *What is the* AREA?

157,08 = AB,AB
50 – ¼ AB,

7854,00 *Area required,*



By the SLIDING RULE

SET 4 on the *Slider* to 314,16 on the *Rule*, then againſt 100 on the *Slider* ſtands 7854 on the Rule the Area ſought: *Otheawiſe*, ſet 1 on the *Slider* to 157,01 on the *Rule*, then againſt 50 on the *Slider* ſtands 7854 on the *Rule*, as before.

Having given the Diameter of a Circle to find its AREA.

RULE.

RULE XII.

Multiply the Square of the Diameter by ,7854 and the Product is the Area: *Otherwise*, Divide the Square of the Diameter by 1,2732 and the Quotient is the Area by THEO. 30 and *Example* 31 SECTION 3.

EXAMPLE XV.

If the Diameter of a Circle be 100, *What is the* AREA?

100
100

Multiply	10000	*Square of the Diameter,*
	,7854	*Conſtant Factor,*
	7854,0000	*Area ſought.*

By the SLIDING RULE.

SET 1,2732 on the *Slider* to 100 on the Rule then againſt 100 on the SLIDER ſtands 7854 on the Rule, the Area required: *Otherwiſe* ſet 1 on the Line D to ,7854 on the *Slider* then againſt 1 on the Line D (for 100 is off the *Rule*) ſtands ,7854 on the Slider, which multiplied by the Square of 100 (becauſe 100 was divided by 100 gives 7854 for the Area as before; SECTION 1 *Rule* 4.

NOTE

NOTE. *That by* Prob. 172. *the above Ratio* viz: 1 to .7854 *may be reduc'd to integer Terms the leaſt of which will be* 14 *to* 11 *from whence we have this Rule for finding the Area of a Circle* viz. *As* 14 *is to* 11, *ſo is the Square of any Circle's Diameter to the Area of that Circle which the Learner may prove at his Leiſure.*

Having given the Circumference of a Circle to find its AREA.

RULE XIII.

Multiply the Square of the Circumference by .07958 and the Product is the Area, by Theorem 31 Prob. 167. *Otherwiſe* Divide the Square of the Circumference by 12.566 and the Product is the Area. *per Note* 8 *Section* 3.

EXAMPLE XVI.

If the Periphery of a Circle be 314.16. *What is the* AREA?

OPERATION

Operation.

By Division.

314,16		12.566) 98696,5056 (7854,2 *Area*,		
314,16		87962		
188496		107345		
31416		100528		
125664		6817		see *Section* 3d.
31416		6283		*contracted*,
94248				*Division*.
98696,5056	*Square Periphery*,	534		
85970,	*Inverted Factor*,	502		
690875	*see Note* 6 *Section* 3d.	31		
88827		23		
4634		8		
790				

7854,26 *Area*.

Thus you may see that all these ways of Measuring a Circle are nearly the same. so that you may use that which comes neareſt to Work.

By the Sliding Rule.

Set 12,56 on the Slider, to 314,1 on the Rule, then againſt 314,1 on the Slider ſtands 7854 on the Rule the Area required. Or, let 1 on the Line D, to ,079 on the Slider then againſt 3,14 (for 314 is off the Rule) the Periphery on the Line D, ſtands 7854, on the *Slider* which multiply'd by the Square of 100 (ſee *Rule* 4, *Section* 1.) give 7854 for the Area as before.

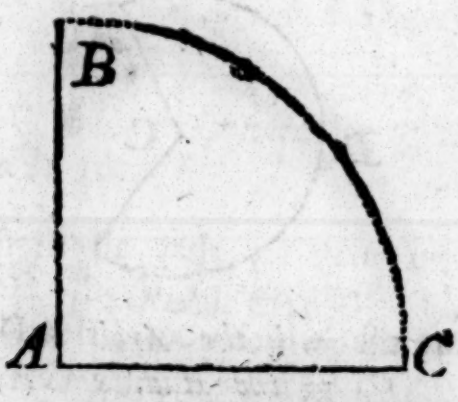
Hou

How to Measure any Sector, Semi-circle or Quadrant.

RULE 14. Multiply the Radius and Arch the one by half the other and the Product will be the Area, by *Theorem* 14 *Problem* 165.

EXAMPLE XVII.

If the Radius or half Diameter be 50 *and the Length of the Arch* 80 $\frac{1}{4}$, *What is the* AREA?



80,25 = *the Arch* BC,
25 = $\frac{1}{2}$ *the Radius*, CA.

40125
16050

2006,25 *Area required.*

By the SLIDING RULE.

SET 2 on the Slider to 50 on the Rule, then againſt 8 $\frac{1}{4}$ on the Slider for 80 $\frac{1}{4}$ is off the Rule ſtands 200,6 on the Rule which multibly'd by 10 (becauſe 80 $\frac{1}{4}$ was divided by 10) gives 2006 for the Area.

EXAMPLE XVIII.

If the Radius CA *be* 50 *and the Arch* ABA 218,56, *What is the* AREA?



109,28 = $\frac{1}{2}$ *the Arch* ABA,
50 = *the Radius* CA,

Area (5464,00)

By the SLIDING RULE.

SET 2 on the *Slider* to 218,56 on the *Rule* then againſt 50 on the *Slider* ſtands 5464 on the *Rule*, the Area required.

Hor

How to Measure any Segment of a CIRCLE.

RULE XV.

It is evident by the Figure that if from the Area of the Sector CABA (found by *Example* 17) you take the Area of the Triangle CAA, (found by *Rule* 5) there will leave the Area of the Segment ABAP.

EXAMPLE XIX.

Let the Arch ABA, *be* 28,656 *the verſed Sine or Height of the Segment* PH = 6, *and the Chord of its Baſe* AA = 24,98 *by which you'll find the Radius* CA *of the whole Circle to be* 16, *and* CP *the Perpendicular of the Triangle* CAA *to be* 10.

Then, 28,656 = *Arch* ABA;
8 = ½ *Radius* CA;

from 229,248 = *Area Sect.* ABAC
Take 124,9 = *Area Triangle* CAA

leaves 104,348 = *Seg.* ABAP *reqd.*

24,98 = AA
5 = ½ CP.

124,90 *Area* △.



N B

By the SLIDING RULE.

FIRIT, *Set* for the Sector as in *Example* 17 *or* 18, and then for the Triangle as in *Rule* 5, the Difference between those two Areas gives the Area of the Segment.

If the Segment be greater than a Semi-circle, the *Rule* is the same, only you must add the Area of the Triangle, to the Area of the Sector as for *Example* 20, let the same Things be given as before but let the Arch ABA be 71,8752, and the Segments height PB = 26.



Then, 71,8752 = *Arch* ABA,
8 = ½ *Radius* CA,

To 575,0016 = *Area Sect.* ACBA
add 124,9 = *Area Tri.* ACA,

6,99,9016 = *Area Seg.* APAB

24,98 = AA,
5 = ⅕ CP

124,90

Or, The Area of any Segment less than a Semi-circle may be had pretty near Truth by *Theorem* 28 without the Arch which in Words gives the following RULE 16.

Multiply the Radius of the Circle by the Difference between the said Radius and Height of the Segment, that Product multiply by 4 *and Divide the last Product by* 3. *to the Quotient add the Square of the foresaid Difference and Subtract the*

the Sum from $\frac{7}{11}$ *of the Square of the ſaid Radius and the Remainder is a Dividend which Divide by* $\frac{3}{11}$ *of the ſaid Radius added to the ſaid Difference this laſt Quotient multiply'd by half the Chord of the Segments Baſe gives the Area of the Segment.*

LET *Example* 9 *be wrought by this Rule,* Thus *Example* 21.

16 = Radius.
16
—
96
16
—
256 Sq: Radius,
7
—
3)1792
—
597$\frac{7}{11}$ Sq: Radius.
313$\frac{4}{11}$ diff: Subt.
—
34)284 Dividend
—
8,353 Quotient.
12,49 Half Cord, Segments, Baſe, Multiply.
—
75177
33412
16706
8353
—
104,32897 Area, which differs but a ſmall matter from Truth, by EXAMPLE XIX.

16 = Radius.
10 Diff. between Radius and Verſed-Sine.
—
160
4
—
3)640
—
213$\frac{4}{11}$
100 Sq: diff. add
—
313$\frac{4}{11}$

16 = Radius.
3
—
2)48
—
24 = $\frac{4}{11}$ Radius.
10 = diff. Add.
—
34 Diviſor.

By either of theſe Methods as alſo by the Series in Step 7 Prob. 167 the Area of any circular Segment may be found. and conſequently

a Table of the Areas of Segments constructed. Thus if the Radius of a Circle being 1, be divided into 1000 equal Parts and thro' every Part Chord Lines be drawn. then the Semi-Circle will be cut into 500 Segments the versed Sine or Height of the first Segment being .001 now the half Chord of any Segment being a mean Proportional between the versed Sine and remaining Part of the Diameter the said half Chord may be easily found and so the Area of this Segment by the above Methods will be found .0000421. In like Manner the Height of the second Segment being ,002 its Area will be found ,00011919 &c. for any other, and having thus got a Table the Area of any Circular Segment may be had thereby, by this *Rule* 17.

Divide the given versed Sine (with 3 Cyphers annexed) by the Diameter of its Circle, seek the Quotient under V. S. and take out the Number against it under Segment Multiply this Number by the Square of the Diameter and the Product is the Area of the Segment.

Note V. S. *Signifies versed-Sine, and Seg. stands over the Areas which answer to the versed-Sine over against them*

The Reason of the above Rule is from Theorems 1 and 3 for circles being similar Figures, as the Diameter of any Circle is to any Part of that Circle so is the Diameter of any other Circle to the like Part of that Circle, by Theo. 1, which proves the first Part of

the

the Rule, and as the Square of any Circle's Diameter is to the Area of any Segment or Part of that Circle, ſo is the Square of any other Circle's Diameter to the Area of its like Part by Theorem 3, which proves the ſecond Part of this Rule.

EXAMPLE XXII.

Let the Radius be 16 *and verſed Sine* 6 *as in* Example 19 *and* 21, *to find the Area by the Table of Segments.*

32)6000(,1875
32
280
256
240
224
160
160

1029 *ſtands againſt* .187½
1024 *Square of the Diameter.*

4076
2038
10190

194,3456 *Area of the Segment ſought.*

" Becauſe the Radius of the Table is only 500 viz. 3 places of Figures under V. S. and this Quotient is ,1875 I therefore take the Segment for 187, and ſay by the Rule of 3 As 187 is to its Segment, ſo is 187½ to 1019 its Segment and ſo may you find a Proportional Part when great exactneſs is required, *Otherwiſe* the Segment for 187 or 188 might ſerve.

By

By the Sliding Rule.

1. Set the Diameter 32 on the *Slider* to the versed Sine 6 on the *Rule*, then against 10 (for 1 is off the *Rule*) on the Slider stands 1,8$\frac{1}{4}$ on the *Rule* which divided by 10 (because the first Term 1 was multiply'd by 10) gives ,18$\frac{1}{4}$ or 1875, this look'd for under *versed-Sine* stands against ,1019 under Segment, so if the Radius of a Circle be $\frac{1}{2}$ and the height or versed of a Segment ,1875 the Area of that Segment is ,1019.

2. Set 1 on the *Line* D, to ,1019 on the *Slider* then against 3,2 (for 32 is off the *Rule*) on the Line D, stands 1,043 on the *Slider* which multiplied by the Square of 10 (because 32 was divided by 10) gives 104,3 for the Area of the Segment required.

☞ If from the Area of the whole Circle you take the Area of this Segment there will leave the Area of the other Segment which is greater than a half Circle, and thus you may find the Area of any circular Segment.

Having the Diameter of a Circle given, to find its Circumference by Theorem 27 Prob. 16. If the Diameter of a Circle be 1, the Periphery of that Circle will be 3,1416 therefore, by Theorem 1, as 1 is to 3,1416, so is any circle's Diameter to its Periphery, but by Prob. 172, as 1 is to 3,1416 so is 7 to 22, not differing ,00[illegible] from Truth, also as 1 : 3,1416 :: 113 : 35[illegible] not erring ,000031, from whence come these ways to this Article, *viz.*

RULE

RULE XVIII.

Multiply the given Diameter by 3,1416 and the Product is the Circumference or, as 113 is to 355 so is the Diameter to the Circumference nearly. Or as 7 is to 22 so is the Diameter to the Circumference which last Proportion may serve for any ordinary Use.

EXAMPLE XXIII.

If the Diameter of a Circle be 32, *What is the Periphery?*

	1	2	3
Constant Factor	3,1416	7 : 22 :: 32	113 : 355 :: 32
Diameter —	32	32	32
	62832	44	710
	94248	66	1065
Periphery =	100,5312	7)704	113)11360(100,5309
			113
		100,57 = P.	60,0
			565
			350
			339
			1100
			1017
			83

Any of these 3 ways may do where Note that the 3d is truest because 3,1416 but 3,141592 &c. Ut per Step 2 Prob. 172.

By the Sliding Rule

Set 7 on the *Slider* to 22 on the *Rule* then against 32 on the *Slider* stands 1005 on the *Rule* the Periphery required.

RULE XIX.

Having the Circumference of a Circle given, to find its Diameter.

By the last Rule, if the Circumference be 1, the Diameter will be ,3184; therefore, Multiply the given Circumference by ,3184: *Otherwise*, As 22 is to 7, *or*, As 355 is to 113, so is the Periphery to the Diameter.

EXAMPLE XXIV.

If the Periphery be 100,5, *What is the Diameter?*

Constant Factor,	,3184	22 : 7 : : 100.5	
Periphery	100.5		7
	15920	22)7035	(31,98 Dia.
	318400	66	
Diameter	31,99920	43	
		22	
		215	
		198	
		170	

By the SLIDING RULE.

SET 22 on the *Slider*, to 7 on the *Rule* then againſt 100,5 on the *Slider*, ſtands 32 on the *Rule*, the Diameter ſought.

How to Meaſure a common Parabola.

RULE 20. Multiply the Axis by the greateſt Ordinate, $\frac{2}{3}$ of that Product is the Area by *Theo.* 26.

EXAMPLE XXV.

If the Axis DC *be* 10,5 *and the greateſt Ordinate* AB 15,6; *What is the* AREA?

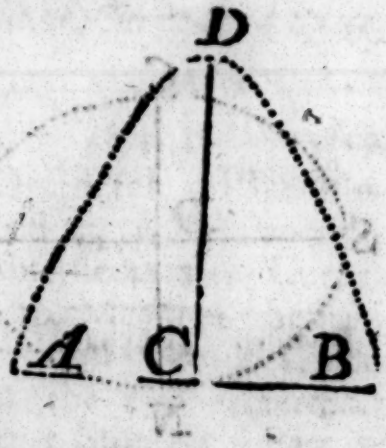
156 = AB,
10,5 = DC,

780
1560

163.80
2

)327.60

109,2 *equal the Area required.*



By the SLIDING RULE.

SET 45 *viz.* $\frac{2}{3}$ on the *Slider* to 10,5 on the *Rule*, then againſt 56 on the *Slider*, ſtands 109,2 on the *Rule*, the Area required.

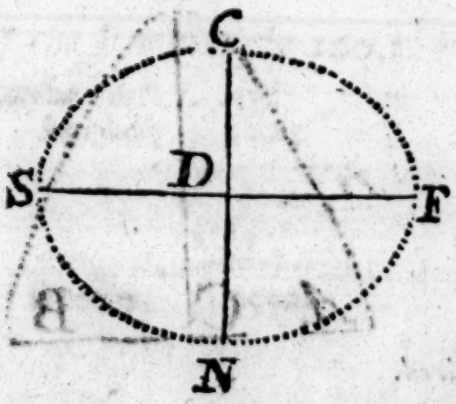
How to Measure an Oval *or* Ellipsis.

RULE XXI.

Multiply the Transverse Diameter by the conjugate, and that Product by .,7854, this last Product is the Area: Or, a mean Proportional between the 2 Diameters of any Ellipsis is equal the Diameter of a Circle, whose Area is equal that of the Ellipsis, By Theorem 29.

EXAMPLE XXVI.

If the Transverse Diameter FS *be* 21 *and the conjugate* Cn 15,6 *What is the Area?*



15,6 = C*n*,
21 = SE,

156
312

327.6
.7854

13104
16380
26208
22932

257,29704 *equal the Area.*

By the SLIDING RULE.

Set 1,27032 on the *Slider* to 21 on the *Rule*, then againſt 15,6 on the *Slider* ſtands 257,29 the Area required on the *Rule*.

I might here apply Theorems 33 and 34, to find the Area of any Eliptical Segment cut off by a line parallel to either of the Diameters. But in common Menſurations when any Segment &c offers it ſelf to be Meaſured it is ſeldom known what the nature of the Curve is, and therefore I ſhall give a general method for Meaſuring any Curve lin'd ſpace whatever, without any regard to the nature of the Curve.

How to Meaſure any Segment or, Curve-lin'd Space,

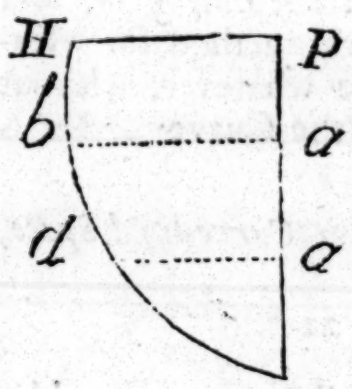
RULE 22.

Take or Meaſure four Ordinates, or Breadths equally diſtant from each other, and perpendicular to the Chord, or Abſciſſa of the Curve-lin'd ſpace which you are to Meaſure, then to the Sum of the two outermoſt Breadths, add three Times the Sum of the two innermoſt Breadths, and Multiply that Sum by $\frac{1}{8}$ Part of the ſaid Chord or Abſciſſa, and that Product will be the Area by 4th. Table of Ordinates Prob. 169.

EXAMPLE 27.

EXAMPLE XXVII.

Let the Semi-segment P H A be that mentioned in Example 19, whose half Chord or Abscissa P A is = 12,245 and Breadth P H = 6 then by Measuring, or truer by the property of the Circle, the Breadths *a b* and *a d* are found 5,46 and 3,88 and the Breadth at the Corner A = *o*.



5,49 = *ab*
3,88 = *ad*,

9,37 *Sum mid. Br.*
Multiply 3

28,11 3 *times that* (*Sum*
6 = HP
o = *Br. at* A, (*Rule*

34,11 *Su. of* &c. *per*
Multiply 1,53 = $\frac{1}{3}$ *of* AP.

10233
17055
3411

52,1883 *equal Area*

Space APH, *as was required.*

Which compared with half the Area found in *Example* 19, you'll find but a small Difference and thus this Method will hold near enough for any ordinary Business, let the Curve be what it will, or if the Figure be made up of Right Lines, or of Curves and Right-lines it is equally the same, I might also express the other Theo-

rem

rems in the forecited Problem *viz.* 5,6,7 *&c.* Ordinates or Breadths are taken, But ſince four Breadths ſo nearly anſwers the Segment of a Circle, I judge the above Rule ſufficient: However, if it be not thought ſo, you may cut, (or ſuppoſed to be cut) the Figure into as many Parts as you pleaſe and take four Breadths in every Part, with which and the Length, Chord or Abſciſſa of the Part, Worſt as before; and the Sum of the Areas of all the Parts will give the Area of the whole, *&c.*

By the SLIDING RULE.

SET 8 on the *Slider* to the Length 12,24 on the *Rule*, then againſt 34,11 (the above Sum) on the *Slider*, ſtands 52,19 nearly, on the *Rule* the Area required: *Otherwiſe*, ſet 8 on the *Slider* to the Length 12,24$\frac{1}{2}$ on the *Rule*, then againſt 6 on the *Slider* ſtand 9,5 on the *Rule*, and againſt 28,11 on the *Slider* ſtands 42,68 on the *Rule* which two Numbers 9,5 and 42,68 added together gives 52,18 as before. This Method by the *Sliding Rule* perhaps may not be underſtood by every one, therefore, take the Reaſon thereof; thus, if you were to Multiply any two Numbers, ſuppoſe 3 and 4 by ſome other particular Number ſuppoſe 6, it will be the ſame Thing whether you Multiply thoſe Numbers 3 and 4 in one Sum, by 6; or ſeperately by 6, and add the two Products together for the Sum of 3 and 4 is 7, which multiply'd by

by 6 gives 42, also 6 times 3 is 18 and 6 times 4 is 24 but 18 added to 24 gives 42, the like also holds true in any Number of Numbers multiply'd by 1 Number. So much is sufficient for Superficial Measure of Planes even the last Rule is sufficient for the whole if rightly applied I come now to take the Superficial Content of Solids which I shall do from a few general Rules *As follows.*

SECTION

SECTION V.

SUPERFICIAL *MEASURE* OF *SOLIDS.*

To find the Superficial Content of any Cube, Parallelopipedon, Prism, Cylinder, &c.

RULE 23. *MULTIPLY the Periphery taken at the Base [or at any other Part] by the Length, and the Product is the Superficial Content of all but the Ends; to which Add the Area of each End (viz.) Double the Area of one End, so have you the whole Superficial Content.*

By THEOREM XVI.

EX: 28. *If each Side of a Cube be* 10,5 *What is its whole Superficial?*

10,5 } a Side of the Cube
10,5 }

525
1050

110,25 Area of one Side.
6

661,50 whole Super. sought.

Note, Because a Cube is contained under 6 equal Squares, If you Multiply the Area of one of those Squares by 6, the Product will be the Superficial Content of the whole Cube, as *per* Margin.

By

By the SLIDING RULE.

SET $\frac{1}{6}$ (*viz.* ,166) on the *Slider* to 10,5 on the *Rule*, then againſt 10,5 on the *Slider* ſtands 661$\frac{1}{2}$ on the *Rule*, the Superficies required.

EXAMPLE XXIX.

If each Side of the Baſe of a Triangular Priſm be 10, *and its Length* 25$\frac{1}{2}$, *What is its Superficial* CONTENT.

,433013 *Tabular for a Trigon.*
100 *Sq. of a Side at Baſe.*

43,3013 *Content of either Baſe.*
2

86,6026 *Content of both Baſes.*

25,5 *Length.*
10 *Side at the Baſe*

25,5 *Curve Superficies.*
Add 86,6026 *Baſes Area.*

341,6026 *whole Superfi.*

By the SLIDING RULE.

Set 1 *on the Slider to* 10 *on the Rule, then againſt* 25,5 *on the Slider ſtands* 255 *the Convex Superficies?*

2. SET 1 on the Line D, to ,433 on the *Slider*, then againſt 1 on the *Line* D (for 10 is off the *Rule*) ſtands ,433 on the *Slider*, ſo 43,3 is the Area at either End of the Priſm &c. as before.

EXAMPLE XXX.

If the Circumference at the End of a Cylinder be 10 *amd its Length* 255, *What is its Superficial* CONTENT.

OPERATION.

OPERATION.

,07958 *ſee Ex.* 16	255 *Length*
100 *Periphery's, Square,*	10 *Periphery,*
7,958 *Area at either End,*	255 *Convex Surface,*
2	15,916 *Area add,*
15,916 *Area at both Ends,*	270,916 *whole Superf.*

By the SLIDING RULE.

SET 12,56 on the *Slider* to 10 on the *Rule* then againſt 20 on the *Slider* ſtands 15,9 on the *Rule*, the double Area or Area at both Ends the reſt of the Work is as in the laſt *Example.*

To find the Superficies of any Pyramid or Cone.

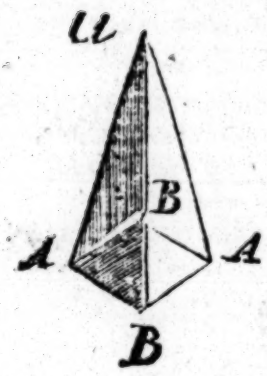
RULE XXIV.

Multiply the Periphery at the Baſe by $\frac{1}{2}$ the ſlant Height and the Product is the Convex Superficies to which add the Area of the Baſe and the Sum is the whole Superficial *Content*, by *Theorem* 19.

EXAMPLE XXXI.

If each Side of the Baſe of a Square Pyramid be 10$\frac{1}{2}$ *and its ſlant Height* 30, *What is it Superficial* Content?

OPERATION.



10,5	= AB a Side of the Base.	10,5
4	Number of Sides.	10,5
42,0	Periphery at the Base.	5,25
15	½UB the slaint Height.	105,0
210		110,25
42		
630	= Convex Surface.	
110,25	Area of the Base Add	
740,25	whole Surface.	

By the SLIDING RULE.

Set 2 on the Slider to 42 on the Rule, then againſt 30 on the Slider ſtands 630 on the Rule, the Convex Surface. And Set to $10\frac{1}{2}$, againſt $10\frac{2}{2}$ ſtands $110\frac{1}{4}$ the Area of the Base as before.

EXAMPLE XXXII.

If the Periphery of ſome Pyramid's or Cone's Baſe be 45 *and that ſlant Height* 380, *What is the Convex Superficies.*

OPERATION.

190 = *half Slant Height,*
45 = *Periphery at the Baſe,*

950
760

8550 *Convex Surface required.*

By the SLIDING RULE.

SET 2 on the *Slider* to 380 on the *Rule*, then againſt 45 on the *Slider* ſtands 8550 on the *Rule*, the Superficial Content of all but the Baſe.

To find the Superficial Content of any Fruſtum of a Cone or Pyramid; or of a Cylinderoid or Priſmoid.

RULE 25. To the Periphery of the greater Baſe add the Periphery of the leſſer Baſe, half that Sum multiply'd by the ſlant Length gives the Convex Surface, to which add the Area of each Baſe and you have the whole Superficies by *Theorem* 65.

EXAMPLE XXXIII.

If the Periphery at the greater End be 502, *the Periphery at the leſſer End* 35, *and the ſlant Length or ſlant Height* 600, *What will be the Convex Superficies.*

502 *Periphery at* GB,
35 — — — *at* LB,

537 *Sum,*
600 *Length,*

2)322200

161100 *Convex Surface required.*

By the Sliding Rule.

Set 2 on the *Slider* to 537 on the *Rule*, then againſt 6 on the *Slider* ſtands 1611 on the *Rule*, which multiply'd by 100 (becauſe 600 was divided by 100) gives 161100 *&c.*

EXAMPLE XXXIV.

There is a Conical Fruſtum, the Periphery at the greater End is 52½ *the Periphery at the leſſer Baſe* 30¼, *the ſlant Length* 40, *required the Superficial Content of all but the greater Baſe upon which it ſtands.*

30,5 *Periphery* GB,
30,75 *Leſſer Baſe*,
83.25 *equal Sum*,
20 *equal half Height*,
166,5 *Convex Surface*,
752,47 *Area Leſſer Baſe*,
918,97 *Superficies reqd.*

30,75,
30,75,
15375,
21525,
22250,
945,5625 *Sq. Periphery* LB,
85970, *inverted Factor*,
66189
85 10
4 72
76
752:47 *Area leſſer Baſe.*

By the Sliding RULE.

Set 2 on the *Slider* to 83,25 on the *Rule*, then againſt 40 on the *Slider*, ſtands 166,5 on the *Rule* the Curve Surface.

2. Set

2. *Set* 1 on the Line D, to ,07958 on the *Slider*, then against 3,075 on the *Line* D, ſtands 7,52½ nearly on the *Slider* ſo 752,½ is the Area at the leſſer Baſe, which added to 166¾ gives 919,07 as before, nearly.

To find the Superficies of any Globe, or Sphere.

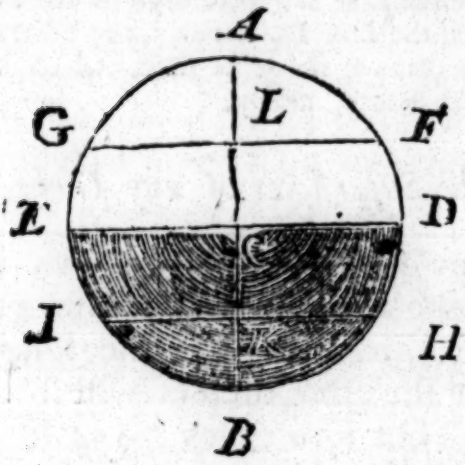
It is prov'd in *Theorem* 55, that the Surface of any Globe is equal to four times the Area of its greateſt Circle, now if the Diameter of a Circle be 1 the Area thereof will be ,7854 (by THEO. 30.) but four times ,7854 is 3,1416 the Periphery of that Circle whoſe Diameter is 1, ſo that, as the Diameter of any Circle is to the Periphery of that Circle, ſo is the Square of the Axis of any Globe, to the Surface of that Globe or Sphere, hence we have as many Rules for finding the Surface of a Globe or Sphere when the Axis is given, as we have for finding the Periphery of a Circle, when the Diameter is given: But I ſhall only uſe the following one.

RULE 26. { Multiply the Square of the Axis of any Globe or Sphere by 3,1416 and the Product is the Area or Surface of the Globe.

EXAMPLE XXXV.

If the Axis of a Globe be 10½, *What is its Curve ſurface?*

OPERATION:



OPERATION.

10,5 }
10,5 } = AB or DE. the Axis

525
1050

110,25 Square of the Axis,
3,1416 constant Factor,

66150
11025
44100
11025
33075

346,361400 Surface required.

By the SLIDING RULE.

SET 1 on the *Line* D to 5,14 on the *Slider*, then againſt 405 on the *Line* D (for 105 is off the *Rule*) ſtands 3,4636 on the *Slider*, ſo 316,36 is the Surface required.

To find the Superficies of the middle Zone of any Globe *or* Sphere.

Rule 27. Multiply the Axis of the Globe by the Length of the Zone and that Product multiply'd by 3,1416 gives the Curve Superficies to which add twice the Area of 1 of its Baſes (the two Baſes being equal) and you have the whole Surface of the Zone. See *Theorem* 55.

EXAMPLE XXXVI.

Let the Axis DE *of the Globe, which is alſo the greateſt Diameter of the Zone, be* 10,5 *and the Length* LK *of the Fruſtum or Zone, be* 7; *given to find the Curve Superficies of the ſaid Zone* FLGIKH.

10,5 = DE *the greateſt Diameter of the Zone,*
7 = LK *the Length of the Zone,*

by 73,5
Multiply 3,1416 *Conſtant Factor,*

157080
94248
219912

230,90760 *the Curve Superficies required.*

By

By the SLIDING RULE.

SET ,318 (*viz.* 3,1416) on the *Slider* to 10,5 on the *Rule*, then againſt 7 on the *Slider* 231 nearly on the *Rule* the Superficial Content of all but the Ends, as was required.

To find the Superficial Content of any Segment of a Globe *or* Sphere.

RULE 28. Multiply the Axis of the Globe by the Height of the Segment and that Product multiply'd by 3,1416 gives the Curve Surface to which add the Area at the Baſe and you have the whole Surface. See *Theorem* 55.

EXAMPLE XXXVII.

Let AB 10,5, *the Axis of the Sphere*, BK 1,75 *the Height of the Segment be given to find the whole Superficies of the Segment*, HKIB.

By the Property of the Circle, or by Menſuration, the Diameter H I *of the Segments Baſe will be found nearly equal to* 8.

OPERATION.

OPERATION.

10,5 = AB *the Globe's Axis*,		0,7854	
1,75 = KB *the Seg. Height*,		64 = *Square of* HI,	
525		31416	
735		47124	
105			
18,375		50,2656 = *Area Base*	
6141,3	*inverted Factor*,	57,726 *Curve Sur. add*,	
55125			
1837			
735			
18			
11			
57,726 *Curve Superficies*.			

By the SLIDING RULE.

SET 0,318 on the *Slider* to 16,5 on the *Rule* hen againſt $1\frac{3}{4}$ on the *Slider*, ſtands 18,375 on he *Rule*, the Convex Surface

2. *Set* 1 on the Line D, to 0,7854 on the *lider*, then againſt 8 on the Line D, ſtands 0,26 on the *Slider* the Area of the Baſe as be-ore.

o find the Surface of any Solid when the Property of the generating Space is not known.

RULE XXIX.

This is the ſame with Rule 22, (ſee *Theo.* 65) ake 4 girths about the Solid equally diſtant from ne another, and to 3 times the Sum of the two iddle Peripheries add the Sum of the two Ex-

treme ones that Sum multiply'd by ½ the Length of the Curve between the two Extremes, gives the Curve Surface of all between the said two Extremes.

EXAMPLE XXXVIII.

In the annexed Solid ABC, *let there be* 7 *Peripheries taken* viz. AB = 15, *a b* = 14 *c d* = 13,3 *e f* = 11, *g h* = 7, *i k* = 3,2 *and the End at* C = 0, *suppose the Length* AC *or* BC *of the Curve*, = 32,8 *required the Curve surface of this Solid.*

I might here Work all the 8 Periphery's at once (by *Prob.* 169, *Table* 7 *&c.*) but to hold to the above Rule, I shall Work 4 at once and so have two *Operations.*



15 = AB,
11 = *ef*,
—
26 = *Sum Ex. Per. P. Aef*
14 = *ab*,
13,3 = *cd*,
—
27,3 = *Sum mid. Prr. Par*
3
—
81,9 = 3 *times the last Sum*
26
—
107,9 *Sum Extremes add*

107

107,9 *laſt Sum*,
2,05 = $\frac{1}{8}$ *of* A*e*, *or* B*f*

5395
21580

221,195 = *the Curve Superficies of the Part* A*e f* B,
7 = *gh* 11 = *ef*,
3,2 = *ik* 0 = *End* C,

10,2 11 = *Sum Extreme Peripheries Part e*C*f*,
3

30.6 = 3 *times Sum middle Peripheries of the ſaid Part*,
11 = *Sum two Extremes*,

41,6
2,05 = $\frac{1}{8}$ *of e* C, *or f* C;

2080
8320

85,280 = *Curve Surface of the Part e*C*f*,
221,195 = *Curve Surface of the Part* A*e f* B,

306,475 = *Curve Surface of the whole Solid* ACB,

By the SLIDING RULE.

SET 8 on the *Silder* to 16,4 (*viz.*) half 32,8 C, or BC) on the *Rule*, then againſt 10,79 107,9 is off the *Rule* on the *Slider* ſtands ,119 on the *Rule* ſo 221,19 is the Curve Sur- ce of the Part A*e f* B, Again, as the *Slider* now nds againſt 41,6 on the *Slider*, you'll find ,28 on the *Rule*, the Curve Surface of the rt *e*C*f*,

To

To find the Superficial Content of any of the Five Regular Bodies.

ALtho', by the last Rule you may Approximate the Surface of any Solid, yet since there are easy and particular Rules for Surfaces of these five Regular Bodies, I shall lay them down in their Order.

1. The *Tetraedron*'s Surface being 4 equal and Equilateral Triangles, if you find the Area of any one of these Triangles and Multiply that Area by 4, you will have the Superficial Content of the whole Tetraedron (or being a Pyramid it may be had by Rule 24) now if the Side of an Equilateral Triangle be 1, its Area will be ,433013, by the Table of Polygons; which multiply'd by 4 gives 1,732051, for the Surface of that Tetraedron, whose Side is 1 or Unity.

2. The Surface of the Octaedron being composed of 8 equal and Equilateral Triangles if its Side viz. a Side of one of these Triangles be 1 its Superficies will be double to that of the Tetraedron (*viz.*) 3,464102.

3. The Hexaedron or Cube is already done in *Example* 28, if a Side thereof be 1 its Superficies will be 6.

4. The Surface of the Icosaedron is contained under 20 equal and Equilateral Triangles so if a Side of one of these Triangles be one

its Area will be ,43013 (as above) which multiply'd by 20 gives 8,660260 for the Surface of that Icosaedron, whose Side is Unity.

5. The Surface of the Dodecaedron being 12 Pentagons, and the Area of a Pentagon whose Side is 1, being 1,720477 multiply'd by 12 gives 20,645724 for the Superficies of a Dodecaedron whose Side is 1 or Unity, and thus is this Table constructed, which shews the Superficial Content of any of the 5 regular Bodies whose Side is 1 or Unity.

A TABLE.

SHEWING *the Superficial Content of any of the five Regular Bodies, the side being* 1, *or Unity.*

The Names of the Bodies.	*Superficies.*
TETRAEDRON.	1,732051
OCTAEDRON.	3,464102
HEXAEDRON.	6,
ICOSAEDRON.	8,66026
DODECAEDRON.	20,645724

By help of this Table the Superficies of any of these Bodies may easily be had by the following Rule, from *Theorem* 3.

Multiply

RULE 30. { Multiply the Square of the Side of the proposed Body, by the Tabular Number and the Product will be the Superficies of that Body

EXAMPLE XXXIX.

If the Side of a Tetraedron be 10, *What is its Surface?*

1,732051 Number against Tetraedron,
100 Square of the given Side,

173,2051 Superficial Content required.

By the SLIDING RULE.

SET 1 on the Line D, to 1,732 on the *Slider* then against 10 on the *Line* D, stands 173,2 on the *Slider* the Surface required, See *Rule* 4, SECTION 1.

EXAMPLE XL.

If the Side of a Dodecaedron be 13, *What is its Superficies?*

20,645724	Tabular Number	13 } given Side.
169	Square of a Side,	13 }
185811516		39
123874344		13
20645724		169 its Square.

3489,127356 Superficies required.

By

By the SLIDING RULE.

SET 1 on the *Line* D, to 20,64 on the *Slider* then againſt 1,3 on the *Line* D, (for 13 is off the *Rule*) ſtands 34,89 on the *Slider* which multiply'd by the Square of 10 (becauſe 13 was divided by 10) gives 3489 for the Superficies as above nearly.

SEC. VI.

SECTION VI.

ARTIFICERS *Works.*

ARTIFICERS, have different Ways of Measuring, according to the Custom of each Trade: Some give the Content in Feet, some in Yards, some in Squares, some in Rods &c. They also differ in their Measures according to the Custom of the Country, some Measure one Way and some another, &c. Now as a General Rule for all the various Methods amongst these Workmen, It is best to take the Dimensions in Feet and Decimal Parts of a Foot; and having found the Content in Feet. you may reduce it into what kind of Measure you please, by RULE I. SEC. V. For it is seldom that Artificers use ought but Superficial or Flat Measure, and because Floors, Walls, Windows, Doors, Roofs, &c, are Rectangles, the Areas of such Forms are always found by RULE II. SEC. V. But if any other Form offer it self, it cannot but come under some of the foregoing RULES.

So that what is to be done in this SECTION, is only to repeat some of the foregoing Labours. An EXAMPLE or two in every Case will make the whole more easy.

Of

Of BRICKLAYER's Work.

The Principals *are* WALLING, TILEING, PARTITIONING, *and* CHIMNEY-Work.

I. *Of* WALLING.

BRICKLAYERS commonly Measure their Work by the Rod Square of 16 Feet and a half; so that one Rod in Length and one in Breadth, contain 272,25 Square Feet; for 16,5 multiply'd into it self produces 272.25 Square Feet. But in some Places the Custom is to allow 18 Feet to the Rod; that is, 324 Square Feet, and in some places the usual Way is to Measure by the Rod of 21 Feet long and 3 Feet high, that is 63 Square Feet; and here they never Regard the thickness of the Wall, but the usual Way is to moderate the Price according to the Thickness.

When you Measure a Piece of Brick-work, the first Thing is to enquire by which of those Ways it must be measured; then having multiply'd the Length and Breadth in Feet together, Divide the Product by the proper Divisor, either for Rods or Roods, and the Quotient will be the Measure required.

But in *England*, commonly Brick-walls that are measur'd by the Rod, are reduc'd to a Standard Thickness, this is THE

RULE.

Multiply the Number of Superficial Feet contain'd in any Wall, by that Number of half Bricks which that Wall is in Thickness: one third Part of that Product shall be the Content thereof in Feet, when reduc'd to standard Thickness.

EXAMPLE I.

If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick, how many Rods of Brick-work are contain'd therein when reduc'd to the Standard?

19,25
72.5
9625
3850
13475
1395,625
11
15351,875
272) 5117,291 (18 Rods.
239479
68) 21679 (3 *quar: of a Rod.*
18,8 on the Rule, so the Answer is 18,8 Rods.

Note 1, That 68 is $\frac{1}{4}$ of 272.

2. That in reducing of Feet into Rods, they usually reject the odd Parts, and divide only by 272, as in the last EXAMPLE.

By the SLIDING RULE.
Set $\frac{1}{4}$ *viz.* 272 on the Slider, to 72.5 on the Rule, then against 19,25 on the Slider, stands 5117,29 on the Rule the Answer in Feet

2d. Set 272 on the Slider, to 5117:29 on the Rule, then against 11 on the Slider stands

Note 3

3. NOTE, *The uſual Way to Meaſure Buildings is to Meaſure with a Line half about the Building on the out Side, and half about in the middle thereof, and theſe two Dimenſions are the Compaſs, and by this Way of Meaſuring the thickneſs of the Walls is accounted for as you may eaſily perceive.*
4. *The Pikes on the End Walls being Triangles meaſured, but commonly Wallers compute two of thoſe Pikes to be a Rectangle, whoſe Height is equal to the Height of the Pike, and Breadth equal to the Breadth of the Building.*

EX 2. *If each Side Wall of a ſtone Building be* 15 *Yards long on the out Side and each End Wall* 5 *Yards broad on the inſide, the Pike at either End Wall* 2 *Yards high, how many Square Yards of all is in that Building, the Height thereof being* 6,25 *Yards.*

30 Sum of the Side Walls
10 Sum of the End Walls
—
40 Compaſs of the Building,
6,25 Height,
———
250,00 Square Yards without Pikes,
10 Pikes add,
———
260 Square Yards in the Building.

5 Br. of the End Walls
2 Height of the Pikes,
—
(two Pikes,
10 Sq. Yards in the

By

By the SLIDING RULE.

SET 1 on the *Slider* to 6,25 on the *Rule*, then against 40 on the *Slider* stands 250 on the *Rule*, to which add 10, and you have the Content as before.

Stone Buildings are sometimes reckon'd amongst Mason's *Work.*

Of CHIMNEYS.

IN Measuring a *Chimney*, if it stand alone without leaning against any Thing, girt it about below the Mantle and take that for the Length, and the Height of the Room, or Chimney; so far as it keeps the same Girt, for the Breadth and Multiply them together for the Content, But if it stand against a Wall then you must Measure it round to the Wall, for the Girt and take the Height as before.

For Arches some take the Breadth and half Breadth of the Arch and add them together and Multiply that Sum by the Length of the Arch for the Content, some make deduction for Doors Windows and the vacancy in Chimneys, between the Hearth and Mantle &c. and some make none.

Of TILING *and* SLATING.

TILING is measured by the Square of 10 Feet long and 10 Feet broad so that one Square contains 100 Square Feet Slating sometimes by the Yard Square and sometimes by the Square Rod of 49 Square Yards *&c.* the Way to Measure both. is to Measure along the Ridge of the Building with a Line for the Length, and then having a small Weight at one End of a Line, put it over the Ridge and let it go down to the Eave and take the other End, to the other Eave, so have you the Breadth of the Roof, where observe, that double Measure is commonly allow'd for Hyps, Vallies, Gutters *&c.* In Tiling it is usual to allow double Measure at the Eaves so much as the Projecture is over the Plate, which is commonly about 18 or 20 Inches.

Ex. 3. *If the Breadth of a Building's Roof with the usual allowance at the Eaves be* 24½ *Feet and the Length along the Ridge* 45 *Feet, how many Square Yards Squares and Rods are in that Roof?*

Feet.
245 Breadth,
45 Length,

1225
980

100(1102,5 Square Feet,
11,025 Squares,

9)1102,5 Square Feet,
49)122,5 (2½ Square Roods, or 2 Rood and 24,5 Yards.

B.

By the SLIDING RULE, *for* Yards.

SET 9 on the *Slider* to 45 on the *Rule*, then against 24,5 on the *Slider* ſtands 122 $\frac{1}{2}$ on the *Rule* the Content in Square Yards: And if inſtead of ſetting 9 on the *Slider* you ſet 100, you'll have the anſwer in Squares, or ſetting 441 (*viz* 9 times 49) you'll have the anſwer in Yards.

Of CARPENTER's Work.

CARPENTERS Works, as Flooring, Partitioning and Rooffing are meaſured by the Square of 100 Square Feet like Tiling and in ſome Places by the Square Yard.

Ex. 4. *If a Floor be* 57 $\frac{1}{4}$ *Feet long, and* 28 *Feet broad, how many Squares?*

57,25 Length,
28,5 Breadth,

28625
45800
11450

100)1631,625 Square Feet,

16,331625 Squares.

By the SLIDING RULE.
SET 100 on the *Slider*, to 57¼ on
the *Rule* then againſt 28½ on
the *Slider*, ſtands 16½ *feet* on
the *Rule*, the Squares required.

Ex.

Ex. 5. *If a Partition between two Rooms be in Length* 80,5 *Feet, and in Height* 12,25 *how many Squares are contain'd therein?*

Feet.	By the *SLIDING RULE.*
12,25	Set 100 on the Slider to 80,5 on the
80,5	Rule, then againſt 12,25 on the
6125	Slider, ſtands 9,9 *fere* on the
98000	Rule, the Squares required.
100) 986,125 Square Feet.	*Note*, That to divide by 100, is only to make two more Places of Deci-
9,86125 Squares.	mals.

Ex. 5. *If a Houſe within the Walls be* 40,5 *Feet long and* 20,5 *Feet broad, how many Squares are contain'd in the Roof thereof?*

Feet.
40,5 Length,
20,5 Breadth,
2025
8100
100)830,25 Square Feet,
8,3025 Squares in the Flat,
4,1512 half the Flat
12,4537 Squares in the Roof,

By the SLIDING RULE.

SET $\frac{200}{3}$ (*viz.*) 66,66, on the Slider to 40,5 on the Rule, then against 20,5 on the Slider stands 12,45 on the Rule the required Squares in the Roof if its truly pitch'd, there are other Works belonging the Carpenter, which are measured by running Measure (*viz.* Length only without Breadth) such as Cornices, Windows, Frames, Lintles *&c.* Also in Flooring Partitioning, and Rooffing deductions are made for Stairs, Chimneys, Doors *&c.* Roofing in some Places excepted.

'If a House's Roof be truly pitch'd, The 'Flat and half Flat thereof taken within the 'Walls (*viz.* the Area and half Area of the 'Floor) is equal to the Measure of the Roof 'on which Footing this Example is wrought 'or as 2 is to 3, so is the Area of the Floor to 'that of the Roof.'

Of PLAISTERER's Work.

PLAISTERING, Rendering and Cieling are all measured by the Square Yard of three Feet long and three Feet broad, when a Room is plaister'd or rendered, take its Compass in the inside guting in Chimney's *&c.* where the Work comes, which multiplied by the Height of the Room gives its Content.

Ex. 6.

Ex. 6. *If the Compaſs of a Room be* 141,5 *Feet and its Height* 11,25 *Feet, how many Square Yards.*

Feet Circuit.		*By the* SLIDING RULE.
141,5		Set 9 on the Slider to 141,5 on the
11,25	Height,	Rule, then againſt 11,25 on the
		Slider, ſtands 176,87 on the
7075		Rule, the ſquare Yards reqd.
2830		
1415		NOTE, If you have a Rule a
1415		Yard long Decimally divided
		to take Dimenſions with, you
9)1591,875	Square Feet.	need not divide by 9. See the
		Beginning of the laſt Sect.
176,875	Square Yards.	

NOTE, *That* Whiting *and* Colouring *are both meaſured by the Yard as* Plaiſtering, *alſo when any of theſe Works are to be meaſured enquire what their Cuſtom is for deductions, and how they are allow'd for? not only in theſe but in all Artificers Works, ſuch Cuſtoms are variable.*

Of JOINER's Work.

JOINERS Work is alſo meaſured by the Square Yard, But in taking Dimenſions they dent in the Line wherever the Plane comes, ſo that in taking the Height of any Room where there is a Corniſh about, and ſwelling Pannels and mouldings, with a ſtring, be-

gin at the Top and girt over the mouldings which makes the Room higher than it is and for Measuring about the Room they only take it as it is upon the Floor.

Ex. 7. *If a Room of Wainſcot (being girt over the mouldings) be* 16,25 *Feet and the Compaſs thereof* 137,5 *Feet; how many Square Yards are therein?*

Feet.
137,5 Circuit.
16,25 Height.

By the SLIDING RULE.

6875.
2750
8250
1375

Set 9 on the Slider to 137,5 on the Rule, then againſt 16,25 on the Slider, ſtands 248,3 *fere* on the Rule the Square Yards required.

9)2234.375 Square Feet.

248,264 Square Yards.

Note, *That Doors, Window-ſhutters, and all ſuch Works as are wrought on both Sides are called Work and half Work and in all ſuch Caſes you muſt find the Content as before, and then take half that Content and add to itſelf, ſo have you the Content of Work and half. Otherwiſe, Multiply the Content by* 3 *and divide that Product by* 2 *ſo have you the Content at Work and half.*

Ex. 8

Ex. 8. *If the Window ſhutters about a Room be 70 Feet Broad (viz. when all the Breadths are taken together) and each 6,5 Feet high how many Yards are therein at Work and half?*

Feet.
6,5 Height of a Window,
70 Breadths,

455,0 Square Feet,
Multiply by 30

2)1365 (and half,
9)682,5 Square Feet at Work
75,8 Square Yards at Work (and half,

By the SLIDING RULE.

Set 6, (viz. 9 times 2 divided by 3) on the *Slider* to 70 on the *Rule* then againſt 6,5 on the *Slider* ſtands 75,8 on the *Rule* the anſwer in Square Yards at Work and half.

☞ In Doors &c. if the Heights be all equal you may add all the Breadths, or if the Breadths be all equal (and the Heights unequal) you may add all the Heights together, as in this *Example* the Heights are all equal, ſo the Sum of all the Breadths is multiplied by the common Height for the Content which is much eaſier than to find the Content of each window by it ſelf &c.

Of

Of PAINTER's Work.

PAINTERS Work is the same with Joiners both in taking Dimensions and Measure.

EX, 9. *If a Globe be Painted whose Axis is* 6,2 *Feet, how many Yards of Painting is thereon.*

6,2 } Axis.
6,2 }

124
372

38,44
3,14 see *Rule* 26, *Sect.* 5.

15376
3844
11532

120,7016 Square Feet, which divided by 9, gives 13,4 &c.

By the SLIDING RULE.

Set 3 (Square Root of 9) on the Line D. to 3,1416 on the Slider, then against 6,2 on the Line D, stands 13,4 on the Slider, the Square Yards required.

Of MASON's Work.

MASONS Work is sometimes measured by the Foot Square, and sometimes by the running Foot (*viz.* Length without Breadth) where observe that for every Foot in Length running Measure there ought to be a Foot in Breadth or girt if it be a Solid, for this Metho

Method of running Meaſure ought to anſwer to Superficial Meaſure; If a Stone be hew'd into any Solid Form you may find its Superficial Content by ſome of the Rules in the laſt Section. Maſons (like Joiners *&c.*) always Meaſure as far as their Tools come.

Ex. 10. *If a Wall be* 112,25 *Feet long and* 16,5 *Feet high, how many Rods are contain'd therein; allowing* 21 *Feet in Length and* 3 *Feet in Breadth* (*viz.*) 63 *Square Feet to a Rod.*

Feet.	By *the* SLIDING *RULE:*
112,25 Length,	*Set* 63 on the Slider to 16,5 on
16,5 Height.	the Rule, then againſt 112,25
	on the Slider ſtands 29,4 *fere*
56125	on the Rule the Rods reqd.
67350	
11225	*Note,* Paving is commonly reckon-
	ed in with *Maſon's* Work.
63)1852,125 Square Feet,	
29,4 *fere,* Rods or 29 Rods 25 Feet.	

Of GLAZIER's Work.

GLaziers Meaſure by the Foot Square, and take Dimenſions nearer than regarded by any of the foregoing, for they commonly take to $\frac{1}{4}$ of an Inch, but if you take Dimenſions by a Decimally divided Foot Rule, you may

may Measure to $\frac{1}{11}$ of an Inch, which is less than half a Quarter of an Inch, And here take Notice that all Curve-lin'd Windows, as Rounds Ovals, Arches *&c.* are measured as if they were full Square *viz.* the greatest Length and Breadth taken and multiplied together as if it were a Rectangle, on account of Labour and waste of Glass.

Ex. 11. *If there be* 4 *Panes of Glass each* 4,23 *Feet high, and* 1,25 *Feet Broad, how many Square Feet is contain'd therein?*

Feet.	*By the* SLIDING *RULE.*
4,23 *Height of each Window,*	
6 *Sum of their Breadths,*	Set 1 *on the Slider, to* 4,23 *on the Rule, then against* 6 *on the Slider stands* 25,38 *on the Rule the ans.*
25,38 *Square Feet required.*	

Ex. 12. *If the Diameter of a round Window be* 2,32 *Feet, What is its Content.*

Feet.	*By the* SLIDING *RULE.*
2,32 *Diameter or Br.*	*Set* 1 on the *Slider* to 2,32 on
2,32 Ditto,	the *Rule*, then against 2,32
464	on the *Slider*, stands 5,38
696	on the *Rule*, the Content
464	required.
5,3824 Content required.	

I might have stor'd this SECTION with more *Examples*, but I thought it needless for if the Customs and Methods of taking Dimensions be known

known there can be no difficulty in finding any Superficies if the laſt Section be carefully obſerv'd, for more Particulars, *See the Queſtions.*

To Meaſure BOARD *and* PLANK.

BOARD and *Plank* are ſometimes ſold by the Foot, running Meaſure, which is only the Plank meaſured along its middle, and ſometimes by the Foot Superficial, which is had by multiplying the Length, taken along the middle in Feet and Decimal Parts of a Foot, by the Breadth at the middle in Inches that Product divided by 12, gives the Flat or Superficial Feet, ſometimes by Solid Meaſure which is moſt reaſonable, becauſe it multiplies Length, Breadth and Thickneſs: And if theſe three Dimenſions be all taken in Inches, you muſt divide by 1728 the Solid Inches in a Solid Foot; but if any of theſe 3 Dimenſions be Feet as moſt commonly the Length is, then you muſt only divide by 144 the Square Inches in a Square Foot, *&c.* But it will be the eaſieſt and trueſt way, and will require no Diviſion, if you take the Dimenſions all in Feet or *Decimals* of Feet, as Taught at the Beginning of the laſt *Section.*

EX. 13.

Ex. 13. *If a Board be* 16 *Inches broad and* 13 *Feet long, How many flat Feet are therein?*

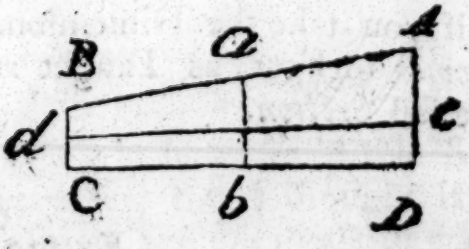
Vulgarly.	*Common Way.*	*Decimally.*
Inches	Feet.	Feet.
156 *Length*	13 *Length*	13 *Length*
16 *Breadth*	16 *Breadth*	1,333 *Breadth*
936	78	3999
156	13	1333
144)2496 *sq. Inches.*	12)208	17,329 *sq. Feet*
17 $\frac{48}{144}$ *sq. Feet,*	17 $\frac{4}{12}$ *square Feet.*	

By the Sliding Rule.

1. As 144 on the Slider, is to 16 on the Rule; ſo is 156 on the Slider, to 17$\frac{1}{3}$ ſquare Feet, *Anſwer.*
2. As 12 on the Slider, is to 13 on the Rule; ſo is 16 on the Slider, to 17$\frac{1}{3}$ on the Rule, as before.
3. As 1 on the Slider, is to 13 on the Rule; ſo is 1$\frac{1}{3}$ on the Slider, to 17$\frac{1}{3}$ on the Rule, as before.

Any of theſe three Ways (as alſo by Feet and Inches) may be taken, but the laſt being moſt expeditious, and conſiders the Dimenſions taken Decimally. I ſhall in the following *Examples* inſiſt chiefly thereon.

Ex. 14. *Let it be required to find the Superficial Content of the Board* ABCD, *whoſe Length* de *along its Middle is* 15,2 F, *and Breadth* ab *croſs in Middle* 0,32 *Feet,*



Feet.	
15,2	Length *de*
,23	Breadth *ab*
304	
456	
Anſ. 4,864	8q1 Feet.

By the SLIDING RULE.

Set 1 on the Slider to 15,2 on the Rule, then against 0,32 on the Slider ſtands 4,87 *ſcre* on the Rule, the Square Feet required.

The Ends *AD* and *BC* being parallel, and *ab* taken parallel to *Cd* in the Middle between them muſt be = half their Sum; and the Sides *AB* and *CD* being not parallel, *de* taken in the Middle between them, muſt be Perpendicular to *ab*, conſequently, *de* multiply'd by *ab* = Area of *ABCD*, by *Theo.* 11, The ſame holds good if the Sides be parallel and the Ends not parallel, but if neither the Sides nor Ends be parallel, it muſt be meaſured as a *Trapezia*.

Ex. 15. *If the Length of a Plank be* 15,2 *Feet and its Breadth* 0,32 *Parts of a Foot, as in the laſt* Ex *and its Thickneſs* 3 *Inch, or* 0,25 *Parts of a Foot, What is its Solid Content?*

Feet.

15,2 Length *de*.
,32 Breadth *ab*.

304
456

4,864 *ſuperficial Content*.
,25 Thickneſs.

24320
9728

1,21600 Solid Content.

NOTE, If any one of the 3 Dimenſions be ſtill the ſame however it be taken, as here the Depth is every where 3 Inch, the Figure muſt then be a *Priſm*, and its Content Solid will be found by THEO. 15, PROB. 166, for *ABCD* repreſents the Baſe of the *Priſm*, and 3 Inch its Height &c.

By the Sliding Rule.

By Rule 5*th*, Sec. 1*ſt*, Find a mean Proportional between any 2 of the 3 Dimenſions; The Breadth and Depth are commonly uſed for this Purpoſe: Then ſet 1 on the Line D, to the Planks Length on the Slider, and againſt the mean Proportional on the Line D, ſtands the Solid Content on the Slider ſo in this Example ,25 on the Slider ſet to ,25 on the Line D, againſt ,32 on the Slider ſtands the mean Proportional on the Line D, Then ſet 1 on the Line D, to 15,2 on the Slider, and againſt the mean Proportional on the Line D, ſtands 1,22 *fere* the Solid Content on the Slider.

Otherwiſe, without a mean Proportional or the Uſe of the Line D, turn any one of the 3 Dimenſions into a Diviſor, and then it will be as this Diviſor on the Slider is to one of the Dimenſions on the Rule, ſo is the third Dimenſion on the Slider to the Solid Content on the Rule, ſo in this Example if you turn ,25 to a Diviſor it may be done thus Set ,25 on the Rule to 1 on the Slider, then againſt 1 on the Rule ſtands 4 on the Slider the diviſor requir'd; then, Set 4 on the Slider to ,32 on the Rule, then againſt 15,2 on the Slider ſtands 1,22 *fere* on the Rule the Content as before.

Note, That as the Menſuration of Square Timber requires Length, Breadth and Thickneſs, it muſt be exactly the ſame with Plank meaſure Solid,

Solid; ſo that the Rules to this Example hold as well for Square Timber, as Solid Plank: Therefore, when you are to Meaſure a Piece of Square Timber; take the Length of the Tree, and the Breadth and Depth in the Middle thereof, and multiply thoſe 3 Dimenſions together for the Solid Content as before: This Method gives the true Content if the Baſes be equal, but varies a little from the Truth if the Timber be tapered, as is proved in meaſuring the Fruſtum of a *Pyramid.*

SOLID Meaſure.

Ex. 16. *If a Piece of Square Timber be* 13,2 *in Length, and* 1,52 *Feet Square* (viz.) 1,52 *Feet broad and* 1,52 *Feet deep, how many Solid Feet is contain'd therein?*

Feet.	
1,52	Depth
1,52	Breadth
304	
760	
152	
2,3104	
13,2	Length
46208	
69312	
3104	
49728	Solid Content.

By the SLIDING RULE.

Set 1 on the Line D to 13,2 on the Slider, then againſt 1,52 on the Line D ſtands 30,5 on the Slider, the Content required.

Ex. 17

Ex. 17. *If a Piece of Square Tapering Timber be* 15 *F. long* 1 *F. broad* 0,7 *F. deep at the leſſer End, and* 2 *F. broad,* 1,4 *F. deep at the greater End, how many Solid Feet is therein.*

Half the Sum of the two Breadths 1 and 2 is 1,5 the Breadth in the Middle and half the Sum of the two Depths 0,7 and 1,4 is 1,05 the Depth in the Middle of the Tree.

Feet.		
1,5	Breadth }	taken in the Middle of the Tree.
1,05	Depth }	
75		
150		
1,575	Area in the Middle.	
15	Length	
7875		
1575		
23,625	Solid Feet.	

Note, If the Tree be not contain'd under ſtreight Planes from Baſe to Baſe, ſo that you judge its Breadth and Depth taken in the Middle, not exact, you may take theſe Dimenſions at each Baſe, or End and half their Sum will give that in the Middle, as in this *Example.*

By the Sliding Rule.

1. Set 1,05 on the Line D, to 1,05 on the Slider, then againſt 1,5 on the Slider ſtands 1,25 on the Line D, a mean Proportional between 1,05 and 1,5.
2. Set 1 on the Line D to 15 on the Slider, then againſt 1,25 on the Line D ſtands 23,62 on the Slider the Content required.

Otherwiſe

Otherwiſe.

1. Set 15 on the Rule to 1 one on the Slider, then againſt 1 on the Rule ſtands 1,066 on the Slider, which is 15 the Length turn'd to a Diviſor.
2. Set 1,066 on the Slider to 1,05 on the Rule, then againſt 1,5 on the Slider ſtands 23,62½ on the Rule the Content as before.

Ex. 18. *If a Piece of Square Tapering Timber be 18 Feet long, 6 Inches deep and 1,5 Feet broad in the Middle, how many Solid Feet are contain'd therein?*

Feet.
1,5 Breadth
,5 Depth } taken in the Middle of the Tree.

,75
18

13,50 Solid Content.

By the SLIDING RULE.

Firſt 5 turn'd to a Diviſor as before, is 2; ſo Set 2 on the Slider to 18 on the Rule, then againſt 1,5 on the Slider ſtands 13,5 on the Rule.

☞ Some Meaſurers of Timber take half the Sum of the Breadth and Depth taken in the Middle of the Tree, for the Side of a mean Square; which Squared and multiply'd by the Length, they ſay, gives the Content: But this Method gives the Content too much, and if the difference between the Breadth and Depth be conſiderable, the Error is great as in the laſt *Example*; the Breadth 1,5 Feet added to the Depth ,5 gives 2, half whereof is 1, whoſe Square is 1 which multiply'd by 18 the Length, gives 18 for the Content too much by 4,5 Feet.

To

To Measure Round Timber.

THE common way is to Gird the Tree in the Middle, and take a fourth of the Girth for the Side of a mean Square, which Squared and multiply'd by the Length gives the Content; but if the Tree be uneven in the Middle so that a true Girth cannot be taken, you must Gird it at each End and take half the Sum of those Girths for the mean Girth, one fourth whereof is the Side of a mean Square as before.

The Girth is commonly taken with a String and the String being doubled is Measured in Inches by a Foot Rule, and half those Inches taken and Squared, multiply'd by the Length in Feet, and divide by 144 gives the Content in Feet,

From whence we have this General Rule upon the Sliding Rule.

Set 12 *on the Line* D *to Length of the the Tree in Feet on the Slider, then against* $\frac{1}{4}$ *of the Girth upon the Line* D, *stands the Content on the Slider.*

But if you take the Girth Decimally you need not divide by 144, and the Rule is the same on the *Sliding Rule* only set 1 on the Line D instead of 12. This is the Method generally Practised, because of its Ease and Expedition, but it always

gives

ives the Content too little: For it is Prov'd rom what goes before, that if the circumference f a Circle be 1 its Area will be ,07958 but ¼ of is ,25 which Squar'd will be ,0625 the Area of Circle whoſe circumference is 1. By the above ay of meaſuring, ſo that if the Timber be equal aſed, *viz.* in Form of a *Cylinder*, the true Con- nt will be that found by this Method,

As ,07958 to ,0625 which is nearly as 22 to 18. o that thoſe who buy Round Timber by this Meaſure have nearly one fifth Part allowed for Chips, and if the Timber be tapering the Error r allowance is greater, as appears by meaſuring e Fruſtum of a Cone in the following SECT.

x. 19. *If a Piece of round Timber be* 20 F. 3 I. *Long, and* 24 *Inches Girth in the Middle, how many Solid Feet are contain'd therein?*

The Common Way.		Decimally.	
Inches.		*Feet.*	
6	¼ Girth.	,5	¼ Girth
6		,5	
36	Square ¼ Girth.	,25	Square ¼ Girth.
20,25	Length.	20,25	
12150		10125	
6075		4050	
Feet.			
4) 729,00	(5,06 Solid Cont:	5,0625	Solid Content.

By the SLIDING RULE.

Set 12 on the Line D to the Length 20,25 on the Slider, then inſt 6 on the Line D ¼ of the Girth, ſtands 5,06 the Content the Slider. Again,

Set 1 on the Line D to the Length 20,25 on the Slider, then inſt ,5 on the Line D ſtands 5,06 the Content on the Slider.

Ex. 20.

Ex. 20. *If a Piece of round Timber be* 18 *Feet long* ,92 *Feet Girth at the leſſer End, and* 1,8 *Feet Girth at the greater End, how many Feet of Timber are therein.*

Here ,92 + 1,8 = 2,72, $\frac{1}{4}$ = 1,36 the Girth in the Middle, $\frac{1}{4}$ = ,34 = $\frac{1}{4}$ of the mean Girth in Feet.

Common Way,

Feet.

,34 $\frac{1}{4}$ Girth
,34
136
102
,1156 Square $\frac{1}{4}$ Girth.
18 Length.
2,0808 Content.

By the Sliding Rule.

Set 1 on the Line D to 18 on the Slider, then againſt 3,4 on the Line D, (for ,34 are off the Rule) ſtands 208 on the Slide which divided by 100 gives 2,08 Content. For the true Content ſee Ex. 15. Sec. 7.

Ex. 21. *If the Length of a Stick be* 3,5 *Feet and its Girth* 2 *Inch, What's the Content?*

,5 $\frac{1}{4}$ Girth.
,5
,25
3,5
125
75
744),875 (,0061 Feet.
1728 Inches in a Solid F.
110
1728
10368
10,5408 Content in Inches.

By the *Sliding Rule.*

Set the Gauge Point 12 on the Line D to the Length 3,5 on the Slider, then againſt 50 on the Line D (for ,5 is off the Rule) ſtands 61 *fere*, on the Slider, which divided by the Square of of 1 gives ,0061 Feet, the Content.

To Meaſure a Growing TREE.

BY PROB. 129, 130, 131, you may find the Height of any Tree, and by PROB. 161, you may find the Girth in the Middle or any other Part, and from thence may compute the Content of the Tree, as in the laſt EXAMPLES.

EX. 22. *If the Height of a Tree be* 19,2 *Feet, and a fourth of the Girth* 1,5 *Inches, What's the Content?*

N. B. For the Learner's practice the following EXAMPLES are contracted, and alſo for the Advantage of the Book from ſwelling too much beyond the bounds intended.

The Common Way.		*Decimally.*	
Inches.		Feet.	
1,5		,125	
1,5		,125	
2,25	ſquare ¼ Girth.	,015625	ſq. ¼ Girth.
19,2	Feet Length.	19,2	Height.
(44)43.200(,3	Feet, Content.	,3000000	Content.

By the SLIDING RULE.

Set 12 on the Line D to 19,2 on the Slider, then againſt ,5 (for 1,5 is off the Rule) on the Line D ſtands 30 on the Slider, which divided by the ſquare of 10 gives 0,3 Feet for the Content.

NOTE, If the above method of measuring growing Trees be not thought sufficient, you may take the height and girth with a ladder a long staff and a string, and work out the Content as before, and having a paper ruled as follows. The first tree you measure in the forrest &c. mark it with the number 1, which set down in the colum under N°, which signifies number, with its length and girth under their proper titles. The second tree you measure mark with the number 2, which set down in your book with the length and girth and their names as before: And so may you measure as many trees need requires, and cast up their contents either by the pen or rule, or both; but the rule commonly serves for all, and having the contents set them down as you see below; by which means you may not only have the list of any number of trees, but also the length, girth and content of each tree, and so be enabled to sell any tree either by the foot or by the tree, and that as well at home as where the timber grows. And if you sell any tree or trees, give their respective numbers in your book a dash with your pen, by which means you will know by your book what trees are sold and what unsold, as for *EXAMPLE*,

If the Trees marked 3, 5 *and* 6 *are Sold, dash those Numbers in your Book, as you see in the* TABLE *following.*

N°.	Length.	¼ of the Girth.	Content.
	Feet.	Inches.	Feet.
1	20,25	6	5,06
2	18	4,08	2,08
—3	20	7	6,8
4	32	8	14,2
—5	15	6,5	4,4
—6	30	18	67,5
—7	30	18	67,5
8	18	24	72
9	20	21½	64,2

Remains of Timber unsold

N°.	Leng	¼ G.	Con.
	Feet	Inch	Feet
—7	22,5	20	62,

Also

Alſo, if you ſell a part of any tree daſh its number as if the whole tree had been ſold, and ſet down the ſame number again both on the tree (if the marked part of the tree be ſold) and in another part of your book with its reſpective content as if it were a new tree; Thus, if you ſell 5 Foot of timber from the tree mark'd 7, whoſe length is 30 feet ¼ of the girth 1,8 inch, and content 67½ Feet. Now becauſe the whole content is 62½ there muſt remain 62½, therefore make a daſh at 7 in your book, and ſet down 7 in your new book under N°, (as before,) you may alſo for proof meaſure the part left, and ſet down their reſpective names &c.

And thus you may keep an account of timber or any ſuch thing, and if you deal largely and keep all your accounts in one book, it will be beſt only to take up about ⅓ part off from the beginning of every page, with an account of timber &c. and then the remaining part of the page may be taken up with remains of timber unſold, the names of thoſe that buy, how much, what number, what price, the time of payment, &c. And for your ready finding any perſon's name; Let your book be numbered in the pages with 1, 2, 3, &c. and have an Index prefix'd thereunto, with the names of the articles contain'd in your book, having the number of the pages where the articles are to be found writ againſt the article in the index, ſo that looking the index for any article you may readily find it.

Note, When you ſell timber to any perſon, it is uſual to mark the two firſt letters of his name thereon, with the firſt lettter of the places name where he lives, to prevent it being ſold twice, &c. Likewiſe, the firſt tree he buys number it 1, the ſecond 2, &c. and keep a liſt of all thoſe numbers in your book, daſhing the old ones; ſo that whether he buy whole trees or parts it will make no difference in your accounts.

SEC-

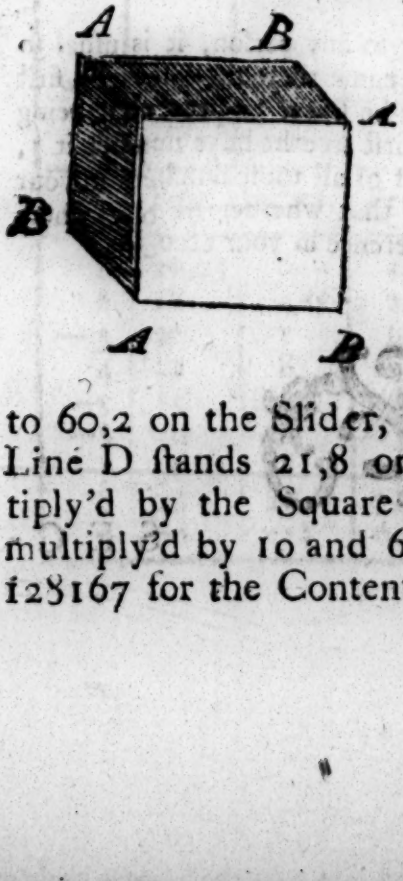
SECTION VII.

SOLID MEASURE.

To find the Solidity of any CUBE.

RULE I. CUBE the Side, that is, multiply the Side by it self, and that Product by the same Side, the last Product is the Content. By THEOREM 15.

EX. 1. *If each Side of a Cube be* 60,2 *What is its Solidity?*



60,2 } given Side AB.
60,2 }

3624,04
60,3 = AB.

218167,208 Solid Content.

By the SLIDING RULE. Set 10 on the Line D to 60,2 on the Slider, then against 6,02 on the Line D stands 21,8 on the Slider, which Multiply'd by the Square of 100 (because I was multiply'd by 10 and 60,2 divided by 10) gives 128167 for the Content.

To find the Solidity of any Parallelopipedon, Prism, or Cylinder.

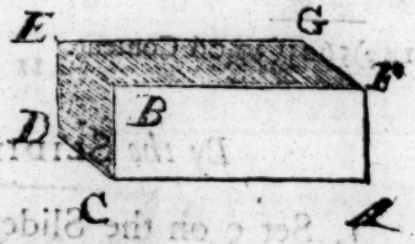
RULE 2. { Multiply the Area of the Base, by the Length of the Solid, and the Product is the Solid Content. *Theo.* 15.

EX. 2. *Let ABCDEFG be a Parallelopipedon or Square Prism, representing a Piece of Timber, Stone, &c. whose Length AC, FB or GE is* 15, *and each Side of its Square Base BCDE* 7,2 *What is its Solid Content?*

7,2 } BC, CD, DE, or EB.
7,2 }

51,84 Area of the Base.
15 Length

777,60 Content requir'd.



By the SLIDING RULE.

Always find a mean proportional between those two things, which being multiply'd together make the area of the base.

Set 1 on the line D to the length on the slider, and against the mean proportional on the line D, stands the content on the slider: But if after the three dimensions be multiply'd together division be required, then instead of 1 on the line D, you must set the square root of the divisor thereon; or you may turn one of the three dimensions into a divisor, and so work as in Ex. 15. SECT. 6. But in this Ex. you need find no mean proportional, because the base of the solid is square. Therefore set 1 on the line D to 15 on the slider, then against 7,2 on the line D stands 777½ on the slider the content solid.

Otherwise.

Set 15 on the rule to 1 on the slider, then against 1 on the rule stands .066 on the slider, which point ,066 on the slider set to 7,2 on the rule, against ,72 on the slider (for 7,2 is on the rule) stands 77¼ on the rule, so the content is 777½ as before.

EX. 3.

Ex. 3. *If a Piece of Timber or Stone be* 25 *Inches broad,* 9 *Inches deep, and* 25 *Ftet long, how many Solid Feet is contain'd therein?*

Common Way.		*Decimally.*	*F. and I.*
Inches.		Feet.	F. I.
25	Breadth.	2,0833	2 1
9	Depth.	,75	0 9
225	Area Base.	1,562475	1 6 9 P.
25	Height.	25	25 0 0
			12 6 0
144)5625(39,06	Content.	39,061875	1 0 6
			0 0 3
		By the SLIDING RULE	39 0 9

1. Set 9 on the Slider to 9 on the Line D, then againſt 25 on the Slider ſtands 15 on the Line D, a mean proportional between 9 and 25.

2. Set 12 on the Line D, (becauſe 12 is the Square Root of 144) to 15 on the Slider, then againſt 15 on the Line D ſtands 39 on the Slider the Content in Feet.

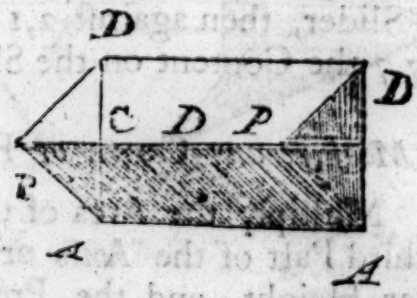
Otherwiſe.

Set 0,75 on the Rule to 1 on the Slider, then againſt 1 on the rule ſtands 1,33 on the Slider, a Diviſor for 0,75: Now ſet 1,33 on the Slider to 25 on the Rule, then againſt 2,08 on the Slider ſtands 39 on the Rule.

Ex. 4

Ex. 4. *Let AA, PB, BP, be a Triangular Prism the greateſt Side of each Baſe being AB* 13,5 *Inches, the Perpendicular CP* 12 *Inches, and the Length of the Solid being AA, BB, or PDC* 20 *Feet, What's its Solidity?*

Common Way.	*Decimally.*	*Feet and Inches.*
Inches.	Feet.	F. I. P.
13,5 = AB.	1,125	1 1 6 = AB,
6 = ½ CP.	1	1 = CP.
81,0 = Area Baſe.	8,125	1 1 6 = CP,
20 Height.	10	10 = ½ BB.
144) 1620 (11,25 Content.	11,25	11 3 0



By the SLIDING RULE.

1. Set 1 on the ſlider to 1 on the line D, then againſt 1,125 on the line D ſtands 1,06 on the ſlider a mean proportional between 1 and 1,125,
2. Set 1 on the line D to 10 on the ſlider, then againſt 1,06 on the line D ſtands 11¼ on the ſlider, the ſolidity in feet.

Otherwiſe,

1. A mean proportional between 6 and 13½ will be found to be 9.
2. Set 12 on the line D to 20 on the ſlider, then againſt 9 on the line D ſtands 11¼ on the ſlider, as before.

Ex.

Ex. 5. *If AD the Diameter of a Cylinder's Base be* 2,1 *and AC its Length be* 8, *What's its Solidity?*

2,1 } = AD.
2,1 }

4,41
,7854

3,463614 Area Base.
8 = AC Height,

27,708912 Content.



By the SLIDING RULE.

Set 1,128 on the Line D, (the Square Root of 1,2732 see *Ex.* 15, *Sect.* 5th, and *Ex.* 2, *Sect.* 7th) to 8 on the Slider, then against 2,1 on the Line D ſtands 27,7 the Content on the Slider.

How to Meaſure any CONE *or* PYRAMID.

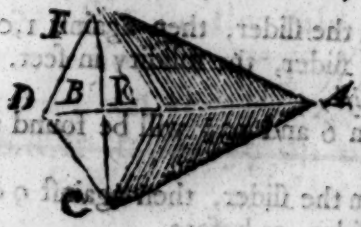
RULE 3. { Multiply the Area of the Baſe by a third Part of the Axis or Perpendicular Height, and the Product is the Solidity. *Theorem* 18.

Ex. 6. *Let AD, FC be a Square Pyramid whose Axis AB is* 15, *and each Side of its Square Baſe CE, FD* 10,1 *What's its Solidity?*

10,1 } = CE = EF = FD = DC.
10,1 }

102,01 Area of the Baſe.
5 ⅓ of AB.

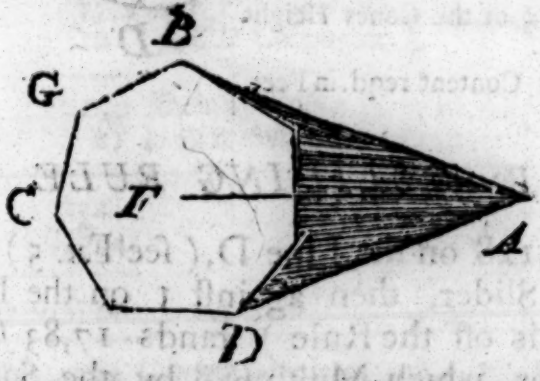
510,05 Solid Content required.



By the SLIDING RULE.

Set 1 on the Line D to 5 on the Slider, then againſt 1,01 on the Line D (for 10,1 is off the Rule) ſtands 5,12 on the Slider, ſo 512 is the Content required.

Ex. 7. *Let A B C D be a Pyramid whoſe Baſe B G C D, is a Regular Heptagon, each ſide whereof as B G, G C, &c. is* 10, *and the Axis A F of the Pyramid is* 68,1, *what is the Solid Content?*



10 } a ſide of the Baſe
10 }

100 ſq. of that ſide
3,633956 Tabular Number

363,3956 Area of the Baſe
22,7 a third of the Axis

8249,08012

By the SLIDING RULE.

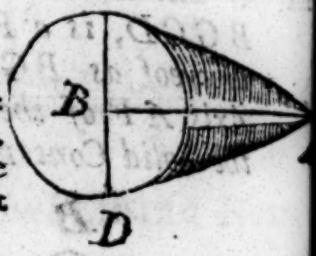
1. Set 0,275182 (ſee *Rule* 10 SECT. 5.) on the Slider to 10 on the Rule then againſt 10 on the Slider, ſtands 363,4, *fere*, on the Rule, the Area of the Baſe.

2. Set 1 on the Slider to 363,4 on the Rule, then againſt 22,7 on the Slider, ſtands 8250 *fere* on the Rule, the Solid Content required.

OTHERWISE.

Set the Square Root of 0,275182 on the Line D, to 22,7 on the Slider, then againſt 10 on the Line D, ſtands the Content on the Slider.

Ex. 8. *Let ACBD be a Cone, whose Axis A is 68,1 Feet, and the Diameter CD of i Base 10 Feet, how many Solid Feet is that Cone?*



Feet.		
10 }	Diameter of the Base	
10 }		
100	its square	
0,7854	Ar. of a Circle whose [Per. is 1.	
78,54	Ar. of the Cones Base	
2,27	$\frac{1}{3}$ of the Cones Height	
178,2858	Content reqd. in Feet.	

By the SLIDING RULE.

Set 1,128 on the Line D, (see Ex. 5) to 2,2 on the Slider, then againſt 1 on the Line D (for 10 is off the Rule) ſtands 17,83 *fere*, o the Slider, which Multiply'd by the Square o 10 (becauſe 10 was divided by 10) gives 178, Feet for the Solid Content.

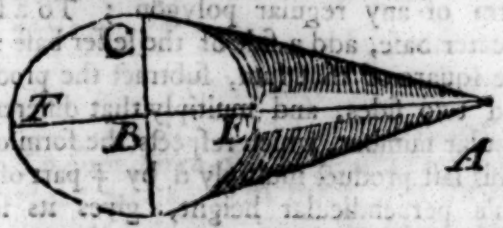
OTHERWISE.

1. Set 1,2732 on the Slider, to 10 on th Rule, then againſt 10 on the Slider, ſtands 78,5 on the Rule, the Baſe's Area.

2. Set 3 on the Slider, to 78,54 on the Ru then againſt 68,1 on the Slider, (for 6,81 is o the Rule) ſtands 178,3 *fere* on the Rule, whi Multiply'd by 10 (becauſe 68,1 was divided b 10) gives 178,3 as before.

Ex.

Ex. 9. *Let A C F D, be an Elliptical Cone, viz. a Cone whose Base is an Ellipsis, whose Longer Diameter E F is* 15,2, *Shorter Diameter C D* 10, *and A B, the Axis of the Cone* 22, *what's its Solidity?*



15,2 = *E F* Transverse } Diameter.
10 = *C D* Congugate }

152 their Product.
22 = A B the Axis.

3344
,7854

3)2626,3776

875,4592 Content required.

Or thus,

,7854
172 Prod. of the two Diameters.

119,3808 Area of the Base.
22 the Cone's Axis.

3)2626,3776

875,4592 Content required.

By the SLIDING RULE.

ct 1,128 on the Line D, to $\frac{1}{2}$ on the Slider, then against (a mean Proportional between 152 and 10,) on the Line stands 875,4 on the Slider, the Solidity required.

To

To find the Solid Content of a Fruſtum, of any Pyramid or Cone.

Rule 4. For the fruſtum of a pyramid, if its baſes be in form of any regular polygon; To a ſide of the greater baſe, add a ſide of the leſſer baſe: and from the ſquare of that ſum, ſubtract the product of the ſaid two ſides, and multiply that difference by the tabular number, which reſpects the form of its baſe; This laſt product multiply'd by $\frac{1}{3}$ part of the fruſtum's perpendicular height, gives its ſolidity by THEOREM 20.

Otherwiſe.

Rule 5. To the product of two ſides of the two baſes, add $\frac{1}{3}$ part of the ſquare of their difference, multiply that ſum by the tabular number as above, this product multiplied by the fruſtum's height, gives the ſolid content, by THEOREM 22.

Otherwiſe.

Rule 6. Multiply the difference between two ſides, of the two baſes by ,52, that product added to a ſide of the leſſer baſe, reduces the fruſtum to a priſm nearly which ſide being ſquar'd, multiply'd by the tabular number, and then by the fruſtum's height, gives its ſolidity nearly, by what follows by *Theorem* 24.

☞ Theſe three rules hold true in conical fruſtums, only the diameters of the baſes of the cone's fruſtum, inſtead of the ſides in the pyramid's fruſtum, and multiply by ,7854 inſtead of the tabular number. But if the baſes of the fruſtums be not regular polygons, as rectangle, ellipſis, *&c.* the two following rules are general.

Rule

Rule 7. Multiply the area of the greater base, by the area of the lesser base; and to the square root of this product, add the area of the said two bases, this sum multiply'd by $\frac{1}{3}$ part of the frustum's height gives its solidity, let its bases be of any form whatever, provided they be parallel and alike, by THEOREM 21.

Otherwise.

Rule 8. To four times the area, taken in the middle of the solid parallel to its bases, add the area of each base, that sum multiply'd by $\frac{1}{6}$ part of the height of the frustum gives its solidity, by THEOREM 58.

☞ This Rule holds not only in Frustum's of Cone's and Pyramid's, but also in Cylindroid's and Prismoid's; And if any such Figure offer itself to be Measured, the Dimensions may be as easily taken in the middle, as at the ends, and so the middle Area found; But in Frustum's of Cones and Pyramid's the Dimensions in the middle, may be had without Mensuration, for if to any part of the greater Base, you add the like part of the less Base, half that Sum, will be a Simular part in the middle between the two Bases, by THEOREM 1st.

This last Rule being general and by the Sli-ing Rule easier than any of the rest, I shall herefore in the seven following Examples, ap-y it only to the Sliding Rule.

EX. 10.

Ex. 10. *Let A B A B, a b a b be the Fruſtum of a Squart Pyramid, each Side of the Greater Baſe being A B* = 13, *and each Side a b of the Leſſer Square Baſe a b a b* = 8, *the Perpendicular Height or Length C D of the Fruſtum* = 24: *What is its Solid Content?*

By RULE 4.

13 = A B
8 = *a b*
21 } Sum of
21 } the Sides
441 ſq. of that
104 [Sum.
337
8 = ⅓ of C D.
2696 Solidity.

By RULE 5.

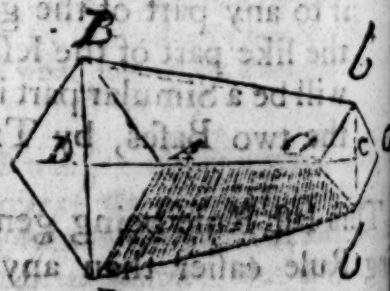
13
8
104 Prod.
5 } Diff.
5 }
25 ſq. Diff.
8 ⅔ = ⅓ ſ. dif.
104 Pr. add
112 ⅔
24 = C D.
2696 Solidity.

By RULE 6.

,52 Multip. Rule 6.
5 Diff. between A B [and *a b*.
2,60 Prod.
8 *a b* add.
10,6 } Side of a Priſm.
10,6 }
112,36 its Square.
24 = C D.
2696,64 Solid Content.

By RULE 8.

2) 21 = ſum of 2 ſides.
10,5 } = 2 Side
10,5 } in the mid
110,25 ſq. mid. ſid.
4
441,00 ſq. 4 tim.
add 169, = ſq. A B.
64, = ſq. *a b*.
674,
4 ⅙ C D.
2696 Content.



By the SLIDING RULE.

In Fruſtums of Square Pyramids, always ſet 1 on the Line D, to a ſixth Part of the Length on the Slider, then againſt twice the Middle Side and againſt a Side of each Baſe on the Line D, ſtands 3 Numbers on the Slider, which Added together gives the Solidity. So in this Example, Set 1 on the Line D, to 4 ($\frac{1}{6}$ of 24) on the Slider, then againſt 21, 13 and 8 on the Line D, ſtands three ſuch Numbers on the Slider, as being Added, makes 2696 for the Solidity.

NOTE. That twice the Middle Side, is always equal to the the Sum of two Sides of the Baſes, otherwiſe having found 10,6 the mean Side, ſet 1 on the Line D, to 24 on the Slider, then againſt 10,6 on the Line D, (for 10,6 is off the Rule) ſtands 26,966 on the Slider, ſo 2696,6 is the Solidity: but this you ſee is not exact, for the true Content is 2696, the reaſon is becauſe 0,52 is not a true Multiplier, nor can there be any true Multiplier fix'd for this purpoſe, as appears from what follows THEO. 24: Yet the ſaid Multiplier can never differ ,08 from truth, and therefore if you chuſe to work by it, it may ſerve with the Word nearly; The common way to work theſe Examples, is after the Methods of Fourth and Fifth Rules, which being none of the Eaſieſt, I ſhall decline them.

☞ The foregoing Rules for Fruſtums, make mention of a Particular Number, but Remember, the Tabular Number for the Square being General, makes no alteration in this Example, and ſo is not uſed.

Ex. 11.

Ex. 11. *Let A B a b be the Fruſtum of a Trigonial Pyra-mid, whoſe Perpendicular Length D d is* 24, *each Side of the Greater Baſe A B G be* 13, *and each Side of the Leſſer Baſe a b c be* 8, *Required the Solid Content?*

13 = A B = 13
8 *ab* = 8

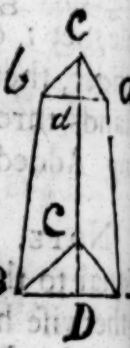
21 Sum. 104 Prod.
21

441 Sqr. 5 } Difference.
104 Prod. 5 }

337 3) 25 Sq. Difr.
,433 Tab. No. 8,333 = ⅓ Sq. dif.
104 Prod. add

145,921 112,333
8 = ⅓ D *d*. ,433 Tab. No.

1167,368 Solidity. 458,640189
By RULE 4. 24 = D *d*.



By RULE 5. 1167,364536 Solidity,

10,5 } a Side in
10,5 } the mid.

110,25
4

441,00 ſ. 4 tim.
add 169, — ſq. A B.
64, — ſq. *ab*.

674
,433 Tab No.

191,842
4 ⅙ D *d*.

1167,360 *By* RULE 8.

NOTE. The tabular number for trigon is ,433013, but ,44 may ſerve for any ordina uſe, this Example may alſo wrought by Rules 6 and 7.

By the SLIDING RULE.

If you divide 6 by ,433013 and t the ſquare root of that produ it will be 1,15; then always 1,15 on the Line D, to the he 24 on the Slider, then a ſide each end 13 and 3, and twice a in the middle 21 on the Line ſtands 3 ſuch numbers on the Slid as being added together, will the Solidity 1167,368.

Ex.

EXAMPLE 12.

If each Side of the greater Base of the Frustum of a Hexagonal Pyramid be 13, *each Side of the lesser Base* 8, *and the Frustum's height* 24, *What's its Solidity?*

By RULE 4th.		By RULE 6th.	
13 greater 8 lesser	Base.	,52	Multiplier.
		5.	Difference.
21		2,60	Product.
21		8	lesser Side add.
441	Square.	10,6	mean Side.
104 = 13 multiply'd by 8.		10,6	
337		112,36	Square.
2,598	Tabular Number.	2,598	Tabular Number.
875,526		291,91128	
8 = ⅓	Height.	24 =	Height.
7004,208	Solidity.	7005,86972	Solidity.

By the SLIDING RULE.

If you divide 6 by 2,598376 the Tabular number for an Hexagon, the quotient will be 2,309 whose square root is 1,518 Therefore, Set 1,518 on the line D to the height of the Frustum 24 on the slider, then against twice a side in the middle 21, the side of the greater base 13, and a side of the lesser base 1, on the line D, stand three such numbers on the slider as being added together make the Solidity 7004,208.

OTHERWISE,

Without the square root of 2,309 and the line D take the sum of the squares of the three sides 8, 10, and 21, whose squares are 64, 169, and 441 which added together make 674. Then set 2,309 on the slider to 674 on the rule; then against 24 on the slider stands 7004.208 the content on the rule.

Ex. 13. *Let there be a Frustum of a rectangular Pyramid, whose Perpendicular Length is* 15, *the Breadth is* 1, *Depth* ,7 *at the lesser End, and* 2 *Broad,* 1,4 *Deep at the greater End, What is its Solidity?*

By RULE 7*th*.

1 Breadth, ,7 Depth } lesser Base

1,4 Depth, 2 Breadth } greater Base.

,7 Area at less End.

2,8

1,96 (1,4 Square Root.
1

2,8 Area at greater End.
,7 Area less Base.
1,4 Square Root.

4,9 Sum.
5 = $\frac{1}{3}$ of the Length.

24) ,96

24,5 Solidity.

By RULE 8*th*.

Add { 2,8 ,7 6,3 } = Area { greater lesser } Base.
6,3 = 4 times the Area in the Middle.

Sum 9,8
2,5 = $\frac{1}{8}$ of the Length.

24,50 Solidity.

Note, Four times the area in the middle, is had by multiplying 2,1 the sum of the two depths by 3 the sum of the two breadths.

By the Sliding Rule

Set 1 on the slider to 9,8 on the rule, then against 2,5 on the slider stands 24,5 on the rule, the solid content.

This Example is the same with *Ex.* 17. *Sect.* 6th, where the content is but 23,625, and here the true content is 24,5: By which it appears that in measuring square tapering timber &c. by *Ex.* 17. *Sect.* 6th, the breadth and depth should be taken somewhat nearer to the greater base than the middle.

Ex. 14.

EX. 14. " Let *AB ab* be the Fruſtum of a Cone, whoſe " Perpendicular Length *CD* is 24, *ab* the diameter of the leſſer " Baſe 8, and *AB* the diameter of the greater Baſe 13, What " is its Solid Content? "



By RUEE 5.

13 AB
8 *ab*
104
8,33 $\frac{1}{3}$ ſq. Diff.
112,33 Sum.
,7854
88,223982
24 = CD.
2117,375568 Solidity required.

13
8
5 Diff.
3)25 Sq.
8,33 $\frac{1}{3}$ ſq. Diff.

By RULE 6.

,52
5
2,60
ab = 8
10,6 mean Diameter.
10,6
112,36 ſq.
,7854
88,247544
24 = CD.
2117,941054 Content

Divide 6 by ,7854 and the quotient is 7,6395 whoſe ſquare root is 2,764, then in all Fruſtum's of Cones, ſet 2,764 on the line D to the height of the fruſtum on the ſlider, then againſt the diameters of the greater and leſs baſes and their ſum (twice a diameter in the middle,) on the line D, ſtands 3 ſuch numbers on the ſlider, as being added gives the ſolid content. So in this Ex. Set 2,764 on the line D to 24 on the ſlider, then againſt 8,13 and 21 on the line D ſtands 201.26, 532.9, and 1383.2 which added together gives 2117.36 content.

Otherwiſe,

Set 1,128 on the line D to 24 on the ſlider, then againſt the mean diameter 10,6 on the line D, ſtands 2118 *fere* on the ſlider for the content. But when accuracy is required, never work by the mean diameter.

EX. 15

Ex. 15. *Let there be a Fruſtum of a Cone, whoſe perpendicular Height is* 18, *the Periphery at the* { *greater* / *leſſer* } *baſe* { 1,8 / ,92 } *What's the Solid Content?*

1,8 } Peripheries.	1,8	,92.	7,3984	ſq: 2,7
,92 }	1,8	,92	3,24	ſq: 1,82
			,8464	ſq: ,92
2,72 } Per. doubled.	3,24	,8464		
2.72 }			11,4848	
			,07958	
7,3984 four times ſq: Per. middle.				
			,913960384	
			3	
By RULE 8th,				
By the SLIDING RULE.			2,741881152	Solidity,

If you divide 6 by ,07958 the quotient will be 75,89 whoſe ſquare root is near 8,7; ſo in all fruſtums of cones, ſet 8,7 on the line D to the fruſtum's height on the ſlider, then againſt the peripheries of the greater and leſſer baſes, and twice the periphery in the middle on the line D ſtands 3 ſuch numbers on the ſlider as being added together gives the ſolidity, Thus, ſet 8,7 on the lineD to 18 on the ſlider, then againſt 2.72, 1.8, and .92 on the line D ſtands 3 numbers on the ſlider, which added together make 2,74 for the ſolid content.

By comparing this Ex, with Ex. 20 Sect. 6th, appears the great error in meaſuring round timber, where the falſe way makes 2,0 feet, and the true way 2,7 feet; that is, the falſe content is to the true one as 20 to 27 nearly, as in this *Example*.

To find the Solidity of any PRISMOID, *or of any* CYLINDEROID.

By RULE 8th, *all ſtreight-lin'd tapering Solids may be meaſured whoſe Baſes are parallel, whether the ſaid Baſes be equal or unequal, alike or unalike, the ſaid*

Rule

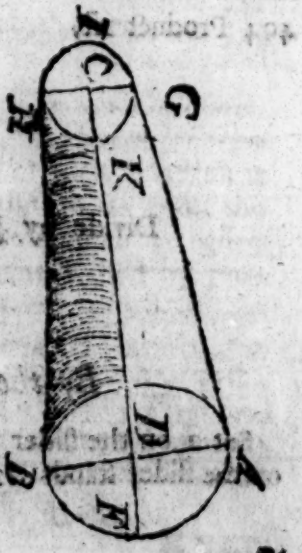
Rule *is General, and alſo for Fruſtums as appears furtheron: Yet for varieties ſake I ſhall give the following* Rule *for Priſmoids and Cylinderoids.* RULE 9.

To twice the length of the greater baſe, add the length of the leſſer baſe, multiply the ſum by the breadth of the greater baſe and reſerve the product: Alſo to twice the length of the leſſer baſe, add the length of the greater baſe; multiply that ſum by the breadth of the leſſer baſe; add this product to the firſt product, and the ſum multiply'd by ,7854 and then by $\frac{1}{3}$ of the ſolid's perpendicular height gives its ſolidity, if the baſes be circular or elliptical; but if the baſes be ſtreight lines, you need not multiply by ,7854.

What is here meant by length and breadth of the baſes, you are to underſtand ſuch things as being multiply'd together gives the area of the baſe if right-lin'd; but if curve-lin'd, the ſaid length multiply'd by the breadth, and that product multiply'd by ,7854 gives the area at either baſe.

EX. 16. "Let *AFGH* be a Cylinderoid whoſe height "*CD* is 20, its baſes being parallel and both ellipſes "but not ſimilar ones, the longeſt diameter *AB* of the "greater baſe 13, the ſhorter diameter *EF* 8, the "longer diameter *IH* of the leſſer baſe 10, and its "conjugate diameter *GH* 5,2: What is its ſolid "content?"

13 AB	20 twice IK	
2	13	
26 twice AB	33	
10 IK	5,2 GH	
36	171,6 Product	
8 EF	288 Product 1ſt.	
288 Product 1ſt.	459,6 Sum	
	,7854	
	360,96984	
	20 CD	
Solidity,	6)721939680(1203,2328	



By the Sliding Rule.

The sum of the two last mentioned products, viz. 459,6 is equal to 6 times the square of a true mean diameter. Therefore Set 1,2732 (see Ex. 15. Sect. 5th,) on the slider to 459,6 on the rule, then against $\frac{1}{6}$ of 20 on the slider stands 1203,2 on the rule, the solid content.

Example 17.

Let ABHF *be a Prismoid, whose Height* CD *is* 20, *the greater Base* AEBE *being a Square whose Side* AE *or* EB *is* 13, *the lesser Base is* FGHI *a Rectangle whose Length* FI *or* GH *is* 12, *and its Breadth* FG *or* HI 5, *What's its Solidity?*

26	twice AE	24	twice GH
12	GH	13	EB
38		37	
13	EB	5	FG
494	Product 1st,	185	Product 2d,
		494	Product 1st,
		679	Sum
		20	CD
	Divide by $\frac{1}{6}$)	13580	
		2263,33	Solidity.



By the Sliding Rule.

Set 1 on the slider to 679 on the rule, then against $\frac{1}{6}$ of 20 on the slider stands 2263,3 on the rule, the solidity.

How

How to find the Solidity of a HOOF, CONE *or* PYRAMID.

First, For the two elliptical HOOFS *of an upright* CONE.

RULE 10.

From the ſquare of the diameter of the hoof's baſe, take the ſquare root of the product of the two diameters, multiply the remainder by the diameter of the hoof's baſe, and that product multiply by the hoofs height, and the laſt product multiply'd by ,2618 and divide by the difference between the diameters gives the ſolidity of the greater hoof.

Theorem 23.

RULE 11.

Multiply the ſquare root of the product of the two diameters by the greater diameter, and from that product take the ſquare of the diameter of the hoof's baſe, multiply this remainder by the diameter of the hoof's baſe, and that product by the hoofs height, and the laſt product multiply'd by ,2618 and divided by the difference between the two diameters, gives the ſolidity of the leſſer hoof.

Theorem 24.

The hoofs of ſquare Pyramids are meaſured by *Theorem* 62.

'Theſe are the true Rules for meaſuring thoſe kinds of hoofs, But if the cone be cut parallel to its ſide, the hoofs are then called parabolic hoofs, becauſe the plane of ſuch a ſection is a parabola. And if the cone be cut parallel to its axis, the plane of that ſection is an hyperbola, ſo thoſe hoofs are called hyperbolic hoofs &c. whoſe ſolidities might be had by proceeding with the property of thoſe curves inſtead of the ellipſe's in *Theo.* 23 and 24; but ſuch things being more for theory than practice are here out of my way, ſee *Ex.* 35th. And ſince the content of a hoof may be had nearly by *Theo.* 62. but nearer by *Theo.* 23 and 24. I ſhall here expreſs thoſe two laſt *Theorems* in words, and give an Example to each, becauſe theſe Rules are much eaſier than the two foregoing.'

RULE

Rule 12.

To the greater diameter add its ſquare and half the product of the two diameters, from this ſum take the leſſer diameter, and multiply the remainder by the height and ,2618 ſo have you the ſolidity of the greater hoof.

Theorem 63.

Rule 13.

To the leſſer diameter add its ſquare and half the product of the two diameters from this ſum take the greater diameter, and multiply the remainder by ,2618 and then by the height for the ſolidity of the leſſer baſe.

Theorem 64.

Example 18.

" Required the ſolidity of the greater hoof *bAB* of the
" conical fruſtum *ab AB*, the diameter of the hoofs baſe
" being *AB* 60, its height *bF* (=*GD*) 36, and *ab* the
" diameter of the fruſtum's leſſer baſe 38,4, What is
" the ſolid content of the ſaid elliptical hoof *bAB*."



60 AB	38,4 *ab*	
3600 □AB	60 AB	
3660 ſum	2)2304,0	Product
	1152	half Product
	3660	ſum of AB□AB
	4812	ſum
	38,4	*ab* ſubtract
	4773,6	diff.
	36	*bF*
	171849,6	
	,2618 =	*π*
	44910,22528	Solidity.

Ex.

Ex. 19. " Required the ſolidity of the leſſer hoof *A b a*, " whoſe height is *DC* 36, the diameter *a b* of its baſe 38,4 " and the other diameter *AB* of the fruſtum 60 ? "

38,4 = *a b*.
30, = ½ AA.

1152,0 } = { ½ Prod.
1474,56 } = { ſq. *a b*.
38,4 } = { *a b*.

2664,96 Sum.
60, = AB ſub.

2604,96 Difference.
36 = Height.

93778,56 Prod.
,2618 Mult.

24551,237000 Solidity.

38,4 }
38,4 } = *a b*.

1474,56 = ſq. *a b*.

Note, By RULE 10.
The Solidity of the greater Hoof will be } 45992,92

By RULE 11,
The Solidity of the leſſer Hoof will be } 23548375

Sum 69541,295

By *Rule* 12 is 44990,225
By *Rule* 13 is 24551,462

Sum 69541,462

And by *Ex.* 14, the ſolidity of the whole fruſtum is found to be = 69541 ; ſo that both theſe methods make the true content of the fruſtum, but to have the true contents of theſe hoofs, work by the foreſaid 10 and 11 *Rules*,

By the SLIDING RULE.

Set $_{26}1\frac{1}{8}$ on the Slider, to the abovementioned difference 3,6 in *Ex.* 18, but 2604,96 in *Ex.* 19 on the Rule, then inſt 36 on the Slider, ſtands the content on the Rule.

Note, If 60 were a ſide of the baſe of a ſquare pyramid's f, 36 the hoof's height, and 38,4 the ſide of the leſſer of the fruſtum, the ſquare of 60 is 3600 which added 152 (half of the product of 38,4 by 60) gives 4752, h multiply'd by ⅓ of 36, gives 28512 for the ſolidity of hoof, by *Theorem* 62.

To find the Solid Content of any of the five regular Bodies.

The tetraedron being a triangular pyramid, is already measured by *Rule* 3 ; and so are the other 4 bodies, (except the cube which is measured by RULE 1,) for it is but finding the solidity of one pyramid, and then multiply that solidity by the number of pyramids which compose the body, so have you its solid content ; from whence we have this following Table.

If the Side of a	be 1,	its Solidity is
Tetraedron		0,11785
Octaedron		0,47140
Hexaedron		1,00000
Icosaedron		2,18169
Dodecaedron		7,66312

By this Table and Prob. 162, if the side of any of these bodies be given, the solidity thereof may be had by this

RULE 14. Multiply the cube of the given side and the tabular number one by the other, so have you the solidity of the body required.

EXAMPLE 20 *and* 21.

If the side of a TETRAEDRON *be* 20, *and the side of a* DODECAEDRON *be* 20, *what is the Solid Content of each ?*

For the *Tetraedron*.		For the *Dodecaedron*,	
,11785	Tab. N°	7,66312	Tab. N°
,8000	Cube of 20 given [side.	8000	C. given side
9428,0000	Solidity req.	613049,6000	Solidity,

By the SLIDING RULE.

Turn the Multiplier ,11785 (for the tetraedron) into a diviſor, and take the ſquare root of that diviſor ; then ſet this ſquare root on the Line D, to the given ſide on the Slider, and againſt the given ſide on the Line D, ſtands the content on the Slider ; this do for any of the other bodies, ſee the Cube whoſe Diviſor is 1.

To find the Solid Content of any Globe or Sphere.

RULE 15.

Multiply the cube of the Axis by 0,5236 (*viz.* $\frac{2}{3}$ of ,7854) and the product is the ſolid content required. By *Theorems* 38 and 39.

Otherwiſe.

RULE 16.

As 21 is to 11, ſo is the cube of the axis to the ſolid content required, for as 21 is to 11, ſo is 1 to 0,5236 nearly by Problem 172.

Otherwiſe.

RULE 17.

Multiply the circumference and ſquare of the axis one by the other and divide that product by 6, gives the ſolidity, by *Theorem* 56.

Ex. 26. *If the Axis* A B *of a Globe be* 20, *what is its Solidity ?*

By *Rule* 18. *Sect.* 5. If the axis or diameter be 20, the circumphery will be 62,832.

By RULE 15.

20 } = A B.
20 }

400 } = { ſq. A B.
20 } = { A B.

88,8000 Solidity.

By RULE 17.

62,832
400 = ſq. Axis.

)251328,000

41888, Solidity.



By

By the SLIDING RULE.

Set 1,382 (*viz.* the ſq. root of $\frac{1}{1,1,3,6}$) on the Line D, to the axis 20 on the Slider, then againſt the axis 20 on the Line D. ſtands 41888, on the Slider, the content, regard had to *Rule* 4. *Sect.* 1.

Note, If the circumference of a globe be 1, its ſolidity will be found = ,016887, from whence we have the Rule following for the ſolidity, when the periphery is given.

Rule 18. { Multiply the cube of the circumference of the greateſt circle, of any globe or ſphere by ,016887, and the product is its ſolidity.

To find the Solidity of any Seg. of a Globe or Sphere.

RULE 19.

From three times the axis of the whole globe, ſubtract twice the height of the ſegment, multiply the remainder by the ſquare of the ſegment's height, and that product multiply'd by 0,5236 gives the ſolidity of the ſegment, by *Theorem* 40.

This is when the height of the ſegment, and axis of the whole Globe is given, and ſince the ſemidiameter of the ſegment's baſe is a mean proportional between its height, and the remaining part of the globe's axis ; (by Prob. 36,) if the height of the ſegment and its diameter be given, its ſolidity may be had thus,

RULE 20.

To three times the ſquare of the ſemidiameter of the ſegment's baſe, add the ſquare of its height, multiply that ſum by the height of the ſegment, and that product multiply'd by 0,5236 gives the ſolidity.

EXAMPLE 23.

Let AHYI be the Segment of a Globe, whoſe Height Ay is 7, *and HI the Diameter of its Baſe* 16, *what is its Solid Content ?*

OPERATION

OPERATION.

By RULE 20.	*By* RULE 19.
8 } = H Y, or I Y.	20 = AB Axis Prob. 36.
8 }	3
64 = ſq. ſemidiameter.	60
3	8 = A Y Height 2,
192 ſq. = 3 times.	52
16 = ſq. A Y Height.	16 ſq. Y A Height.
208	832
4 = A Y Height.	,5236
832	435,6352 Solidity.
,5236	
35,6352 Solidity.	

By the SLIDING RULE.

Set 1,383 on the Line D, (ſee *Ex.* 22.) to the ſegment's eight on the Slider, then againſt the ſemidiameter, and ſegent's height on the Line D, ſtands two ſuch numbers, that to three times the firſt you add the ſecond, that ſum will ew the ſolidity of the ſegment. So in this Example, ſet ,383 on the Line D, to 4 on the Slider, then againſt 4 (for is off the Rule) on the Line D, ſtands 33,51 on the Slider, hich multiply'd by 4 (*viz.* the ſquare of 2, becauſe 8 was ivided by 2) gives 134,04 and multiply'd by 3, gives 02,12 for the firſt number ; alſo, againſt the ſegment's eight 4, ſtands 33,51 on the Slider, for the ſecond number, hich added to the firſt number 402,12, gives 435,63 for the lidity of the ſegment as before, nearly,

How to Meaſure the Fruſtum or Middle Zone of a Globe.

By RULE 21.

To twice the ſquare of the axis, add the ſquare of the diaeter of the other end ; multiply that ſum by the height or ickneſs of the fruſtum, and that product multiply'd by ,2618 ves the ſolidity, by *Theorem* 45.

Ex. 24.

By the Sliding Rule.

Set 1,382 (*viz.* the sq. root of $\frac{1}{1338}$) on the Line D, to the axis 20 on the Slider, then against the axis 20 on the Line D. stands 41888, on the Slider, the content, regard had to *Rule* 4. *Sect.* 1.

Note, If the circumference of a globe be 1, its solidity will be found = ,016887, from whence we have the Rule following for the solidity, when the periphery is given.

Rule 18. Multiply the cube of the circumference of the greatest circle, of any globe or sphere by ,016887, and the product is its solidity.

To find the Solidity of any Seg. of a Globe or Sphere.

RULE 19.

From three times the axis of the whole globe, subtract twice the height of the segment, multiply the remainder by the square of the segment's height, and that product multiply'd by 0,5236 gives the solidity of the segment, by *Theorem* 40.

This is when the height of the segment, and axis of the whole Globe is given, and since the semidiameter of the segment's base is a mean proportional between its height, and the remaining part of the globe's axis ; (by Prob. 36,) if the height of the segment and its diameter be given, its solidity may be had thus,

RULE 20.

To three times the square of the semidiameter of the segment's base, add the square of its height, multiply that sum by the height of the segment, and that product multiply'd by 0,5236 gives the solidity.

EXAMPLE 23.

Let AHYI be the Segment of a Globe, whose Height Ay is 7, *and HI the Diameter of its Base* 16, *what is its Solid Content ?*

Operation

OPERATION.

By RULE 20.		*By* RULE 19.	
8 } 8 }	= H Y, or I Y.	20	= AB Axis Prob. 36.
3		3	
64	= ſq. ſemidiameter.	60	
3		8	= A Y Height 2,
192	ſq. = 3 times.	52	
16	= ſq. A Y Height.	16	ſq. Y A Height.
208		832	
4	= A Y Height.	,5236	
832		435,6352	Solidity.
,5236			
435,6352	Solidity.		

By the SLIDING RULE.

Set 1,383 on the Line D, (ſee *Ex.* 22.) to the ſegment's height on the Slider, then againſt the ſemidiameter, and ſegment's height on the Line D, ſtands two ſuch numbers, that if to three times the firſt you add the ſecond, that ſum will ſhew the ſolidity of the ſegment. So in this Example, ſet 1,383 on the Line D, to 4 on the Slider, then againſt 4 (for 8 is off the Rule) on the Line D, ſtands 33,51 on the Slider, which multiply'd by 4 (*viz.* the ſquare of 2, becauſe 8 was divided by 2) gives 134,04 and multiply'd by 3, gives 402,12 for the firſt number ; alſo, againſt the ſegment's height 4, ſtands 33,51 on the Slider, for the ſecond number, which added to the firſt number 402,12, gives 435,63 for the ſolidity of the ſegment as before, nearly,

How to Meaſure the Fruſtum or Middle Zone of a Globe.

By RULE 21.

To twice the ſquare of the axis, add the ſquare of the diameter of the other end ; multiply that ſum by the height or thickneſs of the fruſtum, and that product multiply'd by ,2618 gives the ſolidity, by *Theorem* 45.

Ex. 24.

EXAMPLE 24.

Let it be required to find the Solidity of the Half Zone M H I R, *whose Height or Thickness* Y C *is* 6, *the Diameter* M R 20, *and the Diameter* H I 16 ?

20 } = M R Axis.
20 }

400
2

800 = 2 sq. M R.
256 = sq. H I.

1056
2618

2764608,
6 = Y C.

1658,7648 = the Solidity.

16 } = H I.
16 }

256 = sq. H I.

By the SLIDING RULE.

Set 1,954 on the Line D, to the height of the frustum 6 on the Slider. then against the axis 20, and the diameter 16 on the Line D, stands 2 such numbers on the Slider, as if 20 twice the first, you add the second, the sum will be 1658,7548 as before.

Note, If K L be = H I, and Z C = Y C, then the above number 276,4608 multiply'd by 12 or, twice 1658,7648, gives the solidity of the whole Zone H K L I.

To find the Solidity of any Spheroid, either Right or Oblate.

RULE 22.

Multiply the square of the axis of the greater circle, (viz. the sq. of the { transverse / conjugate } axis { a { oblate / right } spheroid } by that axis upon which the ellipsis revolves, when the spheroid is described, and that product multiply'd by ,5236, gives the solidity, by *Theorem* 38.

Ex. 25

x. 25. " Let *G H I K* be a right ſpheroid, the diameter " of whoſe greater circle is *G H* 30, and the length *I K* " 40, what is its ſolid content ? "



30 } = G H. 5336
30 } 36000

900 = G H. 18849,6000
40 = I K. for the Solidity.

6000

By the SLIDING RULE.

Set 1,383 on the Line D, to the ranſverſe 40 on the Slider, then againſt the conjugate axis 3 (for 30 off the Line) ſtands 188,5 on the lider, which by removeing the decimal point two places towards the right hand, (becauſe 30 was divided by 10) gives 8850 for the ſolidity nearly, as before.

To Meaſure the Segment of a right SPHEROID.

RULE 23.

With the tranſverſe axis and height of the ſegment, find the olidity exactly (by *Rule* 19.) and this ſolidity multiply'd by e ſquares of the conjugate, and divided by the ſquare of the ranſverſe, gives the ſolidity of the ſegment, by *Theorem* 37.

x. 26. " Let *A B G* be the ſegment of a right ſpheroid, " whoſe height *I y* is 4, the tranſverſe axis *I K* 20, and " the conjugate axis *G H* 15, what is its ſolidity ? "

20 = I K. 435,6
3 225 = ſq. con G I 15.

60 Product. 4100) 980 | 10,0
16 = twice Y y,

245 = the Solidity ſought.

32,0
2,5 Inver Factor

By the SLIDING RULE.

35,6 as by *Ex.* 23. Set the tranſverſe 20 on the Line D, to 435,6 (the like ſegment of the circumſcribing ſphere for *per* Sliding Rule *Ex.* 23.) on the Slider, en againſt the conjugate 14 on the Line D, ſtands 245 on e Slider, the Solidity.

To

To find the Solidity of the Fruſtum of a right SPHEROID.

RULE.

To twice the ſquare of the conjugate or diameter of the greateſt circle, add the ſquare of the diameter at the other end of the fruſtum, multiply that ſum by the fruſtum's height, and that product multiply'd by ,2618 gives the ſolidity of the fruſtum or middle zone. *Theorem* 45.

EXAMPLE 27.

" Let *AB ba* be the middle zone of a right ſpheroid, " whoſe diameters *a b* and *AB* at the ends are each = 24, the " conjugate or greateſt diameter *HG* = 30, and *y z* the length " = 24, What is its ſolidity? "

By the Sliding Rule.

This Rule both by the pen and ſliding rule is the very ſame with Rule 2d, therefore, Set 1,954 on the line D to the fruſtum's or zone's length 24 on the ſlider, then againſt 3 and 2,4 (for 30 and 24 are off the rule) the two diameters on the line D ſtand two ſuch numbers 56,548 and 36,196 that if to twice the firſt 113,096 you add the ſecond 36,196 the ſum 149,292 multiply'd by the ſquare of 10, &c. gives the ſolidity nearly as before (*viz.*) 14929,2.

30 } = GH.	24 } AB or *ab*
30 }	24 }
900 ſq. GH.	576 ſq. AB.
2	
1800 twice ſquare GH.	
576	
2376	
24 *zy* length	
57024,0000	
8162, inverted ,2618.	
114048000	
34214400	
570240	
456142	
14928,8832 Solidity required.	

Ex. 2

EX. 28. Let *GHIK* (foregoing fig.) be an oblate ſpheroid or a turnip-like ſolid, whoſe tranſverſe axis *IK* or diameter of the greateſt circle is 40, and the conjugate *GH* or height 30, What is the ſolid content?

40 } IK.
40 }
—
1600
30 GH
—
48000
,5236
—
25132,8 Solidity.

By the SLIDING RULE.

Set 1,383 on the line D to the ſpheroid's thickneſs 30 on the ſlider, then againſt 4 on the line D (for 40 is off the rule) ſtands 251,33 on the ſlider which by taking away two decimals gives 25133 for the ſolidity nearly as before.

Note, The ſegments and fruſtums of this ſpheroid are meaſured exactly like the laſt, as appears by *Theo.* 37: Only this, the tranſverſe is the diameter of the greateſt circle, and e conjugate the height or thickneſs of the ſolid, and ſo the njugate in this muſt be uſed as tranſverſe in the laſt *&c.*

low to find the Solidity of a Parabolic CONOID.

le 25, { Multiply the area of the greateſt circle by half the axis, and the product is the ſolid content. By *Theorem* 36.

X. 29. *Let ABC be a parabolic Conoid, whoſe axis AD is* 60, *and BC the diameter of its baſe* 40, *Requir'd the Solidity?*



40 } =BC
40 }
—
1600
30 half AD
—
48000
,7854
—
37699,2 Solidity.

By the SLIDING RULE.

Set 1,59577 (viz. the ſquare root of the quotient, 1 divided by half ,7854) on the line D to the conoid's height 60 on the ſlider, then againſt the diameter of its baſe 4 (for 40 is off the rule) on the line D, ſtands 376,99 on the ſlider, ſo 37699 is the ſolidity nearly as before.

To find the Solidity of a Parabolic CONOID'S FRUSTUM.

Rule 26. Multiply the ſum of the ſquares of the two baſes diameters, the height of the fruſtum and 0,3927 into one another and the laſt product is the ſolidity, *Theorem* 41.

Ex 30. *Let A B F E be the fruſtum whoſe height G D* 38,4 *the greater diameter B C* 40, *and the leſſer diameter E F* 24, *Required its Solidity?*

	40	BC	24	EF
Add	1600	ſq. BC.	576	ſquare EF
	576	ſq. EF.		
	2176			
	38,4	GH.		
	13558,4			
	7293,	= ,3927 inverted, ½ of ,7854		
	32813,3	Solidity.		

By the SLIDING RULE.

Set 1,59577 (*viz.* the ſquare root $\frac{1}{1\,9\,2\,7}$) on the line D to the fruſtum's height 38,4 on the ſlider, then againſt the diameters 4 and 2,4 on the line D (for 40 and 24 are off the rule) ſtands 241,27 and 86,86 on the ſlider, which added together gives 328,13, ſo 32813 is the ſolidity.

To

To measure a Parabolic SPINDLE.

RULE 27.

Every parabolic ſpindle being $\frac{8}{15}$ of its circumſcribing cylinder; multiply ,41888 (*viz.* $\frac{8}{15}$ of ,7854) the length of the ſpindle and the ſquare of the diameter of its greateſt circle one into another, and the laſt product is the ſolidity.

Theorem 36.

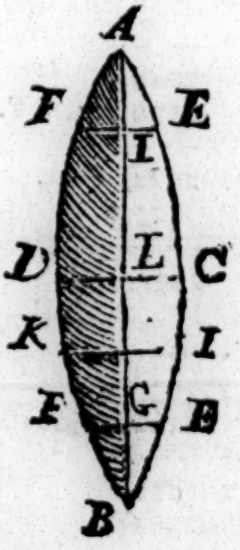
Ex. 31. "Let *ABCD* be a parabolic ſpindle, whoſe length "*AB* 60, and *CD* the diameter of the greateſt circle "40, What is the ſolidity?"

40 } CD
40 }

1600 ſq. CD
60 AB

96000
,41888

40212,48000 Solidity.



By the SLDING RULE.

Set 1,545 (ſquare root $\frac{1}{41888}$) to the length 60 on the ſlider, then againſt 4 the diameter (for 40 is off the rule) on the line D, ſtands 402,1248 on the ſlider, which multiply'd by the ſquare of 10 (becauſe 40 was divided by 10) gives 40212,48 the Solidity as before.

To meaſure the Middle ZONE *of a parabolic Spindle.*

RULE 28.

To twice the ſquare of the diameter of the greateſt circle add the ſquare of the diameter at the end, and from that ſum take ,4 of the ſquare of their difference, and multiply the remainder by the length, and that product multiply'd by ,2618 gives the ſolidity. *Theorem* 42.

Ex. 32.

EXAMPLE 32.

Let EFEF *be the Middle Zone of a parabolic Spindle whose Length* GH *is* 40 *Inches, and the Diameter of the greateſt Circle* CD *is* 32 *Inches, and the Diameter at the End* EF *is* 24, *What is the Solidity?*

32 } CD	24 } EF	32 CD	
32 }	24 }	24 EF	
1024 ſq. CD.	576 ſq. EF	8 } Diff.	
2		8 }	
2048		64 ſq. Diff.	
576 ſq. EF.		,4	
2624 Sum.		25,6 $\frac{4}{10}$ ſq. Diff.	
25,6 $\frac{4}{10}$ ſq. diff. Sub.			
2598,4			
40 GH.			
103936,0			
8162, =,2618 inverted.			
207872			
62361			
1039			
831			
27210,4 Solidity.			

By the Sliding-Rule.

Set 1,954 on D to the zone's length on the ſlider, then on the ſlider againſt the greater and leſſer diameters, and alſo their difference on D, ſtands 3 ſuch numbers, that if to twice the firſt, you add the ſecond, and from that ſum take $\frac{4}{10}$ of the third, there will remain the ſolidity. So 1,954 ſet on D to 40 on the ſlider, then on the ſlider againſt 32, 24 and 8 on D ſtands 10723,3, 6041,9 and 670,2 reſpectively, ſo twice the firſt is = 21446,6 added to the ſecond 6031,9 gives 274785 from which take ,4 of 670,2 *viz.* 268,08 and there reſts 27210,42 the ſolidity.

Theſe are the chief and principal ſolids, yet as the variety of curves is great, ſo that of ſolids muſt be greater, becauſe ſuch ſolids may be obliquely cut into fruſtums, ſegments, ſlices, &c. to enumerate each, would be an endleſs taſk, and

if it could be done would only oblige Theory, for ſince in Practice no ſolid can be meaſured unleſs its dimenſions be taken, which liberty had any ſolid whatever may be meaſured by what is done in PROBLEM 169, either exactly or very near, for it is there proved, that taking only one dimenſion more than common, you have a General and Exact RULE for meaſuring all the foregoing fruſtums, except the laſt which differs but a ſmall matter, as appears by the following EXAMPLE.

A General RULE for meaſuring all *Fruſtums* or *Half Zones*. *This is but exactly to expreſs the 8th, Rule foregoing, and therefore to meaſure any round Fruſtum,* (viz.) *any circular bas'd Fruſtum, By the* Sliding-Rule *take this*

GENERAL RULE,

Set 2,764 (the ſquare root of ,$\frac{6}{7854}$) on the line D to the height or length of the fruſtum on the ſlider, then againſt the lengths of the diameters of the fruſtum's two baſes, and twice a diameter taken in the middle between the two baſes, on the line D, ſtands 3 ſuch numbers on the ſlider as being added together gives the content of the fruſtum or half zone, belong to what curve it will, nearer than can be expected it can be had by gueſſing at the nature of the generating curve, and applying the proper Rules, &c.

Example 33.] Let it be required to find the ſolidity of the foregoing half zone *EFDC*, whoſe height *GL* is 20, the diameter of the greateſt circle *CD* 32, the diameter of the leaſt circle *EF* 24, and *IK* the diameter in the middle between the greateſt and leaſt diameters 30, found by menſuration or by *Theorem* 50.

Operation.

Add, $\left\{\begin{array}{l} 1024 \square \text{ CD } 32. \\ 3603 \square \text{ IK } 60. \\ 576 \square \text{ EF, } 23 \end{array}\right.$

5200 Sum,
,1309 $\frac{1}{8}$ of ,7854.

6806,800
20 G L.

13613,6 $\frac{1}{2}$ Zone CDFE.
2

27227,2 Zone EFE.

By the *Sliding-Rule*.

Set 2,764 on the line D to 20 on the ſlider then againſt 32, 60 (twice 30) and 24 on the line D ſtands 3 ſuch numbers on the ſlider as being added together gives 13613 for the ſolidity.

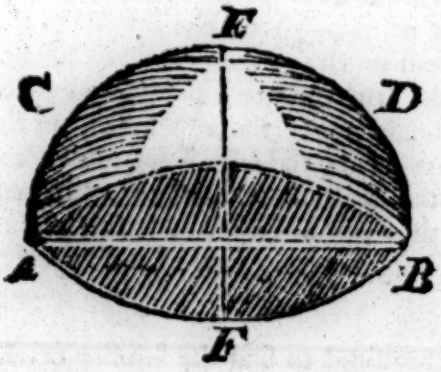
So the difference between this and EX. 32, is very ſmall.

By

By the same Rule you may also find the Solidity of any SEGMENT *neary, as for*

EXAMLPE 34.

" Let *AEB* be a segment of some solid, whose height " *EF* is 4, the diameter *AB* of the base 12, and a dia- " meter *CD* taken in the middle 9, What's the solidity? "



Add { 144 } □ AB.
{ 324 } twice CD □'d
{ 0 } dia. at the end [E

468,0 Sum.
9031, =,1309.

4680
1404
000
42

61,26
4

245,04 Solidity

By the SLIDING RULE.

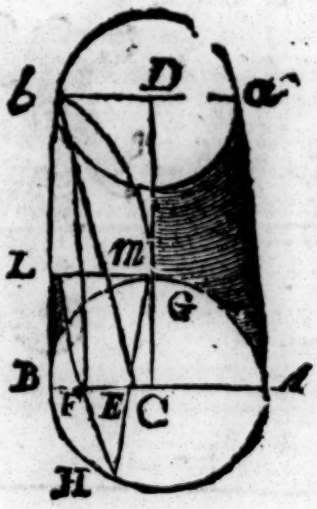
Set 2,764 on the line D to 4 on the slider, then against it on the line D stands 18,25 and against twice 9 (18) stands 226,19 on the slider, whose sum is 245.04 the solidity required.

This Segment is the same with that in Ex. 26, and the solidity by this method is exactly the same with that found there, by the particular Rule.

Ex. 35

EXAMPLE 35.

" Let *AB ba* be the fruſtum of a cone cut by a plane *GbH* parallel to its ſide *aA*, cutting off the parabolic hoof *bEB*, whoſe height *bF* (=*CD*) is 36, the diameters *ab*, *AB*, of the fruſtum's baſes 38,4, 60 conſequently *BE* the verſed-ſine of the hoof's baſe will be 21,6, the difference between *AB* and *ab* half of which is 10,8=*mL*, the verſed-ſine in the middle and 60 added 38,4 is 98,4 half of which is 49,2 the diameter of the fruſtum in the middle, by which (ſee *Ex.* 22, *Sect.* 5th.) the area in the middle *mL*, and alſo of the baſe *EB*, are found. "



299,17 Area in the middle =*mL*.
4

1196,68 four times middle Area.
916,39 Area at the Baſe =*EB*.

2113,07 Sum.
6 $\frac{1}{6}$ of 36.

2678,42 the Solidity required.

The true ſolidity of this hoof is 12851,3436 &c, which ffers but little from that found by the above method, and you be nice in finding the area of the two ſegments (*viz.*) that in the middle, whoſe diameter is 49,2 and verſed-e 10,8: And that of the baſe whoſe diameter is 60 and rſed-ſine 21,6, the difference will be nearly equal, for I ve taken theſe two areas here in a round manner, only ſhew how ſuch things may be done.

And

And to make it appear to any one's reason, whether this general method, (which has no regard to the nature of the figure) or the common way, which guesses at the nature thereof with a particular rule, is more fit for practice, and so concludes SOLID *Measure*.



SEC. VIII.

SECTION VIII.

SURVEYING.

HERE in surveying I mean measuring plotting and dividing of ground, and if what goes before be well understood, this will be easy. For in the First Part, the chain and other needfull instruments in measuring land, taking distances *&c.* are described and apply'd to practice. Also, land being ways computed by superficial measure, you have that done ready in *Sect.* 5th, where besides the general method for easuring surfaces you have general rules, for all the usefull rms of figures particularly exemplifi'd. So that let a field e in what form it will regular or irregular, its superficial conent may be had by some of these rules in the said *Sect.* 5, hich rules I shall repeat no more but by way of reference, you may see in the following examples.

NOTE, *that*

12 Inches	make	1 Foot.
3 Feet		1 Yard.
5 ½ Yards		1 Pole, Perch, or Rod.

But in *Ireland* 7 yards make 1 pole *&c.* also 40 square les perches or rods, make 1 rood, 4 roods make 1 acre, at is, 40 poles in length and 1 in breadth make 1 rood,

4 roods in length and 1 in breadth make 1 acre *&c.* see the tables of measure at the beginning of *Sect.* 5th. After which is shewn the reduction of one measure to another, which rules are also observ'd in the following examples of reducing chains, links *&c.*

EXAMPLES in *Long Measure.*

To reduce Chains *into* Links.

Ex. 1.] *In* 15 *Chains* 25 *Links how many Feet?*

C. L.
15 25
66 Feet in 1 Chain, by (Rule 1. Sect. 5.
1006,50 Feet as required.

Ex. 3.] *In* 27 *Chains* 30 L. *how many Rods Poles* &c.

27 30
4 = Rods in 1 C.
106,26 Rods or Poles.

Ex. 2.] *In* 29 *Chains, how many Feet?*

29 Chains.
66
1614 Feet.

Ex. 4] *In* 80 *Links, how many Rods Poles* &c.

,80
4
3,20 Rods, Poles &c.

To reduce Links *into* Chains.

Ex. 5.] *In* 2580 *Links how many Chains?*

100) 2580 (25,8 Chains.

N. B. If you cut two figures from the number of links towards the right hand, they will be links, and the remainder will be chains.

Ex. 6.] *In* 2580 *Links how many Chains?*

Answer 25,80 Chains, or 25 Chains 80 Links, as above.

To reduce Feet *into* Perches.

Ex. 7.] *In* 1320 *Feet how many Perches?*

1320
2
33) 2640 (80 Poles.

Multiply by 2 to bring it into half feet, and divide by 33 because 33 half feet make 1 Pole or Perch.

Ex. 8.] *Divide* 1320 *Feet, by* 16,5 *the Feet in* 1 *Pole?*

16,5) 1320 (80 Poles as above.

To reduce Chains *into* Links.

Ex. 9.] *In* 35 *Chains, how many Links ?*

35 Chains.
100

3500 Links Anſwer.

Ex. 10.] *In* 87 *C.* 56 *L. how many Links ?*

87,56
100

8756 Links.

Of Square Meaſure.

Ex. 11.] *In one Square Acre how many Square Yards ?*

1 Acre.
10 the ſq. Chains in 1 ſq. Acre.

16 the ſq. Poles in 1 Chain.

160 the ſq. Poles in 1 ſq. Acre.
30,25 the ſq. Yards in 1 ſq. Pole.

4840,00 ſq. Yards in 1 ſq. Acre.

Ex. 12. " If you would turn ſquare chains into ſquare links " annex four cyphers. So 159 ſquare chains will be " 159,0000 ſq. links, which is the ſame as to multiply " by 10000. "

Ex. 13. " In 1671925 ſquare links, how many ſquare " chains ? "

Anſwer 167,1925 ; or, 167 ſquare chains, and 1925 ſquare links as required.

Note, This is done by cutting off 4 figures to the right hand decimals or ſquare links, and the remainder will be chains.

Ex. 14. *In* 167 *Chains, how many Acres ?*

Anſwer, 16,7 ſquare acres ; for 10 chains make 1 acre. Therefore in any number of ſquare chains, cut off one figure to the right hand, for a decimal, and the remainder will be ſquare chains.

Ex. 14.] *In* 15,99 *Acres, how many Chains, and Links ?*

10 the chains in a ſq. acre.

159,9 ſq. chains in 1599 ſq. Acres.
10000 ſq. links in 1 ſq. chain.

15990000 ſq. links in 15,99 ſq. acres.

Ex. 15.

Ex. 15.] *In* 358,75 *C. how many Acres, Roods, and Perches?*

35,875 Acres by *Ex.* 12.
4 Roods in an Acre.

3,500 Roods.
40 Poles in one Rood.

20,000 Poles or Perches.

Ex. 16.] *In* 0,99 *Acres, how many Roods, and Poles?*

0,99 Acres.
4

3,96 Rood.
40

38,40 Poles.

NOTE, That in ſeveral parts of England there are poles uſed 18 feet in length, and ſome 21 and 24 feet, call'd cuſtomary meaſure: But Gunter's Chain giving the content always in ſtatue meaſure $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet to the pole. Now if after any piece of ground is meaſured, you wou'd have its content in cuſtomary meaſure; enquire how many feet they have to the pole in that place? Then (by Theo. 3.) it will always be As the ſquare of the feet in a cuſtomary pole, is to the content of the field in acres and decimal parts of an acre ſtatute meaſure; ſo is the ſquare feet in a pole ſtatute meaſure, to the acres and decimal parts of an acre in that field cuſtomary meaſure. Or inſtead of feet in each ſort of poles, you may uſe half yards.

Ex. 17.] *In* 172 *Acres Statute Meaſure, how many Acres Cuſtomary Meaſure of* 18 *Feet, or* 6 *Yards to the Pole?*

172. given Acres Statute.
121 Square of 11 the half Yards in a Pole Statute.

144)20812(144 Acres.
76
4 Roods in an Acre.
144)304(2 Perches.
16
144)640(4 Poles.
Rem. 64

Note, The half yards in 6 yards are 12, whoſe ſquare is 144 the diviſor. Alſo, that in a cuſtomary acre as well as a ſtatute acre, there are 4 roods and in each rood 40 perches; ſo that the diſſerence is only in the length of the pole.

Anſwer A. P. P. P.
144. 2. $4\frac{64}{144}$ or $4\frac{4}{9}$ of Yard

to

to the pole, are equal to 172 acres of $5\frac{1}{2}$ yards to the pole. In like manner you may turn one kind of cuſtomary meaſure to another; or cuſtomary meaſure to ſtatute meaſure, &c.

In ſome places the inhabitants know not the length of their cuſtomary pole, but ſay, that ſo many gallons of oats will ſuffice one of their acres for ſeed. Now if this be known, the length of a pole may be gueſs'd at; for in many places, it is ſaid that 60 gallons of oats, will ſow an acre ſtatute. Therefore, it will be as 60 gallons is to 121 (the ſquare of 11; the half yards in a ſtatute pole) ſo is the number of gallons that ſow a cuſtomary acre, to the ſquare of the half yards, contain'd in a pole of that cuſtomary meaſure, whoſe ſquare root gives the length of the required pole, in half yards.

Ex. 18. If 97 Gallons of Oats be ſufficient for Sowing ſome Cuſtomary Acre, How many Yard is to the Pole in Length of that Acre?

G. $\square\frac{1}{2}$ Y. G.
If 60 — 121 — 97 ?
97
6lo)1173l7
195,6 — whoſe Square Root is = 14 nearly, the half Yards requir'd.

Ex. 19. If 120 Gallons ſow an Acre, how many Yards to the Pole in that Acre?

If 60 — 121 — 120 ?
120
6lo)145210
242 (15,5 $\frac{1}{2}$ Y.
25)142
305)1700
175

To reduce Acres into Square Links

Ex. 20. In 560 Acres, how many Links?
Anſwer 56000000 Links.

Ex. 21. In 5,23 Acres, how many Links?
Anſwer 523000 Links.

That is always multiply the given acres, by 100000, the ſquare links in one ſquare acre.

In ſurveying, it is common to ſet down the dimenſions in chains, and links; but ſince links are decimal parts of a chain, you may as well ſet down no links, but ſo many chains and parts,

parts, and ſo work by decimals, thus 50 Chains, 12 Links may be writ 50,12 Chains, and 12 Chains 50 Links, thus 12,5 Chains, alſo 08 Chains, thus 0,8 Chains, and ſo of any other; which method being uſed, you will find no difference between finding the areas of the following figures, and of ſuch like figures in *Sect.* 5. And in the following examples, where ever a rule is mentioned, you muſt ſearch the ſaid *Sect.* 5, for ſuch a rule.

To find the Area of any Square Piece of Ground.

Ex. 21. Suppoſe the Side of a Square Field 7 Chains 60 Links; how many Acres, Roods, and Perches, are contain'd therein?

7,6 } by Rule 2d.
7,6 }

57,76 ſq. Chains.

5,776 Acres.
4

3,104 Roods.
40

4,160 Perches.

NOTE, 10 ſquare chains make 1 acre; therefore, always divide the area in chains by 10; or, remove the decimal point one place nearer the left hand, &c. as in *Ex.* 15.

	A.	R.	P.	P.
Anſwer	5.	3.	4.	160.

Ex. 22. In a Rectangular piece of Ground; the Length 10 Chains 75 Links, the Breadth 2 Chains; required the Content in Acres, Roods, and Perches?

10,75 by Rule 3d.
2

2,150 Acres.
4

,600 Roods
40

24,000 Poles. Anſwer 2 A. 0 R. 24 P.

Ex. 23.

Ex, 23. Suppoſe a Field in form of a Rhomboides, whoſe Length is 2 Chains 10 Links, and breadth 90 Links, required the Area ?

2,1 by Rule 4.
,9

,189 Acres.
4

,756 Roods.
40

30,240 Poles. Anſwer 0 A. 0 R. 30 P.

Ex. 24. Suppoſe a Triangular Field; whoſe Baſe is 7 Chains 10 Links, and Perpendicular 2 Chains 30 Links, how many Acres, Roods, and Perches ?

3,55 by Rule 5.
2,3

8,165 ſq. Chains.

,8165 Acres.

3,2660 Roods.
40

A. R. P. P.
10,6400 Perches. Anſwer 0. 3. 10. 64.

Ex. 25. If a Piece of Ground, lay in Form of a Trapezia; the Diagonal 28 Chains 20 Links, the Perpendiculars 10 Chains 50 Links, and 8 Chains, what is the Area ?

10,5 by Rule 7.
8

18,5 ſum Perpend.
14,1

60,85 ſq. Chains.

26,085 Acres.
4

,340 Roods.
40

13,600 Perches.

Anſwer 26 Acres, 0 Roods, and 13 Perches.

Bur

But if any two sides of a trapezia be parallel, then half the sum of these two parallel sides, multiply'd by their distance assunder, gives the area of the trapezia, (by Theo. 11.) which comes easier, than to measure out a diagonal and two perpendiculars.

Example 26.

Suppose two parallel Sides in a Trapezia, measure 3202 Links, and 4008 Links, their Perpendicular distance 2500 Links, required the Content ?

```
 40,08
 32,02
 ------
2 ) 72,10 Sum.
 ------
 36,05 half Sum.
   25
 ------
 881,25 sq. Chains.
 ------
 88,125 Acres.
      4
 ------
  ,500 Roods,
     40
 ------
 20,000 Perches.
```

Note, Some surveyors, do no more with four sided figures, but multiply half the sum of two opposite sides, by half the sum of two opposite ends, and say that product gives the area of the figure, which is a gross error, and always gives the content too much, for Instance.

In the same Trapezia, the sides 3202 *Links, and* 4008 *Links ; the ends* 2800 *and* 3400 *Links ?*

```
  3202 }
  4008 } Sides.
  ----
  7216
  3605 = half Sum.
  3100
 ------
 360500
 10805
 -------
 1117,5500
  881,25
 -------
 236,30 sq. Chains too much.
```

```
 2800 }
 3400 } Ends.
 ----
 6200
 ----
 3100 half Sum.
```

Ex, 27

Ex. 27. *Suppose the Diameter of a Circle* 20,02 *Yards, Requir'd its Area in Acres, Roods, &c.*

20,02 } Diameter.
20,02 }

400,8004
,7853 By Rule 12.

314,78863416 Area in □ Y.

4840) 314,79 (0 Acres.
4

4840) 1259,16 (0 Roods.
40

Poles.

4840) 50366,40 (10,406.

That is, having found the area of the circle (by Rule 12. Sect. 5th.) I divide it by 4840 the ſquare yards in one acre, (*viz. I divide* 314,79 *becauſe it is nearly equal to the true area in yards*) and it goes no times. So I multiply by 4 the roods in an acre, and find it to be no roods. Which again, multiply by 40 the poles in a rood, and I find the quotient to be 10 Poles and ,406 &c. for the anſwer. Or,

If the Dimenſions be taken in Yards, you may reduce them to Chains. Thus,

Y. L. Y.
As 22 —— 100 —— 20,02 ?
100

22) 2002,00 (91 } Diameter in Links.
91 }

8281 Diameter in ſquare Links
,7854 Rule 12.

10000) 6503,8974 } Content in { Square Links.
,65038974 Square Chains.
,065038974 Acres.
4
,260155896 Roods.
40
10,406235840 Perches, as before.

Becauſe 22 yards is 1 chain or 100 links, ſay by the rule three; As 22 is to 100, ſo is any number of yards, to the

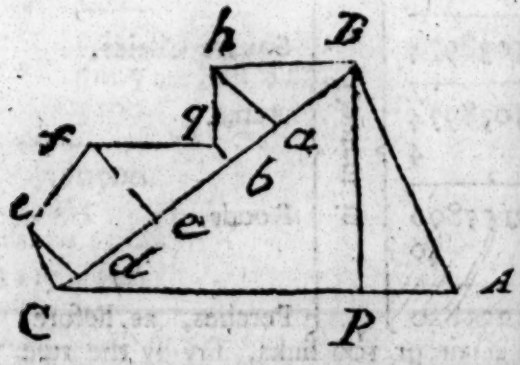
links contain'd in that number of yards, see the work. —— And thus, If the dimensions be given in ought but chains, &c. it will be best to reduce them to chains &c. for then it will work easiest. I think it needless to give any examples in ovals or in regular polygons, since few or no inclosures are in any such form, or if by chance you meet with any such forms, you cannot be at a loss and understand sect. 5th.

Ex. 28 *If a A B C represent any Curve-lin'd portion of Ground, whose Length AB* 2 C. 3 L. *and greatest Breadth D C* 18 L. *What Acres, Roods, and Perches are contain'd therein?*



2,03	= A D B.	
,12	= $\frac{3}{5}$ of D C.	
,02436	Acres,	
,09744	Roods.	
89760	Perches.	

In rule 22, sect. 5th, you have a general method for squaring any curve-lin'd space, but surveyors when they meet with a curve-lin'd hedge &c. as *A C B*, they as a general rule multiply the chord or streight-line *A B* by $\frac{3}{4}$ of the width *D C* for the area; as per work, because the true rule is tedious.



But if you meet with a hedge that hath several bends in it as the hedge *B C* of the triangular inclosure *A B C*. Measure in a right line from *B* to *C*, and as you measure along observe to take off-sets to every bend in the said hedge *B C*, and take each right-angles to the ranging line (*viz.* the prick'd line *B C*). Thus, suppose the

distance *Ba* to be 8 C. and the off-set *ah* 3 C. the distance *ah* 2 C. and the off-set *bg* 2 C. the distance *bc* 6 C. and the off-set *cf* 4 C. the distance *cd* 5 C, and the off-set *de* 1 C. and the distance *dC* 2 C. and off-set 0 C. To keep account of these distances and off-sets, some rule paper and set all down as they take them in the field: But if you draw a figure at random to represent the field, and so set down all the dimensions as you take them, the distances along the ranging lines, and off-sets along the off-setting lines as in the foregoing figure; it will be better to understand than any book that can be contriv'd, for comming to view this figure you have a round view of the fields form, and so may with more ease and certainty either cast up its content, or map the same &c. The areas of those off-sets are found by Ex. 26, all except the triangles at the ends as *dCe* and *Bhb* which are found by Ex. 24. Now in order to find the content of the above figure, let the base *AB* be 30 C. 20 L. and the perpendicular *BP* 15 C.

½*aB* 4 C.	multiply'd by	*ah* 3 C.	gives	12	area	△	*aBh.*
ah	+	*bg* = 5	× by	*ab* 1	gives	5	trapezia	*ahbg.*
bg 2	to	*cf* 4 = 6		*bc* 3		18		*bgcf.*
cf 4		*de* 1 = 5		*cd* 2½		12,5		*cfde.*
de 1	multiply'd by	½ *dC* 1	----	gives		1	area	△ *deC.*
C.						48,5		
15,1	½ AC.							
226,5	area	△ ABC.	in chains.					
48,5		off-sets - -						
27,50	acres in the whole.							
	A.	R,						
Answer,	27	2.						

Note, If the hedge BC had bended inward your own notion would teach you to subtract the area of the off-sets instead of adding, &c. And when you are to survey any field, take a view of it that so you may the better draw its form upon a sheet of white paper; and as you measure any line in the field draw the like line in your figure, and set down the length thereon as taken in the field &c. as above directed: where observe, that it is best to take as large a trapezia in the field as you can and about its sides take triangles &c. ever observing to leave as little for off-sets as you can, unless you have some instrument that will direct you to take the off-sets at right-angles

to

to the ranging line, for they ought always to be taken perpendicular to the said line, otherwise you will fall into the error mentioned in Ex. 26.

$B = 0$
$ab = 3$
$bg = 2$
$cf = 4$
$de = 1$

5) 10

2 mean off-set.
21 = BD,

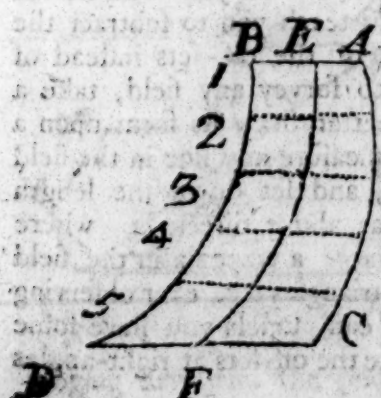
42 } area, { Bde,
1 } { deC.

43 = area off-sets.

NOTE, Some surveyors add all the off-sets together, and divide that sum by the number of off-sets which quotient is called the mean off-set, and multiply by the length between the first and last off-set, gives the area of all between the said first and last off-sets.

Now if the hedge range in a gradual manner, so that the off-sets can be taken at an equal distance from one another, this method holds pretty true: But if the hedge have corners, or bends unequally distant from one another, so that the off-sets cannot be taken at equal distances (for the off-sets shou'd be always taken to every bend); This method is then erronious as appears by this work, for instead of 48,5 you have only 43.

Ex. 30.] *Let ABCD be some piece of bending Ground, as a number of Ridges, &c. whose Content you wou'd know; and suppose the Ridges to turn gradually as by the Figure, and let the prick'd line EF be a line measured in the middle along the top of a Ridge, and several Breadths taken at right-angles thereunto, whose measures are as set down in the Figure.*



1	2	= AB.
2	2,2	
3	3,1	
4	3,8	
5	4,2	
6	5,1	

6) 20,7 sum of the breadths, or [off-sets.

3,45 mean off-set.
16,71 = EF the length.

5,79495 Area *Ans.*

In such like forms this method finds the area very nearly, and is much easier than to reduce the figure into trapeziums, triangles, segments &c.

NOTE 2. What is said concerning one hedge, also holds good in any number of irregular hedges; upon which I have been some what large, in order to spare in the following Example.

EXAMPLE 31.

Let AA BB be a Hill or Mountain, whose circumference at the Top or Summit BB is 10,1 *Chains, at the Middle, CC* 14,02 *Chains, at the Bottom AA* 15,88 *Chains, the Side of the Hill BA* 12,2 *Chains, and its opposite side AB* 14,1 *Chains: What is the area of that Hill?*

10,12	BB.	22,2	BA.
14,02	CC.	14,1	AB.
15,88	AA.		
3)40,02	Sum divided by the number of dimensions	2)26,3(13,15	half sum
13,34			
13,15	half sum of BA and AB.		
17,54210	Acres.		
4			
2,1684	Roods.		
40			
6,,7360	Poles.		



That is, measure round the bottom, middle, and top of the hill. Add those three peripheries together and take a third of that sum; also measure up one side of the hill and down the opposite side thereof. Take half that sum and multiply by the one third last mentioned sum, so have you the area of the hill, according to Mr. *Diggs*. But if you think those dimensions are not sufficient, (as they are not if the hill be very rugged) you may take as many dimensions as you please, and work as in Ex. 29. or 30. for the reason of this Example is the same with those two; as may easily appear if you suppose *BaCh* (Ex. 29.) or *ABCD* (Ex. 30.) to be wrapt about some hill, &c.

Note,

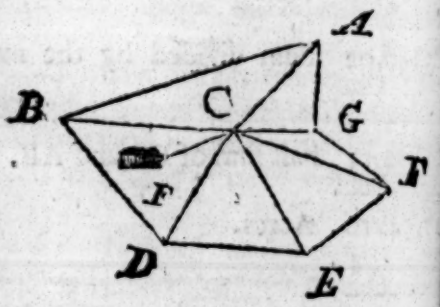
Note, What is said concerning a hill may also be apply'd to a valley, if you consider that the out-side of a hill is the same with the in-side of a valley, &c.

To take the Plot of any Field by the Plain-Table.

EXAMPLE 32.

Let ABDEFG be a Field, and let it be requir'd to take the true Form thereof upon Paper,

1. Cause marks or staffes to be set up in every angle of the field, as at *A*, *B*, *D*, *E*, *F*, *G*, and also if there be any place in the field as a brook, tower, house, tree, &c. that you wou'd have represented in your Map, cause a mark to be set up there; suppose at *F* there be a house that you wou'd have in your plot, set a mark up at the said *F*.



2. Place your table in any place of the field as at *C*, so that from the place *C*, you can see every mark which is set up.

3. Having a ruler or index movable upon the center of your instrument fix the Table fast, and parallel to the horizon, and along the edge or thro' the sights espy every mark one by one, and measure the distances between your instrument or table and every of the said marks; thus, when you espy the mark at *A* along the index, measure the distance *CA* between the mark at *A* and *C* the center of the table, which suppose 2 chains 33 links: Then take 2 chains 33 links from your diagonal scale and set upon the paper which is fastened upon your table under the streight-sided index or ruler from *C* towards *A* and in order to do this, when along the edge of the said index you espy the marks, let your index lay fast and by its edge draw lines quite from the center to the out-edge of the paper, and then those lines will be ready to lay the distances on as you measure them. Again, measure the distance between the mark *F* and your table *C*, which suppose 2 chains 99 links from the same diagonal scale, take 2 chains 99 links and set on your paper from *C* to *F*: also measure the distance between the

inſtrument at *C* and the mark at *F* which let be 1 chain 80 links, take this 1 chain 80 links from the ſaid diagonal ſcale and ſet from *C* to *F*. And thus meaſure the diſtances between your inſtrument *C* and every one of the ſaid marks, and lay thoſe diſtances from a diagonal ſcale upon their reſpective lines on the ſaid paper; which done join all the poiuts where thoſe diſtances end, as *A*, *B*, *G*, *D*, *F*, &c. with right-lines (if the hedges between the ſaid marks was ſtreight), and ſo have you a true and compleat map of the field. Before ever you go out of it only obſerve that ſuch objects as *F*, which are within or without the hedge, muſt not be join'd with thoſe lines which repreſent the ring or outward hedge, as your own kuowledge may eaſily teach you.

NOTE, If the hedge be very crooked, you muſt ſet up ſo many more marks, or you may ſet up marks at the principal corners, and ſo meaſure off-ſets between ſuch corners, and on your paper draw lines from corner to corner and ſet off the off-ſets perpendicularly as you took them, and draw lines or curves over thoſe off-ſets which will repreſent the hedge, &c.

This method of taking a plot by the plain-table (if the weather be fair that ſo your paper be kept dry), is as uſeful and true as any; for in a manner as ſoon as you have taken the dimenſions your map is finiſhed.

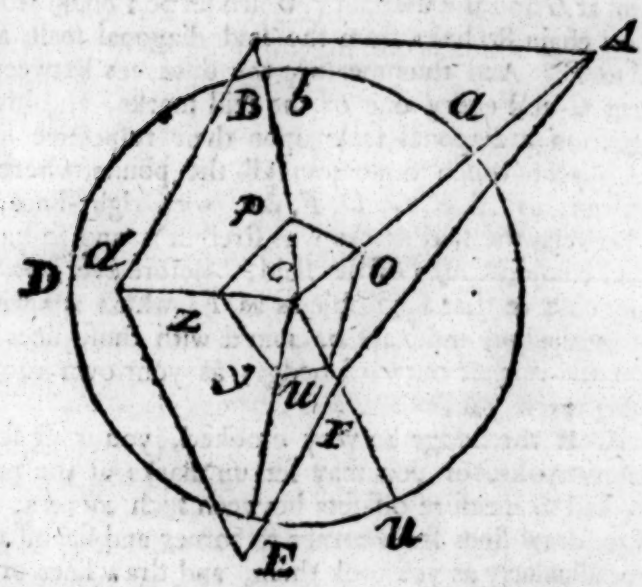
To take the Plot of any Field, by the Semi-circle, *or* Theodilite, *at one Station.*

EXAMPLE 33.] *Let A B D E F be a Field, to be plotted according to its true Form.*

1. Having ſet up marks at every angle of the field and at all the objects you wou'd have in your map, as in the laſt example.

2 Place your inſtrument parallel to the horizon and make it faſt ſo that nothiug can be moved except the index. *Note*, It may be fixt in any part of the field as at *C*, ſo that from the ſaid place, you can ſee all the marks ſet up.

3. Having ſo placed your inſtrument as that along the diameter where the degrees begin you can eſpy any of the marks, ſuppoſe *A*, which meaſure the diſtance between your inſtrument at *C* and the ſaid mark at *A*, which ſuppoſe 4 chains 2 links, ſet this down, (ſee Ex. 29.) Move the index till thro' the ſights



ſights you can ſee the next mark at *B*, and ſee what degrees are cut thereby, ſuppoſe 90 deg. Alſo meaſure the diſtance between *C* and *B*, 2 chains 83 links: again, move the index till thro' the ſights you eſpy the other marks *D*, *E*, and *F*, you'll find the degrees pointed ont by the ſaid index to be 150, 230 and 280; that is, the angles *BCD*, *DCE* and *ECF*, are 60, 80 and 50 reſpectively as *per* figure. Alſo, meaſuring the diſtances *CD*, *CE* and *CF*, 2 chains 3 links, 3 C. 52 L. and 1 chain 52 links, ſet all down in a figure made for that purpoſe. (Ex. 29.)

To Protract the Field laſt taken.

1. Faſten a ſheet of white paper upon a level table &c. then from a ſcale of true chords, take the chord of 60 deg. and with one foot of your compaſſes in the middle of the ſaid ſheet ſweep a circle *a b d e u*.

2. From the center *C* thereof dray a line *CA*, on which from a diagonal ſcale lay 4 chains 2 links, and where *CA* cuts the circle as in *a*, lay 90 deg. of chords upon the circle from *a* to *b*, and thro' *b* draw *CB* making it (by the diagonal ſcale) equal to 3 chains 83 links; alſo, from *b* to *d* upon the arch

or

or circle, lay 60 of chords, aud thro' *d* draw *C D* which make (by diag. ſcale) equal to 2 chains 3 links; and thus go round the circle, till you have laid down all the angles by the chords, and ſides by diag. ſcale; then draw the lines *A B B D*, *&c.* which gives you the true form of the field.

To take the Form of a Field by the Chain only, at one Station.

Ex 34.] *Let it be required to take the Map of the Field A B D E F the laſt Figure?*

1. Having ſet up marks as before, and pitch'd upon *C*, for your ſtation.

2. At *C*, ſet up a ſtaff thro' the great ring at one end of your chain, and ſtand you at the ſaid ſtaff at *C*, ordering ſome aſſiſtant to ſtretch out the chain, moveing with the loſe end in his hand, to the right, or to the left, till you at the ſtaff; where the other eud of the chain remains, can juſt between his feet, eſpy ſomc of the marks ſet up in the angles of the field, *&c.* ſo, when he ſtands at *o*, and you at *C*, will ſee between his feet, the mark at *A*, ſo ſtick down at *o*; alſo, when he ſtands at *p*, you'll ſee the mark *B*, ſo ſtick down another ſtick at *p*; likewiſe, when he ſtands at *z*, *y*, &c. you'll ſee the marks *A*, *D*, *E*, &c. ſo at *z*, *y*, &c. ſtick up ſmall ſticks.

3. Take up your chain, and meaſure the diſtances *o p* 1 chain, 43 links; *p z* 80 links, *z y* 1 chain, 2 links, *&c.* which ſet down (as per fig. next) meaſure the diſtances *A C* 4 chaius 2 links, *C B* 2 chains 83 links, *&c.* ſetting all down in a figure made at random; for repreſentations ſake (ſee Ex 29.) ſo have you done in the field.

To lay down this Field upon Paper.

1. From your diagonal ſcale, take in your compaſſes 1 chain, and ſetting one foot upon *C* as a center, ſweep the circle *o p z y*.

2. From the ſaid diagonrl ſcale, take 1 chain 43 links, and ſet upon this circle from *o* to *p*; ſet 80 links from *p* to *z*, ſetting off all the diſtances upon the circle, that you meaſured between the ſmall ſticks, ſet up at *o*, *p*, *z*, &c.

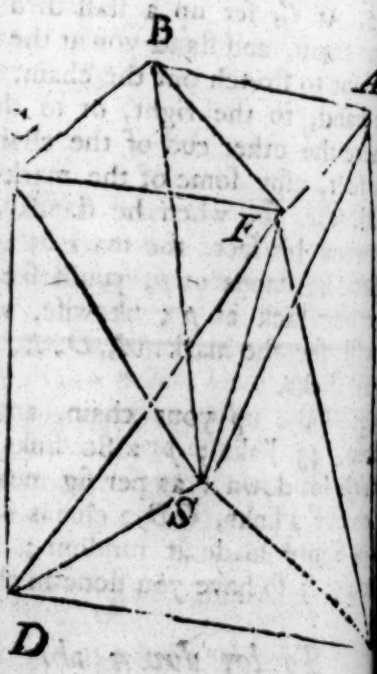
3. Thrc' theſe points *o*. *p*, *z*, *y*, &c. draw the lines *C A*, *B G*, &c. making *C A* = 4 chains 2 links ; *C B* = 2 chains 83 links ; *C D* = 2 chains 3 links, &c. all from the diagonal ſcale.

4. Draw the lines *A B*, *B D*, *D E*, &c. ſo you will have a true map of the field, as before.

To take the Plot of a Field at two Stations, and meaſure only the diſtance between the two Stations, and the ſeveral Angles.

Ex. 35.] *Let A B C D E be a Field to be Map'd ?*

1. This example, is in effect the ſame with Prob. 131 Part Firſt, which being underſtood, this will be gain'd with eaſe.



2. Place your inſtrument in any convenienr place of the field, ſuppoſe at *F*, and there take the angles *A F B* 70, *B F C* 40, *C F D* 60, *D F C* 60, and *E F A* 130 ; by eſpying the marks ſet up at the corners *A B C D E F*, thro' the ſights of the index, as directed in *Ex*. 23.

3. Remove your inſtrument any proper diſtance, ſuppoſe 3 chains 15 links to *S*, which make your ſecond ſtation and there take the angles *A S B* 40, *B S C* 38, *G S D* 98, *D S E* 94, and *E S A* 110, as you did in the firſt ſtation ; but note, whe you are at the firſt ſtation *F*, you muſt obſerve what ang ſome of the marks, ſuppoſe *D*, makes with your ſecon ſtation *S*, *viz*. the angle *S F D* 32 ; and when you come t your ſecond ſtation *S*, obſerve the angle *D S F* 160, made b obſerving the ſaid point, or mark *D*, and *F* your firſt ſtatio or you cannot lay down the figure.

To lay down this Field upon Paper.

1. On a ſheet of large paper, draw a right line rs *SF*, which from your diagonal ſcale make equal to 3 chains 85 links.

2. Upon *F* with the chord of 60, deſcribe a circle, and do the ſame upon *S*.

3. Upon *F*, make the angle *SFD* = 32, drawing *FD* at pleaſure.

4. Upon *S*, (by Prob. 11.) make an angle *FSD* of 160°, drawing *SD* till it cut *FD* in *D*; ſo is *D* one of the angles of the required map.

5. Make the angles at *F*, as you took them in the field, *viz.* by laying their meaſures of chords on the circle deſcrib'd upon *F*, and drawing the lines *FA*, *FB*, *FC*, *FE* at pleaſure. Alſo, upon the circle deſcrib'd upon *S*; lay the chords belonging to the ſeveral angles taken at the ſecond ſtation, drawing the lines *SA*, *SB*, *SE*, till they meet the former lines in the points *ABCE*; laſtly, draw the lines *AB*, *BC*, *CD*, *DE*, *EA*, ſo you have the figure *ABCDE* for the true map, or plot required.

Note, This example requires, that when you are at either ſtation, you muſt ſee the other ſtation and all the marks.—But in the three forgoing examples, if you cannot ſee all the marks at the ſtation, you may ſet lines; and in thoſe lines ſet marks, between your ſtation and required mark; and ſo obſerve from your ſtation to the neareſt mark, which can make no error, if the mark which you obſerve, lye in a right line with your ſtation, and the mark out of ſight; and thus having ſeveral marks, between your ſtation and the mark out of ſight, you will be directed thereby to meaſure from your ſtation, and from mark to mark, till you come to your deſired mark.

Otherwiſe.

Place your inſtrument in ſuch a a part of the field, as that you may ſee as many angles or corners thereof as poſſible; and towards the unſeen part of the hedges, ſet up marks juſt within ſight and no more, and take this as if it were a whole field; then remove the inſtrument to ſuch a place in the field, as that you may ſee the marks laſt ſet up, and the remaining part of the hedge if it can be, if not, ſee up new marks, and ſo remove from place to place, till you have got the whole field; this is ſo eaſy, it needs no example, only taking 2 or more fields adjoining one another.

Ex. 36.] *How to take the plot of a Field, by the Compass, at 2 Stations. Let ABCDE the foregoing figure, be the proposed Field?*

There is no difference between this aud the last example; only, in this case by help of the needle, you have the bearing of the two stations, the one from the other, and also of the marks from each station; now if *F* and *S* represent the two stations, being distant 3 chains 85 links the one from the other; place your compass at *F*, and suppose the north point of the needle exactly to point out *S*, then do the stations *F* and *S* bear north and south from one another; this done, over the center of the compass, look out for the several marks set up at *A B C* &c. and you will find the mark *D* to bear north 32 deg. west, the mark *C* north 92 deg. west, the mark *B* 133 deg. from the north or south 47 deg. west.—Thus you may find all the angles at *F*; then removing your instrument to *S*, over the center of the compass, o. thro' the sights of an index moving thereon, look out for the several marks as before, and you'll find the mark at *D* 160 deg. from the south, or north 20 deg. westerly, and the mark at *E* south, 62 deg. westerly, and the mark at *B* south, 24 deg. westerly; and thus you may find all the angles at *S*,

To Protract this Figure.

Make the line *F S* = 3 chains 85 links, aud upon *F* and *S* with the chord of 60 sweep the two circles, take 32 deg. the bearing of the mark *D* from *F*, and lay it upon the circle describ'd upon *F*, from the line *F S* towards the west, so have you a point in the periphery to draw the line *F D* thro', which extend at pleasure; then on the circle described upon *S*, lay the chord of 20 deg. from *a* to *b*, the bearing of *D* from the second station, thro' *b* draw the line *S D*, cutting *F D* in *D* so doth *D* represent one of the angles in your map; the rest of the angles are laid off exactly, as in the last example.

I have been brief in these two last examples, because they are in effect the same with Prob 138, which see.

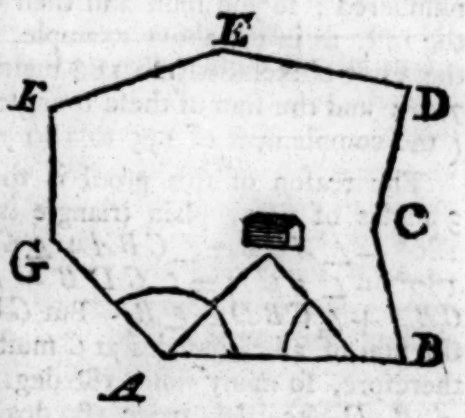
How

How to take the Pot of a WOOD, MORASS, *&c.*

By going about the Out-side thereof; with the Compass, or Theodilite.

Ex. 37. *Let ABCDEFG be a large Wood,* &c. *which you cannot go thro' to Measure, and so must be forc'd to Measure it on the Out-side.*

1. Cause your assistant to go and set up marks at every corner, or any other object you wou'd chuse to have set in the map.



2. Place your instrument at any of those marks, as at *A*,; there observe the two nearest marks *B* and *G*, first one and then the other, aud observe what degrees are contained between pointing at *B*, and pointing at *G*; suppose 146,

3. Remove your instrument to *B* measuring the distance *AB* as you go, which let be 23 chains 40 links; then placing your instrument at *B*, look out for the next two nearest marks *A* and *C*, and mind what degrees are included, suppose 76. Again, measure from *B* to *C* 15 chains 20 links, and placing your instrument at *C*, look as before for *B* and *D*, and in doing this suppose the index moves over 155 degrees: and thus go round the whole wood, &c. taking all the angles and sides, and set all down in a plot drawn at random as you see by the figure, so are your observations finish'd.

To lay down these OBSERVATIONS.

Draw the line *A B*, which from a diagnnal scale make equal to 23 chains 40 links; and upon *A* (by Prob. 11.) make an angle of 146 deg.; also upon *B* make an angle of 76 deg. making *B C* equal 15 chains 20 links; then upon *C*, make an angle 155 deg. to bend inward, because its so in your figure, making *CD* equal to 17 chains 90 links; and thns go round the figure, till you come at *G*, and then see if the distance *GA*, and angle at *C* be what it ought to be, by this means you'll know whether your figure be true.

How

To know whether you have taken the Angles in the Field true, or not.

Take 2 from the number of angles, and multiply 180 by the remainder; and if the product be equal to the sum of all the angles, the observations are truly made, otherwise not. But here observe, if there be any inward angles, they are not numbered; so you must add their complements to the sum of the rest. as in the above example, the number of angles are 6 the augle *C* excluded, so 186 multiply'd by 4 (6—2) gives 720; and the sum of these 6 angles is 695, to which add 25, (the complement of 155 to 180) and it proves, *&c.*

The reason of this proof is thus. Since the sum of the 3 angles of every plain triangle is = 180 deg. Therefore, $180° - \angle ACB = \angle CBA + \angle CAB$ (fig. to Ex. 32.) and $180° - \angle BCD = \angle CDB + \angle CBD$, &c. And an $\angle CBA + \angle CBD = \angle B$. But *C* being the center of a circle, the sum of all the angles at *C* must be equal to twice 180 deg. therefore, so many times 180 deg. as are the number of angles *A*, *B*, *D*, &c. less twice 180 deg. is equal the sum of all the angles of the figure, Q. E. D.——By the same way of reasoning it may also be proved: That the sum of the external angles of all right-lin'd figures are equal (*viz.*) the sum of the external angles of any right-lin'd figure is equal to twice 180 deg. or 360 degrees. 32. *E.* 1

If there were an object that you would have in your map as a house at Z, which may be seen at the angle *A* and also at the point *a*, in the hedge *AB*.

When you are observing at *A*, see what angle the house at Z makes with the hedge *AB*, or mark at *B*, which let be 60 deg. also when you have measured on to *a* 17 chains; plant your instrument at *a*, and observe the mark at *A*, and the house at Z; and suppose between these two observation the index moves over 58 deg. then when you have laid your map make the $\angle ZAB = 60$ deg. and having made $Aa = 17$ ch. upon *a* make the $\angle ZaA = 58$ deg. drawing *aZ* till it meet *AZ* in Z, so is Z the place of the house required.

In the same manner may you put any number of objects in your map, and by this means may survey a county; but the towns, rivers, roads, churches, *&c.* be required in the map, if you cannot by going about the county, *&c.* see such objects

objects, you will be obliged to ſurvey thro' the ſame, 'till you have taken all you want to have; which done, if you do not uſe the needle (which is beſt in ſuch aſſairs) you may by a pocket compaſs, *&c* take the bearing of any two places, by which means you will be enabled to draw the compaſs in your map, to ſhew its ſituation, *&c*.

I might alſo ſhew how any field might be taken by placing the inſtrument in any angle thereof; but if what has already been ſaid be underſtood, there can be no difficulty in thoſe things.

The reaſon of all ſuch methods as is here uſed, is from Theorems Part Firſt; and is only this, what diſtances you take in the field by a chain of 22 yards long; you lay the ſame diſtances down upon paper by a chain, not perhaps an inch long, *viz.* from the diagonal ſcale; and for the inſtrument, which is divided into degrees, you may ſet off the angles upon your paper by the ſame inſtrument, or by a protractor, or line of chords, which all anſwer the ſame end.

So that you do the ſame thing upon paper with ſmall inſtruments, that you do in the field with large ones.——Pauſe a little on this reaſon, and it will do more for you than a multitude of examples.

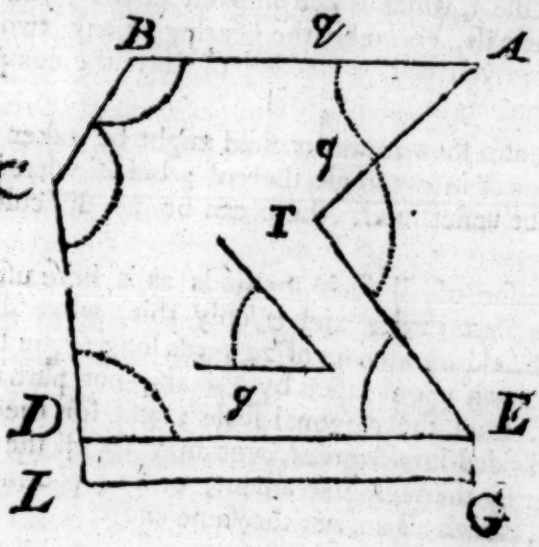
How to take the Plot of a Field by going round the ſame with a CHAIN *only*.——EX. 38

Firſt, Having ſet up marks at every angle, put a ſtick thro' the great ring at one end of the chain againſt any of the marks as at *A*, and ſtretch the other end along the hedge *AB*, and there ſet down a ſmall ſtick as at *q* in a right-line with *AB*, the end of the chain reſting at *A*; ſtretch the other end along the hedge *AF*, and there ſet up another mark at *p* in a right-line with *AF*, then meaſure the diſtance *pq*; which ſuppoſe 50 links.

2. Meaſure the hedge *AB* 30 chains 62 links, then faſtening the end of the chain at *B* as you did at *A*, and take the meaſure of the angle at *B*. And thus go round the field, meaſuring all the ſides and angles.

How

Haw to lay down the Plot laſt taken.



Draw the line *AB*, making it equal 30 chains 62 links, from a diagonal ſcale; take 1 chain in your compaſſes, and ſetting 1 foot in *A* ſweep the arch *g q*, on which from the diagonal ſcale lay 70 links. In like manner, make the angle at *B* = 1 chain 30 links. And thus go round the figure, laying down all the ſides and angles as you took them in the field, till you come to the angle *F*, and then ſee if the ſide *AF* and the angle at *F*, be equal to what they ought to be; ſo will you know whether your figure be truly laid down, or uot.

Note, You muſt lay down all the ſides, by one diagonal ſcale. But it is no matter whether you project the angles by the ſame ſcale or not; only obſerve to ſtrike the arch, and meaſure it by one and the ſame ſcale. It ſometimes falls out that in this way of ſurveying, (by reaſon of buſhes, ditches, *&c.*) you cannot come near the hedge, and in ſuch caſes are oblig'd to meaſure parallel to the hedge; as for example,

Suppoſe DE the hedge you cant come near.

On the outſide of the hedge, meaſure any perpendicular diſtance thereunto, as the diſtance *DL*; then go to the other end of the hedge, and meaſure the ſame diſtance perpendicular to

to the hedge *DE*, (*viz.*) *EG* equal to *DL*, then it is plain if you meaſure in a right-line from *L* to *G*, it will be equal to the length of the hedge *ED*.

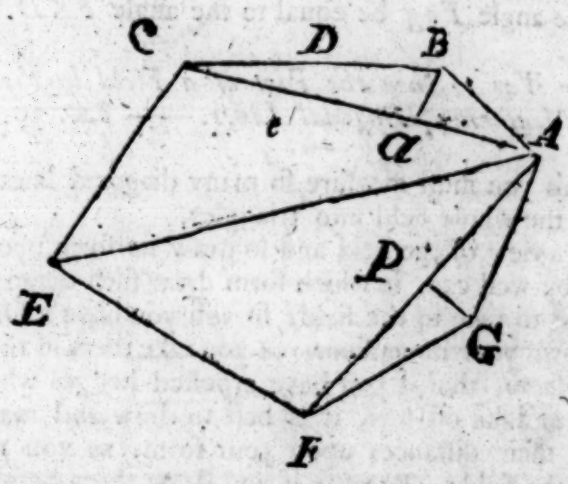
In like manner if you cannot come near the hedges *EF* and *ED* to take the meaſure of the angle at *E*, you may by the ſame method meaſure *eF* parallel to *EF* and *eg* parallel *ED*, then will the angle *Feg* be equal to the angle *FED*.

Another Way to take the Plot of a Field by the Chain, by Meaſuring Diagonal Lines. —— Ex. 39.

To do this you muſt meaſure ſo many diagonal lines as will divide the whole field into triangles.

1. Take a view of the field and ſo draw its form upon paper as near as you well can, in which form draw ſuch diagonal lines as you intend to take in the field, ſo will you have a form ready to ſet down your dimenſions, as you take them in the field; and here obſerve, that if you have crooked hedges which will oblige you to take off-ſets, it is beſt to draw and mark ſuch off-ſets and their diſtances upon your form, as you meaſure them out in the field: Becauſe if you draw them before, it is a queſtion whether you draw the juſt number or not, for you muſt always beſure as you meaſure along the ſtreight line that you off-ſet to every bend or corner in the hedge, at right-angles to the line you are meaſuring on. Alſo obſerve, that as you are meaſuring out any diagonal line if you can ſee the oppoſite angle, which if you can you may take a perpendicular from your diagonal line to the ſaid angle; but if a perpendicular cannot be taken, you muſt meaſure the other two ſides of the triangle. So in this figure the hedges being all pretty ſtreight, except the hedge *ABDC* which bends at *B* and *D*, I therefore meaſure diagonals from *A* to *E* 4 chains 36 links, from *F* to *A* 3 chains 84 links, but in going from *F* towards *A* the oppoſite angle *G* may be ſeen, and I find the perpendicular (by a ſquare apply'd to the line *FA* and the point *G*) will ariſe at *P*, there ſet down a mark being at the end of 1 chain 44 links, and meaſure out *PG* 1 chain 1 link, then come backwards to *P* and meaſure out the line *FA*. And then meaſure from *A* towards *C*, I find *Aa* 1 C. 8 L. and the off-ſet *aB* 35 L. then going 74 links further, as to *e*, I obſerve the bend *D* at right-angles to the ranging line *AC*, which diſtance *eD* meaſures 4 links, and from *e* to *C* 80 links, ſo the whole line *AC* is

2 chains 62 links. Now if no more perpendiculars can be had you must measure the hedge *GE* 2 chains, and the hedge *EF* 3 chains 22 links, so have you dimensions sufficient for laying down the plot or map of the field *ABDCEFG*.



To lay down this Field.

1. Draw the line AC making A*a* = 1,08 *C.* *ae* = 74 *L.* and *e*C = 80 *L.* upon *a* and *e* raise the perpendiculars *a*B 85 *L.* and *e*D 24 *L.* draw AB, BD, and DC representing the bending hedge. Take 4 *C.* 36 *L.* in your compasses and one foot in A sweep the arch E, and with 2 *C.* and one foot in C sweep another arch crossing E in E, and draw the line CE. In like manner, with the distances AF 3 *C.* 84 *L.* and one foot in A strike an arch, and EF 3,22 *C.* and one foot in E strike another arch crossing the former in F draw EF, make FP = 1,44 *C.* and raise the perpendicular PG = 1,1 *C.* draw FG and GA, and the plot is made all from the diagonal scale.

NOTE, When you are in the field, if you take some diagonals or one diagonal more than is useful, as one from C to F when your plot is made you may take this diagonal from the same scale, and setting one foot in C look if the other fall just upon F, which if it do your plot is truly made, &c. Also observe in this and the two foregoing examples, if the ground be hilly &c. so that you cannot see from one mark to another you must set lines and in those lines set marks to direct your

ever take care in all the foregoing exumples about plotting to meaſure in ſtreight lines. —— There is an inſtrument called, *Perambulator*, *Pedometer*, a way-wiſer, or ſurveying wheel, which is much readier than the chain: It is a wheel generally made of wood whoſe periphery is half a pole, with a face divided like a clock, and ſo made that as you drive it before you, the pointer ſhews how many yards, poles, furlongs, and Miles you go &c. I might yet ſhew ſeveral other ways to take a plot, &c. as by placing the inſtrument in an angle of the field, but if what has been done be carefully obſerv'd, thoſe other methods cannot be difficult. And obſerve, that ſince every plane triangle conſiſts of 6 parts, (viz.) 3 ſides and 3 angles, if any method can be found that will take any 3 of thoſe 6, (a ſide being one of the 3) by ſuch a method you may take a plot, &c.

To find the Content of a MAP *when made.*

When your map is laid down by any of the foregoing methods, divide it into triangles, trapeziums, &c. and in each triangle let fall a perpendicular, which perpendiculars and baſes &c. meaſure by the ſame ſcale that you took the ſides of the figure from, and ſo by the form find the area; then all theſe areas added together gives the area of the whole figure or plot.

Ex. 40.] *Let the Area of the laſt Figure be requir'd?* This figure conſiſts of three off-ſets *ABa*, *aBDe*, and *CDe*, one trapezia *ACEFA*, and one triangle *AFG*. Therefore, Firſt for the off-ſet,

	Sq. Chains.
Half *Aa* ,54 multiply'd by *aB* ,35 gives - - - - -	,1890
aB ,36 added to *eD* ,24 is = ,59 which multiply'd by $\frac{1}{3}$ *ae* ,37 gives - - - - - - - - - - - - - -	,1416
And $\frac{1}{3}$ *De* ,12 multiply'd by *eC* ,8 gives - - - - -	,099
Which added together gives the area of the ſpace *ABDEA* (viz.) - - - - - - - - - - - - - - -	,4266

2. For the trapezia *ACEF*

In each of the triangles *ACE* and *AEF* you have given all the three ſides, by which (ſect. 5. rule 6.) you may find the area of each triangle; or with more eaſe, take a perpendicular from *C* to *AE* (the neareſt diſtance) which is 80 L. alſo, the neareſt diſtance from *F* to *AE* is 2 C. 64 L. Then 2,64 added to ,8 gives 3,44 which multiply'd by $\frac{1}{2}$ *AE* 2,18 gives 7,4992 ſquare chains.

Laſtly,

Lastly, AF 3,04 multiply'd by $\frac{1}{2}PG$,55 gives - - - } 1,4520 = area triangle AFG,

Add { 7,4992 ,4266 } trapezia { $ACEF$. $ABDC$.

,93778 = Acres.

4

3,75112 = Roods.

40

30,04480 = Poles.

Note, If you want only to have the area of a field and survey by Ex. 39, you need not plot it, unless to take perpendiculars; because you take dimensions sufficient for finding the area.

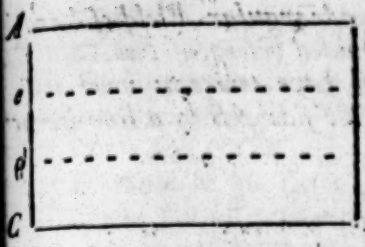
	A.	R.	P
Answer,	0	3	30,0448

How to divide a Plot when laid down.

In PART I. I have shewn how to divide and cut a plot by are every way possible. Yet in this place I shall shew some general and practical Rules for doing the same. And first, when any plot is laid down and its content found, and you know what way the dividing hedges must range, draw a line in the plot so as to cut off so much as you expect, then measure the said part so cut off and see if it answer your intent which if it do not, you must vary the dividing line till it exactly hit, which done measure its limits in your plot, so will you know how to set off such a line in the field. Thus do for all your dividing lines and set them all off in your plot, with their limits or distances asunder at each end, and other places if you think fit. Then going into the field lay out the same distances in chains and links setting up marks or digging little holes at the out-side of every new inclosure, which done measure every of the said new inclosures by its self and if it is not exact, you must vary your marks till it come to its true content; and here note, that since most ground is uneven, it generally measures to more in parcels than it does when measured altogether; by this means you may divide any field at pleasure after its mapped. To make this plainer take the following easy example.

EX. 41. *Let the Rectangle ABCD, whose Area is 3 Acres, and Length AB 20 Chains, be divided into 3 equal Parts by Lines or Hedges drawn Parallel to the Length AB.*

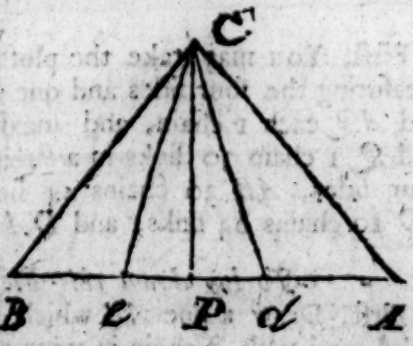
1. Since the length of any rectangle multiply'd by its breadth gives its area; its evident if you divide the area by the length the quotient gives the breadth; or the area divided by the breadth



breadth gives the length; ſo here $\frac{1}{3}$ part of 30 acres or 300 chains being 100 C. the area of one of the equal parts divided by 20 chains the length, gives 5 chains for the breadth of each ſhare.

Therefore, meaſure 5 C. from *C* to *d*, and from *d* to *e*, and from *e* to *A*, at one end of the field making marks at *d* and *e*; then go to the other end meaſure in like manner from *D* to *a* 5 chains, and from *a* to *u* 5 chains ſetting marks, ſo is the field divided into 3 equal ſhares as was required.

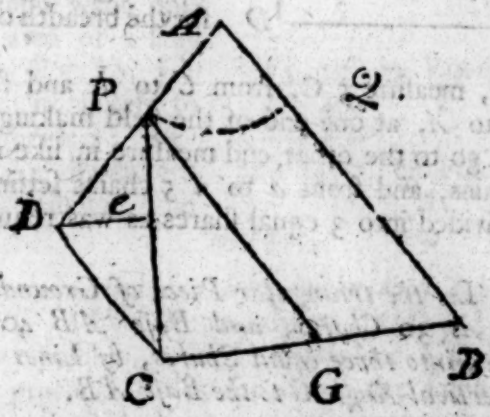
Ex. 42. *Let the triangular Piece of Ground ABC whoſe Area is* 30 *Chains, and Baſe AB* 40 *Chains, be divided into three equal Shares, by Lines drawn from the vertical Angle C to the Baſe AB.*



Its plain from meaſuring a triangle, that if you divide the area by half the baſe, the quotient will be the perpendicular, or the area divided by half the perpendicular gives the baſe; Therefore, if the perpendicular *CP* be 15 chains then a third of 300 the area, is equal to 100 which divided by 15 the perpendicular gives 6 C. 66 L. for the Baſe of the third ſhare of 30 acres; ſo meaſure out *A d* 6,66 C. *d e* 6,66 C. and *e B* 6,66 C ſo will the triangles *ACd*, *dCe*, and *eCB*, be each equal 10 acres as requir'd. - - - - - But in this example the work 40 is eaſier, becauſe the baſe *AB* = 40 C. is given, for you need but take 6,66 a third of 40, and lay or meaſure from *A* to *d*, and from *d* to *e*, &c. as before. By this method you may alſo lay down a triangle to contain any number of acres, &c. *viz.* if you have the content and the baſe, or perpendicular given, by dividing the content by half the one, you'll have the other.

Ex. 43.

Ex. 43. *Let ABCD be a Quadrangular Field of Land, to be Measured and Divided between two Men Z and Y, and that Z must have twice as much as Y, and have it laid towards the side AB by a line drawn from the point P ?*



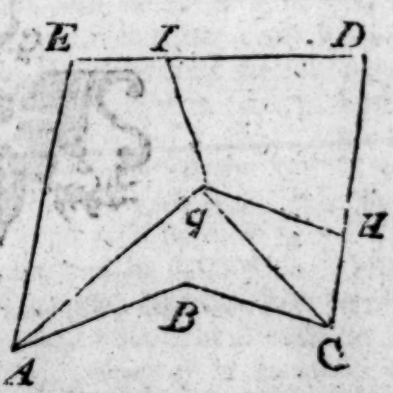
First, You may take the plot of any four sided field, by measuring the four sides and one angle. Thus, measure *AP* and *AQ* each 1 chain, and measure the distance between *P* and *Q* 1 chain 30 links in a streight line; next measure the four sides, *AB* 30 chains 44 links, *BC* 20 chains 4 links, *CD* 10 chains 64 links, and *DA* 22 chains 22 links.

To lay down this Field upon Paper.

First, Draw a line *AB* which make equal to 30 chains 44 links, and with 1 chain in your compasses and one foot in *A* strike the arch *PQ* laying 1 chain 2 links from *Q* to *P*, and draw *AD* making it equal to 22 chains 22 links. —— Then with 10 chains 64 links in your compasses and one foot in *D* sweep an arch *C*, also with 20 chains 4 links and one foot in *B* cross the arch *C* in *C*. draw *BC* and *DE* and the plot is finish'd. —— Now for its area, First, draw a diagonal line from *B* to *D* which measures 40,6 chains, whose nearest distance to *A* is 20 chains, and to *C* 10 chains which added to 10 gives 30, half of which is 15 chains, (half the sum of the perpendiculars) and 15 chains multiply'd by the diagonal *BD* 40,6 chains, gives 609,0 chains the area of the whole field; so the share of *Z* must be 40,6 chains, and the share

of Y 20,3 chains. —— To lay thls off, Draw a line from the given point *P* by gueſs, as the line *PG* which meaſures 20,35 chains, and a perpendicular *D e* therunto 8 chains, by which the area of the triangle *P DC* turns out 81,4 chains, which taken from 203 chains the ſhare of Y, leavas 121,6 chains, which Y wants of his ſhare; ſo if you draw a line from *P* to *BC*, ſo as to cut off a triangle whoſe area may be 121,6, this Line ſo drawn will be the dividing line required, therefore, a perpendicular line from *BC* to *P* is found 20,30 chains, ſo 121,6 divided by half 20,30 (*viz.*) by 10,15 gives 12 chains *fere*, for the baſe of this triangle; ſo take 12 chains and lay from *C* to *G*, and draw the liue *PG* which cuts the ſhares as required.

Ex. 44. *Let ABCDE, be a Field containing* 27 *Acres, to be divided amongſt* 3 *Men equally, and the Hedges to run from g, a Pond in the Field?*



1. Since no point is given in the hedges, draw a line from the angle *A*, which let be one of the dividing hedges to *g* the given point in your plot.

2 Draw a line from *C* to *g*, and meaſure the trapezia *AgCB*, which you'll find to contain 6,65 acres, now if this 6,65 acres had been 9 acres, (*viz.*) $\frac{1}{3}$ of the whole content 27 acres, then the line *Cg* would have been another dividing hedge, but 6,65 wants 2,35 of $\frac{1}{3}$ ſo if you add a triangle to contain 2,35 to *AgCB* 6,65, you will have the ſecond dividing hedge; therefore take the neareſt diſtance from *g* to the line *CD*, and you'll find it 8 chains 62 links a perpendicular to *DC* from *g*. Now the reſpective area 2,35 chains divided by 4,31 chains (half of 8,62 chains,) gives 5,45 chains, ſo ſet 5,45 chains from *C* to *H* and draw *g H* the ſecond dividing hedge; in like manner, by drawing a line from *g* to the angle *D*, you may find the content of the triangle *AIH*, which will want an acre epual to the area of the

the triangle *IgD* of being a third ſhare, ſo taking a perpendicular or neareſt diſtance from *g* to the ſide *DE*, you may by diviſion (as before) find the point *I*, from which drawing the line *Ig* the plot is divided into 3 equal ſhares or parts, as was required. Now go to the field and meaſure out with your chain from *C* to *H* 5 chains 45 links making a mark at *H*, alſo meaſure the diſtance *DI*, make a mark at *I*, and then meaſure the 3 ſhares *IgAE*, *gAPCH*, and *HDIg*, which if the ground was truly ſurvey'd, the plot truly laid down, meaſured and divided, it will anſwer exactly, but if it do not you muſt vary the marks at *H* and *I* till you find it to ſuit your deſire. By theſe means you may divide land at pleaſure.

SEC. IX

SECTION IX.

GAUGING.

GAUGING is to know what quantity of liquor any veſſel will hold, or how much liquor is in it when it is either quite full or partly ſo. And being but ſolid meaſure may be perform'd by the rules in ſect. 7. both by the pen and *Coggeſhal*'s ſliding-rule; where note, I have ſuppoſed the line D to egin with unity and end at 10, but on theſe rules this line ſually begins between 3 and 4, and ends between 4 and 5. ut this can be no leſs if you underſtand its deſcription. (ſect. .) Now becauſe the practice of gauging (for eaſe ſake) is all erform'd almoſt by the ſliding-rule, I ſhall here deſcribe a iding-rule fitted for this purpoſe, which is in form of a rallelopipedon, three faces of which have each a ſlider; but me of theſe rules have but two ſliders and ſome have four iders, viz. 1 on each face or ſide, that which I ſhall here eſcribe, is that of three ſliders; which being underſtood it ill not be hard to apply theſe other kinds, becauſe the lines e uſually mark'd in each with the ſame letters. On one ſide this rule ſtand A on the rule, and B on the ſlider; theſe two es are the ſame with the rule and ſlider. (Sect. 1.) Here is ſo another line mark'd MD, which is the ſame with A and but numbered contrary way, and does not begin at 1 as ey do, but begins at 21,5042 and ſo goes on in an inverted

order, thus, 2, (or 200) 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 9, 8, 7, 6, 5, 4, 3, and the laſt diviſion is 2,1042. Now if from the logarithm of 10 or 100 you take the logarithm of 2,15042 or 21,5042, the remainder ſhews the diſtance of unity or 1, from the end of the line M D, and is exactly the ſame with the diſtance of the points M B (on the line A) from the end of the line A. On another ſide of this rule the lines are mark'd C, D, E, the line C is exactly the ſame with the line A B, and the line D the ſame with that already deſcribed, ſect. 1. The line E has a triple radius, each of which is a third part of the radius of D, or $\frac{1}{3}$ of the radius of C, A or B, and the other diviſions 1, 2, 3, &c. anſwerable, ſo that if the line A be made, all the lines B, C, D, E and MD may be made from it. On a third ſide of this rule are mark'd N on the ſlider, which is the very ſame with A, B, and C, and on the rule is two lines the one mark'd S..L. or ſeg: ly and is for ullaging a lying caſk, the other is mark'd S.S. or ſeg: ſt. for ullaging a ſtanding caſk, (ſee example 29 and 30 following). Theſe lines may be conſtructed thus, firſt for the line S. L. Take a caſk which agrees neareſt to the moſt common ones in practice, and ſuppoſe its bung diameter to be divided into 100 equal parts, and plaines paſſing thro' each of thoſe parts will divide the caſk into two unequal parts or ſlices parallel to the axis of the caſk: it is evident if you find the content of the firſt ſlice in gallons, and add it to the content of the ſecond ſlice or ſegment ſo found, ſo you'll have a number for ,01 and ,02 and adding the content of the third ſlice to the laſt ſum you have a number for ,03 &c. by continually adding the laſt content to the former, you'll have a table of meaſures, by which the line S. L. may be laid down. In like manner for a ſtanding caſk, if you ſuppoſe the axis to be divided into 100 equal parts and plaines paſs thro' each of thoſe parts at right-angles to the axis, the caſk by this means will be divided into 100 unequal parts, whoſe contents being ſeverally found, and the laſt always added to the ſum of the preceeding ones, you'll have a table of meaſures for the line S. S. the lines S. L. and S. S. are fitted to ale gallons, becauſe the gallons are moſt common in practice. —— The line A. L. mentioned in example 31. is conſtructed thus, Let s = a ſegment under ſeg: in the table of ſegments, d = the diameter of any cylinder, and l = the length of the ſame cylinder

cylinder, then $\frac{ddls}{282} = m$ the meaſure of a ſlice thereof paral-lel to the axis (Ex. 9. following) in ale gallons, whoſe end is ſimilar to the ſegment *s*; or $\frac{ddls}{231}$ is the meaſure of the ſame ſegment in wine gallons. But if $\frac{ddls}{282} = m$, then $\frac{282}{1} \times m = ddls$, from whence ariſes this proportion on the ſliding-rule, as $\sqrt{\frac{282}{1}}$ on D, is to *l* on C, ſo is *d* on MD on C. So that if 282 be divided by every number under ſeg: in a table of ſegments made to 1000 verſed-ſines, (the table in this book being taken from Mr. *Shirtcliffe*'s Gauging, is only made to 500 or half the circle) and half the logarithm of each quo-tient, taken from the ſame ſcale of equal parts that the line N was taken from, and thoſe diſtances laid on the line A. L. you'll have the line required: ſee example 31, following for its uſe.---On the out-ſide of the ſliders B and C are plac'd two lines, the one numbered from 13 to 36, and the other from ,42 of a gallon 3,60 gallons, which is only as a table to ſhew the content of any cylinder in ale gallons at 1 inch deep, whoſe diameter is between 13 and 36 inches: ſee its uſe in Ex. 21.----- On the fourth and laſt ſide of this rule ſtands 4 lines mark'd ſpheroid, 2d variety &c. which ſee in Ex. 27. Towards the latter end on this rule are ſeveral other marks, as A *g*. for ale gallons, W *g*. for wine gallons, M. R. for malt buſhels round meaſure, M. S. malt buſhels ſquare meaſure, or right-lin'd figures &c. In the foregoing ſection 1, where the common ſliding-rule is uſed; obſerve, that which is done by the ſlider and rule, is done on this rule by the lines B and A, and what is there done by the ſlider and line D, is here done by the line C and D they being the very ſame on both rules, ſo that being obſerv'd, I need ſay no more to the uſe of theſe lines.

Extraction of the Cube Root by the Sliding-Rule.

Rule.] Set the firſt 1 on D to the firſt 1 on E, then againſt any number on E ſtands its cube root on D; for the line D being triple to the line E, againſt any number on E as 8, will on D ſtand a number 2, which 2 is but $\frac{1}{3}$ of the diſtance be-tween 1 and 8, &c.——Ex. *What is the Cube Root of* 2197? Here I ſet 1 on E to 1 on D, then the greateſt number on E is but

but 1000 (viz. the third 1 is 1000 when the firſt 1 is eſteem'd 1 on E, ſee ſect: 1.) therefore I divide 2197 by 1000 (the cube of 10), and againſt the quotient 2,97 on E ſtands 1,3 on D; ſo 1,3 is the cube root of 2,197, and 1,3 multiplyed by 10 (becauſe 2197 whs divided by the cube of 10) gives 13 the cube root of 2197 requir'd.---*Any three numbers being given, to find a fourth which may be to the third, as the cube of the ſecond is to the cube of the firſt.* Let the 1ſt, 2d, and 3d numbers be 2, 8, and 9; by what is ſaid above it is eaſy to underſtand that as 2 on D is to 9 on E, ſo is 8 on D to 576 on E, the fourth number required. Again, if the three given numbers were 2, 80, and 9. Here when 2 on D is ſet to 9 on E, 80 is off the line D, but againſt 8 (viz. $\frac{80}{10}$) on D ſtnds 576 on E, which multiply'd by the cube of 10 (viz.) 1000 (becauſe the root 60 was divided by 10) gives 576000 the term required,

The uſe of the line M D, *with the lines* B *and* A.

This is for gauging malt on rectangular floors, where note, that when the figure 1 on M D denotes 1, then the braſs pin M B over againſt it on A denotes 2150,42 the cubic inches in a buſhel of malt; by which means the leaſt number that can be on A is 100, (viz.) the firſt 1 on A is 100, but if this firſt 1 ought to be 1, then the line A muſt be divided by 100, and when A is divided by 100, then B and M D muſt be divided by two ſuch numbers as being multiply'd together may make 100, that is the diviſor for A, and the product of the diviſors for B and M D being equal, the fourth number found on B will be the content in malt buſhels; The reaſon of this will appear by conſidering the conſtruction of the rule.

Ex. *If the breadth of a couch, floor, ciſtern, &c. of malt be* 56,2 *inches, its length* 270 *inches, and depth* 5,2 *inches how many buſhels of malt is in that rectangle.*

Rule. *Always ſet the length* 270 *on* B *to the breadth* 56,2 *on* M D, *then againſt the depth* 5,2 *on* A *ſtands* 36,7 *the content on* B.

Here that 5,2 may be found on A, the firſt 1 on A may be $\frac{1}{100}$ or 1, let it be 1; then the ſecond M B on A is divided by 100 (viz.) 2150,42 becauſe 21,5042: ſo the length 270 and breadth 56,2 being each divided by 10 (becauſe 10 times 10 is 100 equal to the diviſor of A) will be 27 and 5,62. then ſetting 27 on B to 5,62 on M D, againſt 5,2 on A is 36,7 on the buſhels required.---Again, if the breadth be 72, length

140 and depth 18,2 inches, then if the first 1 on A denote 1, then the second MB 2150,42 on A will be 21,5042, so the divisor of A being 100 the product of the divisors of the length 140 on B, and breadth 72 on MD must be 100, so let each be divided by 10, and so set 14 on B to 7,2 on MD then against 18,2 on A stands 85,3 on B the bushels required.---- Again, Let the length be 1250 inches, breadth 360 inches, and the height 9 inches. Here let the first 1 on A be $\frac{1}{10}$ then the second MB 2150,42 will be 2,15042, so that A is divided by 1030; therefore let 1250 be divided by 10 and 360 by 100 (because 10 times 100 is 1000) so we'll have 125 and 3,6; therefore, set 125 on B to 3,6 on MD, then against 9 on A stands 1884 on B the bushels in that floor, &c. required

This being added to section 1. I presume will be a sufficient explaination of the sliding-rule in all cases, except in measuring frustums or gauging casks, &c. which I shall explain in the following example: *Let it be required to multiply the sum of the squares of* 2, 4, *and* 5 *by* 10, *and to divide that product by* 36? First, when 1 on C is set to 1 on D, then against any number on C stands its square root on D; therefore when you work with the lines C and D, you work with numbers and square roots, so the divisor being 36 its square root 6 must be set on D to the common multiplier 10 on C, the rule thus standing look for the numbers 2, 4, and 5 on D, and against 2 on D stands 1,11 on C, so that the square of 1 multiply'd by 10 and divided by 36 gives 1,11; and against 4 on D stands 4,44 on C, so the product of the square of 4 by 10 divided by 36 is 4,44. Also, against 5 on D stands 7 on C nearly, so the square of 5 multiply'd by 10 and divided by 36 is 7 *fere*; the sum of these 3 is 12,55 as was required.

For if a number of terms must be multiplied by one number, if you multiply them by one at a time, and add all the products together, the sum will be the same, as if the sum of all the terms had been multiply'd at once. ---- Lastly, as in sect. 1st. if the numbers you are to work with be very unequal, so that they cannot be all had on the lines at once, you must use multiplication or division to make them come upon the lines, where observe that in most cases 10, 100, 10000 *&c.* will do either for such multiplication or division, and this may be done easily in the mind by annexing or taking away cyphers, or removing the decimal point, if mix'd numbers, *&c.* And here note, that if the divisor be decreased or any of the other terms in-

increaſed, then the fourth number found will be more than it ought to be; but if the diviſor be increaſed, or either of the other terms decreaſed, then the fourth term will be leſs than it ought to be; as for inſtance, if the difference between the ſquare of 40 and the ſquare of 2, muſt be multiplied by 10 and divided by 36, this being numbers and ſquares muſt be wrought on the lines *D* and *C*, and the ſquare root of 36 being 6; for this 6 take the figure 6 on the line *D*, which ſet to the ſecond 1 on *C* calling it 10 then againſt 2 on *D*, wil ſtand 1,11 on *C*, now the figure 4 on *D* in this caſe being only 4, 40 cannot be had on this line; but if we divide 40 by 10, the Quotient 4 may be had on *D*, againſt which on *C*, ſtands 4,44 which multiply'd by 100, (becauſe the root 40, was divided by the root of 100) gives 444, from which take 1,11 and there leaves 442,89 for the anſwer. —— *Otherwiſe*, If you call the firſt 1 on *D* 10 then the diviſor 6 will become 60, which being ſet to 10 on *C* as before againſt 40 on *D*, will ſtand 4,44 on *C*, which multiply'd by 100 (becauſe the diviſor was multiply'd by 10) gives 444; now in this caſe 2 is of the line *D*, but a cypher being put to 2, againſt 20 on the line *D* ſtands 1,11 as before, which neither be multiply'd nor divided becauſe the firſt and third terms were each multiply'd by the ſame number, *viz.* 10.——In the preceeding ſections, where the ſliding rule is mentioned, I have in moſt caſes mentioned theſe diviſions and multiplications which being compar'd with this explanation, I think it will be needleſs to make repetition any more on this head, and therefore in the following examples I ſhall allways ſhew the uſe of this rule, as if ſuch diviſion, *&c.* were not needful.

The Deſcription and Uſe of the Gauging-Rule.

1. This rule is commonly 4 foot long, and is made to double in 4 joints for convenience *&c.* it hath alſo 4 ſides on which are ſeveral lines call'd diagonals, the one for ale gallons mark'd A G, and the other for wine gallons mark'd W G. Put the end which is cut aſlope in at the bung hole, and let it touch the bottom of the head, (try it's both heads to know if the bung hole be in the middle) and the number cut at the bung, ſhews the ale or wine gallons this caſk will hold. This gives a pretty good gueſs of the *French* wine hogſhead, or

London

London beer barrel, the rule being made for ſuch like caſks, ſo it muſt err in other forms.

2. On another ſide is a line of inches from 1 to 48 decimally divided, and alſo upon the ſame ſide you have *Oughtred's* Gauge Line, being a line of one thirds of areas of circles in wine gallons, by which you may gauge a caſk; thus, take the diameter at the bung which ſuppoſe 7,1, which double and it makes 14,2, to this add the diameter taken at the head which let be 5,8 and the ſum is 20, which multiply'd by the caſk's length 30 inches gives 600, and this divided by 10 is 60 wine gallons the content of the caſk, this holds pretty true in a ſpheroidical caſk.

3. On a third ſide of this rule is a line of equal parts, from 1 to 96, this line with *Oughtred's* Gauge Liue laſt mention'd, do make a table of areas of circles in ale gallons, ſo that if you find the diameter in this line turn up the other face of the rule, and againſt the diameter you ſhall have the area of the circle in ale gallons. As for example, if the diameter of a circle be 19 inches, the area of that circle upon the other edge of oughtred's line, is a little above 1 gallon; alſo, if the diameter be 30 inches, the area is 2,5 gallons, *&c.* and ſo of any other. —— The uſe of theſe lines thus together, is the ſame with that of a table of areas of circles in ale gallons, to be found in ſeveral old books of gauging. —— *Note*, The reaſon for dividing 600 (in ths 2 above) by 10, is becauſe the numbers 1, 2, 3, 4, *&c.* are tenths.

4. On a fourth ſide of this rule is drawn a line of numbers, the ſame with the lines A, B; and C, on the ſliding rule, on which are the gauge points A. G. and W. G. for ale and wine gallons, which works with compaſſes inſtead of a ſlider. As for example, if the length of any caſk or veſſel be 30 inches and its mean diameter 26 inches, then ſet one foot of your compaſſes in the gauge point and extend the other foot to the mean diameter 26, with that extent and one foot in 30 the depth or length, turn'd twice over will at laſt fall upon the content required, which will be 57 ale gallons or 69 wine gallons, accordingly as you uſed the gauge poins A, G, or W, G.

5. With this line of numbers, there is alſo a line of ſegments (as S. L.) for ullaging, whoſe uſe is this, extend from the bung diameter on the line of numbers to 100, or radius on tha line of ſegments, that extent the ſame way

reaches

reaches from the dry inches on the line of numbers, to a reserv'd number on the line of segments, or which is truer, extend from the bung diameter to the dry inches that extent on the line of segments, will reach from 100 to the reserv'd number; then extend on the line of numbers, from 1 to the content of the cask, that extent the same way will reach from the reserv'd number, towhat the lying cask wants of being full, but if you use the wet inches instead of the dry, you'll have the ale gallons in the cask. Thus much for the gauge rule, which at best serves only for a guess.

How to reduce one kind of Measure to another.

In 1 *Gallon of Wine, how many Gallons of Corn Measure?*

Inches Gal. Inches

If 268,8 : 1 :: 231

1

268,8) 2310000 (0,8594 *fere.*

So that a wine gallon is to a corn gallon as 1 to 0,86 *fere*; and if you multiply any number of wine gallons by 0,8594 the product will shew the number of corn gallons.

Note, Eight gallon is a statute bushel, but 8 times 268,8 is 2150,4 which should be the statute bushel of malt, but 2150,42 solid inches being settled by act of parliament, for a malt bashel the corn or malt gallon being $\frac{1}{8}$ thereof, should be $\frac{2150,42}{8,} = 268,8025$, wherein the decimal ,0025 being small is rejected, and 268,8 cubish inches settled for the gallon of corn or malt, the statute malt bushel shoule be an exact cylinder, whose diameters must each be equal to 18 $\frac{5}{7}$ inches, and its depth 8 inches, whose solid content is so near 2150,42 solid inches that the difference is not regarded, and if you would have a bushel of any form, consult the following questions, sect. 10.

In 5768 *Gallons of Beer, how many Barrels?*

32) 5768 (180 Barrels.

8) 8 (1 Firkin.

In 0,5768 gallons of ale, how many qnarts, pints, gills, &c. ?

0,5768
4 qnarts in one gallon.

2,3072
2 pints in one quart.

,6144
2 gills in one piut.

1,2288
2 noggins in one gill.

,4576 answer 2 quarts, 0 pints, 1 gill, 0 noggins.

In 2520 gallons of wine how many tuns ?

252) 2520 (10 answer
250

[0]

In 67854,2 solid inches, how many gallons of ale, and rundlets of wine ?

282) 67854,2

240,61773 quotient.
4

2,47092
2

,94184
2

1,88368
2

1,76736

4158) 67854,2

16,318949 quotient.
18

2551592
318949

5,741082
4

2,964328
2

1,928656

	G.	Q.	P.	Gill	Nog:		Rund.	G.	Q.	P.
Answers	240	2	0	1	1,76736	—	16	5	2	0

See these multipliers and divisors in the following tables.

Inches.
35¼

A TABLE *of* Beer Measure.

That is, 282 solid inches make 8 pints, or 4 quarts or 1 gallon *&c.*

Pints.	Quarts.	Gallons.	Firkins.	Kilderkins.	Barrel.
70½ — 2					
282 — 8 — 4					
2256 — 64 — 32 — 8					
4512 — 128 — 64 — 16 — 2					
9024 — 256 — 128 — 32 — 4 — 2					

A TABLE *of* Ale Measure.

Pints.	Quarts.	Pottles.	Gallons.	Firkins.	Kilderkins.	Barrels.	Inches solid.
1 —							35¼
1 —	2 —						70½
1 —	2 —	4 —					141
1 —	2 —	4 —	8 —				282
1 —	8 —	16 —	32 —	64 —			2256
1 —	2 —	16 —	32 —	64 —	128 —		4512
1 —	2 —	4 —	32 —	64 —	128 —	256 —	9024

Inches.
28⅛

A TABLE *of* Wine Measure.

Pints.	Quarts.	Gallons.	Rundlets.	Hogsheads.	Tricons.	Pipes.	
57¼ — 2							
231 — 8 — 4							
4158 — 144 — 72 — 18							
14553 — 504 — 252 — 63 — 35							
19404 — 672 — 339 — 84 — 4½ — 1½							
29106 — 1008 — 504 — 126 — 7 — 2 — 1½							
58212 — 2016 — 1008 — 252 — 14 — 4 — 3 — 2 — 1 Ton							

Thefe

Theſe are Mr. *Everard's* tables, to which add that 268,8 ſolid iuches make 1 gallon of corn, 2150,42 ſolid inches 1 buſhel of malt ſtatute, and 217,6 ſolid inches make 1 iriſh gallon, ale, wine and oyl. So that let a ſolid be of any form whatſoever, if the ſolidity of its cavity be 282 inches, it will hold juſt one ale or beer gallon; or if its ſolidity be 231 inches, or 268,8 inches it will hold juſt 1 gallon of wine, or 1 gallon of corn, *&c.* from whence it appears, that if the ſolidity of the cavity in inches of any ſolid whatſoever, be divided

by { 282 / 231 / 268,8 } the quotient will be the anſwer, { ale or beer / wine - - - - / corn - - - - } gallons.

The diviſor for malt buſhels is 2150,42.

And therefore to ſhew the art of gauging, is in effect no more but to repeat ſolid meaſure, (ſect 7.) in which ſection the rules being plainly laid down for all kinds of ſolids, I ſhall in the following examples quote ſuch rules when occaſion requires; but to keep up the faſhion, I muſt firſt ſhew how to find the content of any ſuperficial figure in gallons, and in ſuch caſes, you muſt conſider the figure to have one inch of depth, for a plane can hold no liquor. And becauſe multiplication is eaſier than diviſion to moſt perſons, you may by what is done in diviſion of decimals, turn all the foregoing diviſors into multipliers. So 1,000, *&c.* divided

by { 282 / 231 / 268,8 } gives { ,003546 / ,004329 / ,0037202 } multiplier for { ale or beer / wine - - - / corn - - - } gal.

The multiplier for 2150,42 is ,00046502 corn or malt buſhels.

So that whether you divide by 282, or multiply by ,003549 the quotient and product will be equal, and ſo for the reſt. But ſince circular areas are not had, untill multiply'd by 0,7854, or more exactly by 0,785398; if therefore you make 0,785398 your dividend inſtead of 1,000, *&c.* you will have multipliers for circular areas, or if you divide the foregoing diviſors 282, 231, 268,8, *&c.* by 0,78539S you will have diviſors for circular figures, thus,

282	,785398	,002785	a mult. for	A. or B.	for circular figures.
231	,785398	,003399		W. gals.	
,78539	282	359,05	a divi. for	A. galls.	
,78539	231	294,12		W. gals.	
,78539	2150,42	2738		M. buſhel	

Now

Now to gauge by the ſliding rule. If you take the ſquare root of any of the foregoing diviſors, and mark that number upon the line D, it will be a conſtant gauge point for that ſort of meaſure, ſo

gauge point for right - lin'd figures is { 16,79 / 15,19 / 46,36 } for { ale. / wine. / malt.

the gauge point for circular figures is { 18,95 / 17,15 / 52,32 } for { ale. / wine. / malt.

Now becauſe circular figures are moſt common in gauging, the above numbers 18,95 and 17,15 are mark'd on the line D, with A. G. for ale gallons, and W. G. for wine gallons, but by theſe rules you may find the gauge point to any number, and ſo mark that gauge point on the line D, in the ſliding-rule your ſelf, if it be not on already. Alſo obſerve, that in uſing the above multipliers and diviſors for circular areas, it ſaves you the labour of uſing ,7854, becauſe this decimal ,7854 you ſee is conſidered in ſuch multipliers and diviſors, and in like manner, when you come to figures which have ſeveral multipliers or diviſors, as cones, fruſtums, *&c.* you may reduce them all into one, and the work will be much abreviated.

How to gauge a Square.

Ex. 1. *What's the area of a ſquare whoſe ſide is* 7,6 *inches, in ale and wine gallons, at* 1 *inch deep?*

7,6
7,6

57,76 area by rule 2d ſect. 5th.
,003546 multiplier for ale.

,20482396 ale gallons.
4

,81929584 quarts.
2

1,63859168 pints.
2

1,27718336 *Anſwer*

Or thus by diviſion.

282) 57,79 (,2048 A. G.
— 1,22
64

,249856 wine gall.

G. Q. P. *l.*
0 : 0 : 1 : 1,27 &c. ale meaſure.

Thus

Thus you see it is the same thing whether you multiply or divide, and here observe, that as 231 is to 282 so is 1 to 1,22, therefore if you multiply any number of ale gallons by 1,22 the product will be wine gallons, which is easier than to multiply or divide the area of the figure in inches by the factors for wine measure,

By the *Sliding-Rule.*

Set the gauge point (square measure) on the line D to 1 on the slider, then against 7,6 on the line stands ,205 on the slider the content in ale, if you set 16,79 to 1. But if you set 15,19 to 1 you'll have ,25 the answer in wine gallons.

To gange a Long Square.

Ex. 2. *Required the content of a long square whose length is* 100,5 *inches, breadth* 20 *inches in ale gallons?*

100,5		,003546	multiplier
20		2010	
2010,0	area, rule 2d sect. 5th	7,127460	ale gal. answer,

By the *Sliding-Rule.*

Set 282 on the slider to 100,5 on the rule, then against 20 on the slider stands 7,13 on the rule the content required. Or set 16,79 on D to (a mean proportional between 100,5 and 20) 44,83 on C, then against 1 on D stands 7,13 on C as before.

To gauge a Triangle.

Ex. 3. *Let the base of a triangle be* 260 *and the perpendicular* 110 *inches, what's the content in corn gallons?*

130		
110		By the *Sliding-Rule.*
14300	area.	Set 282 on B to 130 on A, then
,00372	multiplier.	against 110 on B stands 53,2
53,19600	corn gallons, answer.	*fere*, on A the corn gallons required.

To gauge a Trapezia, *as a* Cooler, Floor *of* Malt, *&c.*

Ex. 4. *In a trapezia the diagonal* 2820 *I. the perpendiculars* 105 *and* 80 *inches, required the content in wine and corn gallons?*

Operation.

Operation.

105	260850	area in inches.
80	,00372	multiplier.
185	970,36200	corn gallons answer
1410		

231) 260850 area by rule 7 sect. 5.)
1129,23 wine gallons.

By the *Sliding-Rule.*

1. For wine, Set 231 on B to 1410 on A, then against 185 on B, stands 1129,2 on A the content in wine gallons. 2. For corn, Set 268,8 on B to 1410 on A, then against 185 on B stands 970,36 on A the content in corn gallons. Note, In multiplying for corn gallons, I omit the two last places in the factors because they are but small. I think it needless to give any examples to the rhombus, rhomboides, regular polygons, irregular polygons, &c. for few such forms are to be met with in gauging, and if by chance they be, you have rules in sect. 5th, for such like figures.

Note, When you gauge such a surface as the last, stretch a diagonal line between the longest corners, and let it lie fast, then with a ruler or small cord, take the nearest distance between the opposite angles, and the said line, so you'll get the perpendiculars.

How to gauge a Circle.

Ex. 5. *If the diameter of a circle be* 40 *inches, what is its area in ale, and wine gallons? — see* Ex. 21.

40 } diam.	,002785 A. G.	,003399 W G.
40 }	1600	1600
1600 sq. diam.	4,456002	5,438400 answers

Here the area of the circle (as also in the following oval) in gallons, is as soon had as in the dimensions taken (rule 12 sect. 5.) see the work

By the *Sliding-Rule.*

Set the gauge point (A. G. for ale gallons, or W. G. for wine gallons) on the line D, to 1 on C, then against the diameter on D 40, stands the content on C 4,456 ale or beer gallons, or 5,44 *fere*, wine gallons.—*Otherwise*, Set 359,05 on B to 40 on A, then against 40 on B, stands 4,456 on A the answer as before. But for wine, set 294,12 on B, &c.

How

How to gauge an Oval, *or* Ellipsis.

Ex. 6. *If the tranſverſe diameter be* 72 *inches, and the conjugate diameter* 50 *inches; what is the area in ale and wine gallons?*

72	,002785 for A. G.	,003399 for W.G.
50	3600	3600
3600 inches.	10,126000 anſwers.	12,236400

By the *Sliding-Rule.*

Set (for ale gallons) 359,05 on B, to 72 on A, then againſt 50 on B ſtands 10,136 on A, the ale gallons requir'd. Otherwiſe, Set the gauge point (A.G.) 18,95 on D, to 1 on C, then againſt 60 (a mean proportional between 72 and 50) on D, ſtands 10,126 on C, as before. By the ſame method you may find the content in wine, by uſing the diviſor, and gauge point for wine gallons.

How to gauge a ſegment of a Circle.

Ex. 7. "Suppoſe the ſegment of a circle, whoſe verſed "ſine is 10 inches, and the diameter of the whole "circle 50 inches: Required the area of the ſegment "in ale gallons?"

To work this ſee rule 17. ſect. 5. under V. S. againſt 200 the quotient ſtands ,111823 under ſegment. multip. 2500 ſq. diameter.

279,557500 ar. rule 14. ſect. 5.
,003546 mul. for A. G.

,9913108950 *Anſwer.*

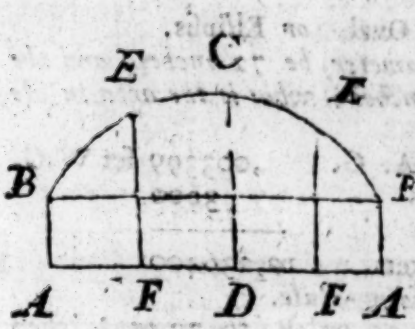
50)10,000(200 inches.

By the *Sliding-Rule* Having divided the verſed ſine (with 3 annex'd cyphers) by the diameter, and by the quotient 200 under V. S. found ,1118 under ſegment. Set 282 on B to 2500 on A, then againſt 0,1118 on B, ſtands 0,9913 on A, the anſwer.

How to gauge a Segment *of any kind, or any* Curve Lin'd *figure.*

Ex. 8. "Let *A B C B A* be ſome curve lin'd ſpace, whoſe ſubtence *A A* (= *B B*) is 20, the leaſt breadth *B B* is 5, the greateſt breadth *C D* is 12, and a breadth *E F* in the middle (between *A B* and *C D*) is 10, (all in inches) what's the area in wine gallons?"

Rule.



RULE.

To the ſum of the greateſt and leaſt breadths, add four times the breadth in the middle between them, this multiply'd by $\frac{1}{6}$ of the length, gives the area.

Theorem 57.

5	AB.
12	CD.
40	EF 4.
—	
57	ſum.
20	AA.
—	
1140	
,000511 = $\frac{1}{6}$ of ,003546.	
,673740	ale gallon.
1,22	
,82196280	wine gallons.

By the *Sliding-Rule*.

Set 1692 (viz. 6 times 282) on B to 57 on A, then againſt 200 on B (for 20 is off) ſtand 6,737 on A, ſo the anſwer is ,6737 ale gal. &c. for wine.

Or, Set 1 on B to ,6737 on A, then againſt 1,22 on B ſtands ,822 on A the wine gallons.

Note, If you think 3 breadths be not exact enough for your purpoſe, you take 4 and work by rule 22, ſect. 5th.

By theſe examples (and thoſe in ſect. 5th) you may gauge any ſurface at one inch deep; and conſequently any equal and ſimilar bas'd ſolid, for having the content at one inch deep, that content multiply'd by the whole depth in inches gives the content of the whole ſolid. And by the ſame method you may alſo find the content of any ſuch ſolid at any propoſed depth. For the area at the ſurface (found as before) multiply'd by any depth in inches gives the content at that depth, as for example 9. *If a cylinder lye parallel to the horizon whoſe length is* 100 *inches, the diameter of each baſe* 50 *inches, and ſo much liquor in this ſolid, as that pntting a ſtick perpendicular to the horizon the wet inches may be* 10. By example 7, the content of this ſegment is found to be 0,9913109 here, which multiply'd by the length 100 inches, gives 99,13109 for the ale gallons in this ſegment of a cylinder. Again,

Ex. 10.

Ex. 10. If there be a floor of malt in form of a trapezia, whose content is there found 970,392 corn or malt gallons at 1 inch deep. Now if the depth of this floor be 10 inches, then the content thereof will be 9703,62 corn gallons, the like is to be underſtood of any ſuch like figures; which thing being obſerved, it will be needleſs to give any examples to the cube, priſm and cylinder.

To gauge the Fruſtum of any Pyramid *or* Cone.

To do this ſee rules 4, 5, 6, 7 and 8 in ſect. 7th; but the laſt of theſe rules being a general one, I ſhall only inſiſt thereon.

Ex. 11. *Let the ſame figure and dimenſions (in inches) be given as in* Ex. 10. Sect. 7. *to find the content in ale and wine gallons?*

13 AB.	169	ſq. AB.
8 *ab*	64	ſq. *ab*.
—	441	4 times ſq. mid. ſide *viz.* ſq. 21
21 twice a mid. ſide.		
	674	
	,000591	$= \frac{1}{6}$ of ,003549.
By the *Sliding-Rule.*		
Set 41,13 on D to 20	,398334	
the length on C, then	24	CD length.
againſt 13, 8 and 21,		
on D ſtands 3 ſuch	9,560016	*Anſwer* ale gallons.
numbers on C as being	1,22	ſection 9th, example 1ſt.
added together makes		
9,56 content.	11,5672	*Anſwer* wine gallons.

Note, If you multiply 282 by 6 the ſquare root of that product will be 41,13 which will be a conſtant gauge point for the fruſtums of ſquare pyramids in ale meaſure. In like manner you may find gauge points for wine, corn, &c. for ſuch fruſtums, the reaſon is; firſt, by rule 8 ſect. 7th, you are to multiply the above ſum by $\frac{1}{6}$ of the height or length, or by the whole height and divide by 6 for the ſolidity in inches, which divided by 282 gives the content in ale gallons: Now theſe two diviſors 6 and 282 being multiply'd together makes 1692, which is ſuch a diviſor as will perform all at once dividing, &c.

Ex. 12. " There is a fruſtum of a rectangular pyramid, " the length of the greater baſe is 27 inches, its breadth " 12 inches, the length of the leſſer baſe is 18 I. its " breadth 8 I. and the length of the fruſtum 100 inches. " How many ale gallons will it hold? "

First 18 + 27 = 45 twice the length in the middle, and 8 + 12 = 20 twice the breadth in the middle.

27	18	45
12	8	20
324 }	144	900
144 }		
900	4 times area middle.	
1368	ſum.	
100	axis or length.	
136800	ſolidity 6 times.	
195000,0	inverted.	

80,8488 ale gallons *Anſwer.*

By the *Sliding-Rule.*

First, Find 18 a mean proportional between 27 and 12 the length and breadth of the greater baſe; and alſo 12 a mean proportional between 18 and 8 the length and breadth of the leſſer baſe, which mean proportionals 18 and 12 added together gives 30 twice a mean porportional in the middle.

Then, ſet 41,13 on D to 100 on C, and againſt theſe 3 means 18, 12, and 30 on D ſtands 19,15, 8,51 and 53,19 which added together is 80,15 the ale gallons required.

Ex. 13. " There is a fruſtum of a cone, whoſe length or " depth is 100 inches, the diameter at the greater " baſe 18 inches, and the diameter at the leſſer baſe " 12 I. conſequently the diameter in the middle will " be 15 inches. How many ale or beer gallons will it " hold? "

324 } ſq. diameter { greater } baſe.
144 } { leſſer }

900 four times the ſquare of the diameter in the middle.

1368	ſum.
100	length.
136800	
,000464 = $\frac{1}{8}$ of ,002785.	

63,475200 ale gallons.

By the *Sliding-Rule.*

If you multiply 359,05 by 6 the product will be 2154,3 whoſe ſquare root 46,41 is a conſtant ale gauge point for all conical fruſtums; and alſo the ſquare root of the product of 294,1 by 6, and of 2738 by 6, will be a conſtant gauge point for wine gal. and malt buſhels, of all ſuch fruſtums.

Se

Set 46,41 on D to 100 the length on C, then againſt the two diameters 18 and 12 and their ſum 30 on D ſtands 14 9, 6,7, and 41,9, which added together gives 63,5 ale gallons for the anſwer.

Ex. 14. " There is a fruſtum of an elliptical cone, whoſe " length is 100, the tranſverſe and conjugate diameters " of the greater baſe 27 and 12, aud of the leſſer baſe " 8 and 18, all in inches: What's the content in ale " gallons.

$$\left.\begin{matrix}324 = 27\\144 = 18\\900 = 45\end{matrix}\right\} \times \left\{\begin{matrix}12\\8\\20\end{matrix}\right.$$

1368 ſum.
100 length.

136800
,000464 = $\frac{1}{8}$ of ,002785.

63,475200 ale gallons.

By the *Sliding-Rule.*
Firſt, find 18 a mean proportional between 27 and 12, alſo 12 a mean proportional between 18 and 8. Then ſet 46,41 on D to 100 on C, then againſt the two mean proportionals 18 and 12, and their ſum 20 on D ſtands 14,9, 6,7 and 41,9, which added together is 63,4 ale gallons requir'd.

To gauge any Globe *or* Sphere.

Ex. 15. *If the axis of a ſphere be* 100 *inches, how many ale gallons will it hold?* See rule 15, ſect. 7th.

1000000 cube of the axis 100.
,00185663 = ,5236 × ,003546.

1856,68 ale gallons.

By the *Sliding-Rule.*
If you divide 282 by 0,5236 the ſquare root of that quotient will be 23,2. Set 23,2 on D to 100 the axis on C, then againſt the axis 100 on D ſtands 1859,68 on C, the ale gallons requir'd.

How to gauge any Segment, as a Bowl, &c.

Ex. 16. *Let ABD be a bowl, bottom of a kettle, &c. whoſe depth CD is* 15, *diameter AB* 60 *at the baſe, and EF* 40 *a diameter in the middle. What is its content in ale and wine gallons?* See rule 29, ſect. 7th.

By the *Sliding-Rule.*] Set 46,41 on D to 15 the depth on C, then againſt the diameter of the baſe 60 and twice the diameter in the middle 80 on D ſtands 25,1 and 44,5 which added together gives 69,6 the content in ale gallons.

Operation.

		Operation.
3600	sq. AB.	
6400	4 times square EF, or twice EF 40 squared	
10000	sum.	
15	CD.	
150000		
,000464	$\frac{1}{8}$ of ,002784.	
69,6 - - -	ale gallons.	
1,22		
84,912	wine gallons *Answer.*	



Thus you may see all the difference between solid measure and gauging, is only on the account of the divisors, &c. and you take the solidity of any vessel in inches, you may always divide by 282, or multiply by ,003546 for ale gallons, &c.

How to gauge MALT.

The duty upon malt is charged by the *Wincheſter* bushel, which contains 2150,42 solid inches; so that what form soever the surface of the malt be in, whether floor, cistern, couch, &c. find the content of that form at one inch deep in malt bushels, which multiplied by the depth in inches, gives the whole content in bushels. And here observe, that malt being a dry substance, will not like liquor have an even surface, also the bottom of the floor, &c. may be uneven; Now to remedy all this you must take the depth or thickness of the malt, in 6, 8, or 10 places or more if needful, and add all those depths, which divided by the number of depths, gives the mean depth.

Ex. 17. *There is a rectangular floor of malt, whose length is* 72 *inches, breadth* 48 *inches, and*

the { 1, 2, 3, 4, 5 } *depth is* { 4,7 ; 5,4 ; 5,6 ; 4,9 ; 4,4 } *inches, requir'd the content?*

Number of depths, 5) 25,0 (5 *mean depth.*

Operation.

Operation.

72 length.
48 breadth.

3456 area of the ſurface.
,000465 mult; for malt buſh.
1,607040 buſhels at 1 I. deep. [right-hand fig.

By the *Sliding-Rule*.

Set the gauge point MS 46,36 on D to the mean depth 5 on C, then againſt 58,7 (a mean proportional between 72 and 48) on D ſtands 8,04 fere, on C, the anſwer in malt buſhels. *Otherwiſe*, Set the length 72 on B to the breadth 48 on MD, then againſt the depth 5 on A ſtand the content 8,04 buſhels on B.

Ex. 18. *There is an eliptical ciſtren of malt, whoſe longeſt diameteo is 72 inches, the ſhorteſt diameter 48 inches, and the mean depth 5 inches: How many buſhels of corn or malt is therein?*

72 tranſverſe } dia.
48 conjugate }

3456
,000365 multiplier for malt B.
1,260440 B. at 1 inch deep. [circular fig.
5 mean depths

6,302200 malt or corn buſhels.

By the *Sliding-Rule*.

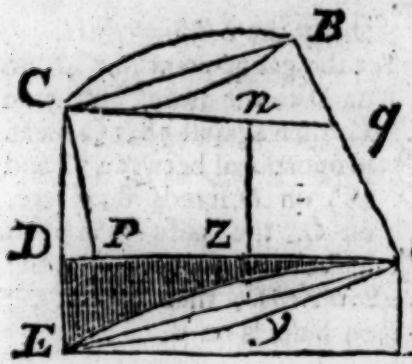
Set the gauge point MR 52,32 on D to the mean depth 5 on C, then againſt 58,7 on D, (a mean proportional between the two diameters 72 and 48) ſtands 6,3 on C, for the malt or corn buſhels required.

To gauge any Tun, Tub, Back, Cooler, *&c.*

Theſe veſſels are of various ſorts ſuch as fruſtums of pyramids, fruſtums of cones, priſmoids, and cylindroids, all of which are meaſured in ſection 7th; ſo in this place I ſhall only ſhew the inclining of ſuch veſſels, (viz.) to find how much liquor they hold at every, or any inch of depth, which being known and ſet down in a table, you may by putting a ſtick into the veſſel, and ſeeing the wet inches, know by your table how much liquor is in the ſaid veſſel. And to make this method general and plain, obſerve the following definitions,

1. When the bottom of any ſuch veſſel is parallel to the horizon, or exactly level, it is ſaid to be an upright veſſel or tub, but otherwiſe, an inclined one.

2. What



2. What liquor is requir'd to cover every part of the bottom of an inclined tun, is called the drip, or fall of the tun.

3. That horizontal plane *DG* that just touches the highest point of the bottom at *G*, is called the horizontal base; and another *Cg* that would just touch the lowest point *C*, of the top or end, the horizontal end.

4. If from *C* any point in the horizontal end, a perpendicular *CP* be let fall on the horizontal base *DG*, it is call'd the height of the tun.

5. The ends of tuns are said to be parallelly-posited, when the length of the one end is either parallel to the length or breadth of the other end, and also the breadth of one end parallel to either, &c. And if it be an elliptical tun, we may for brevity's sake, call the transverse the greatest length, and the conjngate the greatest breadth of the bases or ends.

EX. 19. *Let there be an upright parallel posited tun, whose bases are ellipses, the top* 65, *breadth* 60, *length at the bottom* 110, *breadth* 100, *and the whole depth* 12, *all in inches, to find the content at every inch deep.*

RULE.

First, to find the diameters of the ellipsis at one inch deep, it will always be, (see the *fig.* to *theo.* 20.) 'as the height of the tun 12 is to 45 the difference between the lengths 110 and 45 of the two bases, or to 40 the difference between their breadths 100 and 60, so is any proposed height 1 to 3,75 the difference between the lengths at 12 and 11 inches height, or to $3\frac{1}{3}$ the difference between the breadths at 12 and 11 inches height.' Now these differences being found for 1 inch, because the proportion holds for any depth, you may by continually adding 3,75 the difference of the lengths to 65 the lesser length have all the lengths, and by adding $3\frac{1}{3}$ to 60 have all the breadths, as in the 2d and 3d columns in the following table. And thus having found all the dimensions you may by Ex. 16 sect. 7, find all the contents as set down in the 4th column, which

which respectively taken from 238,9595 the whole content of the tun leaves the content in the 5th column. But having found the first 3 contents 11,4884, 24,2766, and 38,4340, you may with much more ease find the following contents, (than by the foresaid Ex. 16, Sect. 7th.) For put A = 11,4884 the first content, B = 24,2766 the 2d content, C = 38,3440 the 3d content, and D, E, F, G, &c. = the 4th, 5th, 6th, 7th, &c. contents, Then,

$$D = : \frac{A}{1} + 3 \times \frac{C - B}{3 - 2} : \times 4 = 54,0305$$

$$E = : \frac{B}{2} + 3 \times \frac{D - C}{4 - 3} : \times 5 = 71,1355$$

$$F = : \frac{C}{3} + 3 \times \frac{E - D}{5 - 4} : \times 6 = 89,8187$$

$$G = : \frac{D}{4} + 3 \times \frac{F - E}{6 - 5} : \times 7 = 110,1497$$

Inches from the Top.	The Content at
4	
5	
6	
7	

The TABLATURE *for Inching an upright and parallelly-posited Tun, whose Ends are Ellipses.*

Dry Inches.	Lengths	Breadths	Cont. from	Cont. from	Wet Inches.
0	65	60	0	238,95	12
1	68¼	63⅓	11,48	227,47	11
2	72½	66⅔	24,27	214,69	10
3	76¼	70	38,43	200,52	9
4	80	73⅓	54,03	184,93	8
5	83¾	76⅔	71,13	167,82	7
6	87½	80	89,81	149,14	6
7	91¼	83⅓	110,15	128,81	5
8	95	86⅔	132,20	106,76	4
9	98¾	90	156,03	82,92	3
10	102½	93⅓	181,72	57,23	2
11	106¼	96⅔	209,34	29,61	1
21	110	100	238,96	0	0

See the Rule at PROBLEM 180, PART I.

The

The use of this table is this, suppose you come at any time to this tun, and find the wet inches to be 9, then looking for 9 under wet inches, against it, under content from the bottom you'll find 200,52, and under content from the top you'll find 38,48.-----So then there are in the tun 200,52 ale gallons, and it wants 38,43 ale gallons of being full, &c, of any other.

But all such tuns commonly lean a little, and also have a certain point fixt for the dipping place, which is such a point as can be come at with ease; now to shew how such inclined tuns may be inched: Let it be supposed that we have just now inched the tun *gGDC*, whose height *CP* is 12 inches and the several contents as in the above table, and let it be required to inch the inclining tun *GBCE*, or rather *GgCDE*, because the plane *gC* being parallel to the horizon, the vessel can hold no liquor above the the point *g*, and so the hoof *gBC* need not be regarded; suppose the dipping place to be at *n*, and let *ny* the distance between the bottom of the tun and *GD* the horizontal base be 2 inches, and let the hoof *GDE* or quantity of liquor which just covers the base *GE* be 15 ale gallons, so that when the wet inches are 2, there is 15 gallons of liquor in the tun.----- Now in the above table if instead of 0 at the bottom of the 6th column, you write 2 and number the rest 3, 4, 5, &c. to 14, where 12 stands at the top, you will have a column for the wet inches answering this inclined tun; also writing 15 instead of 0 at the bottom of the 5th column, and so adding 15 to each of the numbers above the said 0 you'll have a column for the content at 2, 3, 4, 5, &c. wet inches; so 15 added to 29,61 you'll have 44,61 for the content at 3 wet inches, also 15 added to 283,95 gives 253,95 for the content at 14 wet inches, &c. which is the content of the whole tub, This is so plain I think it needless to give an example. Note, you may find the content of the hoof *GDE* by its rule sect. 7, or as some gaugers, measure it with water and an ale pint, &c.------Also backs, or coolers oft times have their bottoms large in respect of their depths, that so the liquor put into them may the sooner cool, and these bottoms being large are often very uneven, which if they be you must take the depths or dips in several places, and (as in Ex. 17.) get a mean depth, which done fix such a point in the cooler for a dipping place, as that the depth there taken may be just equal to this mean depth, but if this cannot be done you must make a just allowance for the difference. Such large bottomed vessels

veſſels are commonly taken at every tenth part of an inch, which is called tenthing; and if they be ſquare priſms (as often they are) it is very eaſy to do, for you need but find the content by the foregoing examples at one inch deep, (whatſoever be the forms of the ſarface, if the area at the bottom and top be equal) and $\frac{1}{10}$ of the area, will be the area at $\frac{1}{10}$ of an inch deep, and ſo on for any uumber of tenths, in ſuch caſes you need have no table of contents, *&c.* for the area at the ſurface being known you may ſoon multiply that area by the wet inches, and ſo know what liquor is in the tun, *&c.* as for

Ex. 20. *Let there be a Back reſembling a* Parallelopipedon, *whoſe length is* 140, *breadth* 120 *inches, and the depths taken at* 10 *ſeveral places as ſet down below, where the ſum is* 48,9, *which divided by* 10 *the number of depths, gives* 4,89 *for the mean depth. To find the content at* 1 *inch deep?*

5,1
4,8 — 140 = length.
5, — 120 = breadth.
5,2
4,6 — 282) 16800 area of the baſe in inches.
4,9
4,5 — 59,57 content at one inch deep in A. gallons.
4,7 — 0,7 propoſed depth.
5,2
4,9 — 41,999 content at 0,7 inch deep.

48,9 inches.

Again, ſuppoſe you come ſometime to this cooler, and find the depth 1,2 inches,

Then 59,57 area at 1 inch deep in ale gallons.
1,2 inches, mean depth propoſed.

71,484 ale gallons at 1,2 inches deep.

Aud thus you may find the content at any depth you pleaſe.

Note, In ſuch veſſels, there is always a point (the eaſieſt to come at) in the edge of the veſſel mark'd for the conſtant dipping place, and if in this example you find the wet inches 5 at the dipping place, then, from 5 inches take the mean depth 4,89 inches, and there leaves 0,11 inches; ſo that you

muſt always take $\frac{11}{100}$ of an inch from the wet inches, taken at this dipping place, and multiply the foreſaid content 59,57 by the remainder for the true content of worts, or liquor in the cooler; but if the mean depth exceed the wet inches at the dipping place, then the difference muſt be added to the wet inches ſo taken, *&c.* as your ſenſe may direct you better than a multitude of words. This will do for backs, coolers, *&c.* whoſe baſes are like, and equal figures and their ſides ſtreight, but if they be otherwiſe, you muſt find the content to every inch deep (by Ex. 19. if you would be very exact) but the common way is to take or find the dimenſions in the middle of every inch, (but ſome take it in the middle of 2 3 or 4 inches) and ſo get a mean area, which multiply'd by 1 inch gives the content for 1 inch deep, and ſo proceeding with every inch of depth, you'll have a table as in Ex. 19. The area of any circle in ale gallons at one inch deep, may be had from the underſide of the ſliders in the ſliding rule, by inſpection, if the diameter of the circle be any number of inches, and tenth parts of an inch, between 12 and 36 inches, thus,

EXAMPLE 21.

If the diameter of a Circle *be* 13,4 *inches, what is its content in ale gallons?*

1. Draw out the ſlider, and look in the underſide on the lower line for 13,4, and againſt it on the upper line you'll find 0,5 ale gallons, for the anſwer. Do the like for any other number of inches. Theſe lines are calculated by Ex. 5. the lower line is divided into inches and tenths of an inch, and the upper line into tenths and hundreds of an ale gallon, and are very uſefull in inching cylinders, and conical tuns, *&c.* for if the tun be above 12 and under 36 inches diameter, you may by drawing out the ſlider, or ſliders, within the tun, take its greateſt breadth or diameter, and then by looking the under ſide of the ſlider, you have both the diameter in inches, and content at 1 inch deep in ale gallons. Theſe lines and their uſes are ſo plain, they need no farther explination, only, if you inch a conical tun, *&c.* to take the diameter at, or in the middle of every inch, which eſteem as a mean diameter, and ſo get a table of depths and contents, as above directed.

To gauge curve-lin'd Tuns, *as* Coppers, Stills, *&c.*

Rule. Take two diameters (at right angles to each other) at the top *AB*, two at the bottom *gg*, cloſe to *H* the top of the crown, two in the middle *K* between the top of the copper,

and top of the crown, and the diſtance or height *GH*, then work with theſe as directed in rule 8. ſect. 7. to which add the content of the crown (viz. as many gallons to cover the crown, found by Ex. 16. or by meaſuring with water) and you'll have the conrent required.

But if the veſſel to be very large, and you doubt this method will not come ſo near good as you could wiſh, you may take 4, 5, or 6 croſs diameters, and work as directed in the table of ordinates prob. 16,9 ; or by this rnle, gauge the veſſel at twice or thrice, but once as above taught will be exact enough for commou practice : the common way to gauge theſe tuns, is to take the area in the middle ef every 6 inches, or in the middle of 2, 4, 8, *&c.* inches if you pleaſe, and this area multiply'd by its height, gives the content of that 6 iuches nearly, and thus having taken the content of every 6 inches and add them altogether, you'll have the content of the copper above the crown nearly ; take the following examples both ways.

EXAMPLE 22.

' Let the depth *GH* be 30 inches, top diameter *AB* 115 inches, bottom diameter *qq* 96 inches, middle diameter 105,28 inches ; to ſind the content by the above rules and the diameters as the middle of every 6 inches, as in the table to ſind the content the common way, ſuppoſe 92,8 gallons, cover the crown, and the copper circular ? '

Parts of the Depth.	Diam. in the middle of every 6 inches.	Con. of every 6 inches in ale gallons.
6	113,1	213,72
6	109,2	199,26
6	105,28	185,16
6	101,36	171,66
6	97,33	158,22
Sum of *AqqB* - - - - -		928,02
Cover the Crown *CAD*		92,8
Coppers Content - - -		1020,82

Theſe contents in the third column, are eaſily ſound by the ſliding-rule thus, ſet the gauge point A. G. on D to 6 the common depth on C, then againſt the diameter in the ſecond collumn on D ſtands reſpective contents on C, as per third columa, ſo againſt 109,2 on D, ſtands 199,26 on C, *&c.* which added altogether and to 92,8, gives 1020,82 ale gallons for the content required. And becauſe a diameter taken in the middle of 6 inchss, *&c.* is not the true mean diameter, and the

the sides of copper curving, *&c.* This method must give the content too little, consequently the general rule rule 8, sect. 7, is nearer truth, for you see it gives the content 1022,327 *&c.* ale gallons.

$$\left.\begin{matrix}13225\\44335{,}41\\9216\end{matrix}\right\} = \left\{\square \left\{\begin{matrix}AB\ 115\\\text{twice } CF\ 105{,}28\\qq\ 96\end{matrix}\right\}\text{ add.}\right.$$

66776,41

30 = *GH*

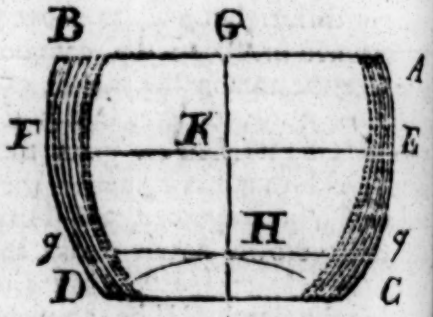
2003292,3

,000464 = $\frac{1}{9}$,002785

929,5276272 cont. *AqqE*

92,8 crown add

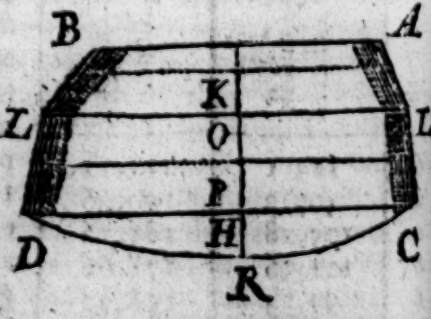
1022,327 cont. segm.



Note, From the content of *CqqD* take *CHD*, and what leaves is call'd the content of the copper.

Ex. 23. *Let ABCRD be an elliptical based* Copper, *whose greatest bulge or widness is at LL below the top AB, and the crown CRD to bend outward ?*

Such coppers or vessels as these, whose greatest cavity is neither at top nor bottom, must be measured at twice by taking them at two frustums, thus, take two cross diameters 89 and 89,5 at the greatest bulge *LL*, two at *G* 80,5 and 80,8 the top, and two at *K* 85,5 and 86 the middle between *G* and *O*, by which and the length *GO* find the content of the frustum *ALLB*; then take the two cross diameters 83 and 83,5 at *H* where the crown begins, also two 86,5 and 87 in the middle between *O* and *H*, and so find the frustum *LCDL*, which two frustums added to the content of the crown *CRD* gives the coutent required; 'Suppose the content of the crown 32 ale gallons, the depth *GO* 15, and *OH* 21 all in inches ?'



The

The prod. of the top diam. 20,5 by 80,1	6504,4	
The prod. of the greateſt diam. 89 by 89,5	7965,5	add
4 times prod. the mid. diam. 85,5 by 89 by 4	29412,0	
Sum =	43881,9	
multiply by the height GO	15	
	658228,5	
ſee Ex. 13.	,004464	

content of the fruſtum ALLB = 305,4180240 A. Gal.

Again,

the prod. diam.	at the bottom H 83 by 83,5	6930,5	rule 8.
	at the gr, bulge 08,9 by 89,5	7965,5	
	at the middle at P 8,65 by 87,4	30102,0	ſect. 7.
ſum.		44998,0	
multiply'd by the height HO		21	
		944958	
,000464 inverted =		464000	
		377983	
		56697	
		3780	

The content of the fruſtum CLLD =	438460	
Content of the fruſtum ALLB =	305418	
Content of the crown CRD =	32	
Content of the whole copper ABCRD	775,878	A, Gal.

If you meet with a ſtill or any veſſel that hath two bulges you muſt conſider it as three fruſtum, &c. I might add more examples to this head if it were not for tautology, for if ſect. 7. be underſtood there can nothing come amiſs to a gauger, even the 8th rule itſelf will ſerve the whole, as appears by inſpecting the foregoing and following examples, for ſince it meaſures all fruſtums, and all ſegments, &c. let a veſſel be ever ſo irregular it may be divided in fruſtums, ſegments, hoofs, &c. whoſe contents

contents taken altogether gives the content of the veſſel itſelf. To inch any curve-lin'd veſſel is but (as in Ex. 19) to find the content at every inch &c. deep, which being ſo eaſy from what goes before, that I think an example needleſs.

CASK-GAUGING.

BY Rule 8, Sect. 7th, any caſk may be gauged without regard to the generating curve, but to confirm this *rule* or *theorem* 60, I ſhall ſhew the common way of caſk-gauging, firſt, (viz.) by aſſuming the caſk's form of which are four, and are diſtinguiſh'd thus,

Variety,	Is ſaid to be two equal Fruſtums of a	the Staves are then	from head to bung
1	Cone	Steight	
2	Parabolic Conoid	Curved little	
3	Parabolic Spindle	Curved more	
4	Spherold	Curved much	

To theſe 4 varieties ſome add 2 varieties more, (viz.) one between the parabolic's conoid and ſpindle, call'd the equilateral hyperbolic ſpindle, and another between the parabolic ſpindle and ſpheroid, call'd the circular ſpindle. But ſince the common way to know theſe varieties is only by gueſs, (unleſs you'll be at the expence of proſecuting ſuch *theorems* as 49, 50, and 51,) theſe 4 varieties will ſerve in this place, I preſume that any one who becomes acquainted with the general method (*theo.* 60) will not bodder his mind with particular rules and ſuppos'd forms.----If the bung be in the middle of the caſk, and the heads thereof equal, then the caſk may be gauged as one fruſtum, otherwiſe it muſt be gaug'd at twice; therefore, when you take the dimenſions of a caſk, obſerve that the bung diameter be taken in the middle between the two head diameters, and that the two head diameters be equal. If (as commonly happens) the bung diameter be about four or five times the difference betwixt it and the head; the multipliers (as prov'd *theo.* 25, 46, 47 and 48,) for the 4 foregoing varieties may be fixt at 0,51, 0,52½, 0,67½ and 0,69.

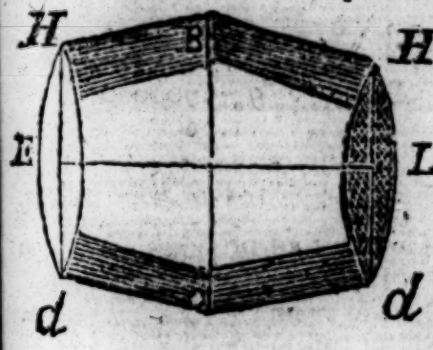
Ex. 24.

Ex. 24. *Let HdDdHB repreſent a caſk of the firſt variety, (viz. of two equal conical fruſtums) whoſe bung diameter DB is 32 inches, head diameter Hd 26 inches, and the length LE 40 inches, to find its content in gallons.*

RULE.

To the head diameter add the bung diameter, and from the ſquare of that ſum take the product of the two ſaid diameters, multiply the remainder by the caſk's length (rule 4, ſect. 7th.) and that product divide by 3 times 359,05 (viz.) 1077,15 for ale gallons, or divide by 882,35 for wine gallons.

Operation.



26 = *Hd* = 26
32 = *BD* = 32

58 } ſum.
58 }

3364
832 product.

2532 diff.
40 *LE.*

1077,15) 101280,00 (94,02 ale gal.
1,22

113,7044 W. G.

By the *Sliding-Rule.*

Set 65,64 (ſquare root of 4 times 1077,15) on D for ale, but 59,4 (ſquare root of 4 times 882,35) on D for wine, to 40 the caſk's length on C, then againſt 58 (the ſum of the diameters) and 6 (their difference) on D ſtands 2 ſuch numbers on C, as 3 times the greater being added to the leſſer gives the content required; ſo for the ale you'll find 31,23 and 0,35, but 3 times 31,23 (viz.) 93,69 added to 0,34 gives 94,03 the content in ale gallons: Alſo, for wine gallons, you'll find 38,12 and 0,41, ſo 3 times 38,12 is 114,36 which added to 0,41 gives 114,77 its content in wine gallons.

Ex. 25. *Given the bung diameter BD 32 inches, the head diameter Hd 26 I. and the length LE 40 I. of a caſk, variety ſecond (viz.) two fruſtums of two equal parabolic conoids, to find its content in ale and wine gallons.*

This

This RULE is the same with Rule 26, Sect. 7th.

Multiply the sum of the squares of the head and bung diameters by the cask's length, and that product divided by 718,1 (viz. twice 359,05) gives the content in ale, or by 588,23 for wine gallons.

The square of {32 / 29} the {bung / head} dia. is {1024 / 676}

sum 1700

length *LE* 40

68000

multiply by ,$^{092}_{\frac{1}{2}}{}^{85}$ (viz.) by ,00013925

Answer in ale gallons, 94,6900000

1,22

By the *Sliding-Rule*. 115,6 wine gallons.

For {ale / wine} set {26, 8 / 24,25} [square root of {718, 1 / 588,23}] on D to 40 the cask's length on C, then against {32 bung / 26 head} diam. on D, stands {57, 1 / 69,63} and {37, 6 / 45,97} on C, which added together gives {94,7 / 115,6} the content in ale and wine gallons.

EX. 26. *Given the bung diameter BD* 32 *inches, the head diameter Hd* 26 *I, and the cask's length LE* 40 *I.* variety third, *(viz.) two equal frustums of a parabolic spindle, to find the content in ale and wine gallons.*

RULE.

To twice the square of the bung diameter add the square of the head diameter, and from that sum take 0,4 of the square of the difference of these two diameters, multiply the remainder by the cask's length, and that product divide by 1077,15 for ale, or by 882,35 for wine gallons. *(Rule* 28, *Sect.* 7*th.)*

2048 twice sq. BD.

676 sq. *Hd.*

2724 sum.

14,4 6×6×0,4.

2709,6

2709,6 40 length LE.

1077,15) 108384,00 (100,62 A. G.

1,22

Answer 122,7564 W. G.

By

By the *Sliding-Rule.*

Set 32,82 (the ſquare root of 1077,15) on D to 40 the caſk's length on C, then againſt the diameters 32 and 26 and their difference 6 on D ſtands 38,03, 25,1 and 1,34 on C, ſo twice 38,03 viz. 76,06 added to 25,1 gives 101,16 which leſſened by 0,4 of 1,34 viz. 0,536 gives 100,624 the content in ale gallons; likewiſe, by ſetting 29,7 (the ſquare root of 882,35) on D inſtead of 32,82 you'll have the wine gallons.

Ex. 27. *Given the bung diameter BD* 32 *inches, the head diameter Hd* 26 *I. and the caſk's length LE* 40 *I.* variety fourth, *(viz.) two equal fruſtums of a ſpheroid, to find its content in ale and wine gallons.*

RULE.

To twice the ſquare of the bung diameter add the ſquare of the head diameter, multiply that ſum by the caſk's length, and divide by 1077,15 for ale, or by 882,35 for wine gallons.

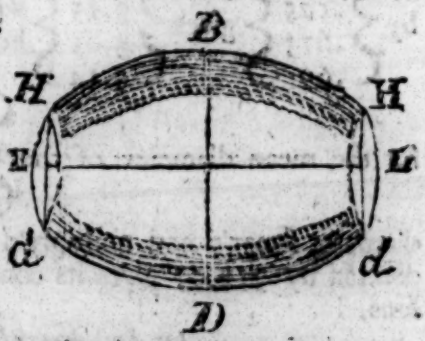
(Rule 24, *Sect.* 7*th.)*

2048 twice □BD.
676 ſq. *Hd*

2724 ſum.
40 length LE.

1077,15) 108960(101,15 AG
1,22

123,4030 WG



By the *Sliding-Rule.*

Set 32,82 on D to 40 on C, then againſt 32 and 26 on D ſtands 38,03 and 25,1 on C, ſo twice 38,03 added to 25,1 gives 101,16 ale gallons: And by ſetting 29,7 on D, *&c.* you'll have the wine gallons.

I have in 3 of theſe 4 laſt examples uſed diviſion, but if you choſe multiplication, you are taught elſe where how to turn diviſors into multipliers, *&c.* I have alſo reduc'd the ale gallons to wine for brevity's ſake, but if wine meaſure only be requir'd it may as eaſily be had as ale meaſure, by uſing the diviſor for wine inſtead of that for ale. Alſo, by the *ſliding-rule* you have the content exactly, which is both eaſier and readier in practice than the common and not true way; by the mean diameter which is as follows,

How to find the Content of a Caſk by the Mean Diameter.

A General RULE.

Multiply the difference between the head and bung diameters by { 0,51 / 0,525 / 0,675 / 0,69 } for the { 1 / 2 / 3 / 4 } variety. { this product added to the head / diameter gives the mean diame- / ter; that is, reduces the caſk / to a cylinder. } Then with this mean diameter, and the caſk's length, work exactly as if you were gauging a cylinder, and you'll have the content perhaps nearly.

By the *Sliding-Rule.*

Set the gauge point AG for ale gal. or WG for wine gal. to the caſk's length on C, then againſt the mean diameter on D ſtands the content on C.

So in the 4 preceeding examples, the difference between the bung and head diameters is 6, which multiply'd

by { ,51 / ,525 / ,675 / ,69 } gives { 3,06 / 3,15 / 4,05 / 4,14 } which added to 26 head diameter gives { 29,06 / 29,15 / 30,05 / 30,14 }

for the mean diameter of the { 1 / 2 / 3 / 4 } variety.--- Which mean diameter ſquar'd and multiply'd by 40 the caſk's length, and divided by 359,05 gives its content in ale, &c. for wine gallons.

Or by the *Sliding-Rule.*

For ale gallons, ſet the gauge point AG 18,94 on D to 40 on C, then

againſt { 29,09 / 29,15 / 30,05 / 30,14 } on D ſtands { 94, / 94,6 / 100,6 / 101,2 } on C the content in ale gallons.

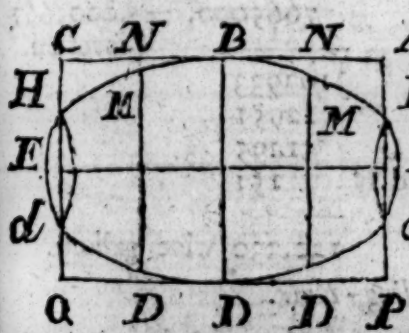
Theſe mean diameters may be had by the *ſliding-rule*, thus: On one ſide of the rule there is a line of inches decimally divided beginning at 0, and ending at 12, under which ſtand ſpheroid, 2d and 3d variety; (and here obſerve, I begin at the cone and end at the ſpheroid in numbering the varieties, but on the ſliding-rule theſe varieties begin at the ſpheroid and end at the cone.) Now to find the mean diameter, take the diffe-

ren

ence between the head and bulg diameters on the line of inches, and againſt it on the line belonging to its variety, you have a number which added to the head diameter gives the mean diameter, ſo againſt 6 on the inches you'll have 4,17 againſt the ſpheroid, 3,78 for the parabolic ſpindle or 2d variety, and 3,39 for the parabolic conoid or 3d variety, which reſpectively added to 26 the head diameter gives 30,17, 29,78 and 22,39 for the mean diameters of the ſpheroid, parabolic ſpindle, and parabolic conoid, according to *Everard*'s Tables page 112, which are not ſo near as theſe found by the above method in this caſe.

How to gauge a full Caſk by help of a fourth Dimenſion, viz. *a Middle Diameter, without regarding the Variety,*

To make this appear plain, it may not be amiſs to ſhew how the dimenſions for this new way of gauging may be taken in an eaſy manner.



Let *HddH* repreſent ſome caſk.

1. Having a long ſtreight ruler as *AC*, with a pin thro' the middle *B*, put this ſmall pin into the bung hole at *B*, move this ruler thereupon as a center untill it cut two other rulers *AP* and *CQ* apply'd to the heads *Hd* and *Hd* of the caſk, and paſſing by *L* and *E* the center of each head; then it is plain by the figure that as the rulers now lye, that if from the bung diameter *BD*, you take the ſum of the two parts *AH* and *CH*, (which *AH* and *CH* are equal if the caſk lye with its axis *LE* parallel to the ruler *AO*) there will leave the head diameter *Hd*. Alſo, if the point *N* be taken in the middle between *B* and *C*, and *B* and *A*, and the parts *NM*, and *NM* be meaſured parallel to each other, and their ſum taken fron *BD* there will leave *MD* a diameter in the middle between the bung and head. Again, if the ruler *AC* be moved till *CH* be equal to *AH*, it is evident *AC* will be equal (the thickneſs of the heads and length of the ſtaves over the heads excepted) to the caſk's length: And if your rulers be

be divided into inches, by these means you may readily have the dimensions of any cask, &c. and must be as good as that by the Callipers, or better.

Ex. 28. " Given the bung diameter *BD* 32 inches, the head diameter *Hd* 26 I, the middle diameter *MD* 30,41 and the cask's length *LE* 40 I. to find the content in ale and wine gallons."

RULE from *Theorem* 60.

To the sum of the squares of the head and bung diameters, add four times the square of the middle diameter (or twice the middle diameter squared), multiply that sum by the cask's length, and that product multiply by $\left\{\begin{matrix},0034642\\,0005667\end{matrix}\right\}$ Or, divide by $\left\{\begin{matrix}2154,32\\1764,71\end{matrix}\right\}$ $\left\{\begin{matrix}\text{ale}\\\text{wine}\end{matrix}\right\}$ gallons,

Operation,

		For Wine.
1024	sq. BD.	
676	sq. *Hd*.	
3696,64	sq. twice MD.	215865,60 product,
		7665000, = ,0005667
5396,64	sum.	[inverted.
40	length LE.	107933
		12951
215865,60		1295
2464000, = ,0004642	inverted.	151
100,204	ale gal. *Answer*.	122,330 wine gallons.

By the *Sliding-Rule*.

For $\left\{\begin{matrix}\text{ale}\\\text{wine}\end{matrix}\right\}$ set $\left\{\begin{matrix}46,4\\52\end{matrix}\right\}$ on D to 40 the cask's length on C, then against 32 and 26 the bung and head diameters, and 60,8 twice the middle diameter on D stands $\left\{\begin{matrix}19, 12, 8, \text{ and } 68,4\\23, 2, 51, \text{ and } 83\end{matrix}\right\}$ whose sum is $\left\{\begin{matrix}102,2\\123,3\end{matrix}\right\}$ $\left\{\begin{matrix}\text{ale}\\\text{wine}\end{matrix}\right\}$ gallons required.

This method being general, it is needless to give any more examples: And if 46,4 and 42 were marked on the *sliding-rules* with A G and W G, instead of the common gauge points, it will hold universally, for casks, frustums, *&c.* as you may easily gather from what's already done in this book. So that in effect the whole art of gauging may be perform'd by one General

General Rule, and that nearly as eaſy as by any of the particular rules, and in general more exact; for what gauger can be aſſured of the caſk's, &c. form, his beſt guide is only ſuppoſition.

To find the ULLAGE *of any Standing or Lying* Cask.

IF a caſk ſtand upright upon one of its heads, it is call'd a ſtanding caſk; but if it lye with its axis [LE] parallel to the horizon, it is call'd a lying caſk. And what I ſhall do in this place, ſhall only be to find the quantity of liquor in a caſk part full in either of theſe poſitions.

And Firſt, of a Standing Caſk

Ex. 29. *Let HddH be a ſtanding caſk filled with liquor to Lq, to find its ullage or quantity of liquor therin.*

RULE.



It is eaſy to perceive even from the fig. that this is only to gauge a ſruſtum, if the caſk be not half full, or two unequal ſruſtums *BHdD* and *BLqD*; if it be more than half full, from hence the old method of ullaging this caſk requires its variety, that ſo by the nature of the generating curve, the ſolidity or ullage may be exactly had; but if we uſe the univerſal rule, or theorem 60, we need have no ſuch ſuppoſition: which is Thus,

To the ſquare of *Hd* 26 the head diameter, add the ſquare of *Lq* 31,05, the ſquare of the diameter at the liquor's ſurface; and alſo, the ſquare of twice *MD*, (*viz.*) twice 29,09; Multiply the ſum by the dry inches *LO* 12, and that product multiply

by { ,0004642 } or divide by { 2154,32 } ale | gallons. Or
{ ,0005667 } { 1764,71 } wine

which is the ſame thing if you eſteem *Lq* as the bung diameter, *MD* as the middle diameter, and *LO* as the caſk's length, then

then the rule and operation are the same with the last example, whose steps being traced you'll have 27,949 ale gallons for the content of the empty part *LHdq*, which taken from 100,6 AG, (found by Ex. 26, this cask and that being equal) the content of the whole cask leaves 72,651 ale gallons for the liquor in the cask, or part *HBLqDd*; or if you had found the content of the half cask *HBDd*, and also of the frustum *BLqD*, their sum would be the same. But if the cask be not half full, use the wet inches instead of the dry, and you'll have the quantity of liquor in the cask at one operation, as easily appears from the fig.----The dimensions of the parts for ullaging this cask may be taken as directed in the last example, or easier Thus, Lay a streight ruler *fo* over the head *Hd* and by the center *L* until a plum-line *QP*, at the end *Q* just touch the bulge or bung *B*; then its evident if the cask be upright, its axis *LE* and plum-line *QP* will be parallel, and *LQ* = half the bung diameter, and *mL* = the difference between half the bung diameter and half that of the liquor's surface, *Qm* being made = 12 = *LO*, and if *Qn* be = 6 = $\frac{1}{2}$ *LO*; then *nM* will be = $\frac{1}{2}$ the difference between the bung *BD* and middle diameter *MD*.

This example may be wrought the old way by the lines N and SS on the sliding-rule, Thus, first you must find the content of the cask, then set the cask's length 40 (in this case) on the line N to the radius of segments 100 on the line SS, then against 28 the wet inches on N stands a reserv'd number 71,6 on SS. Secondly, set 100 on B to the whole content 100,6 on A, then against the reserv'd number 71,6 on B stands 72 on A, the gallons remaining in the cask ale measure, and is nearly $\frac{1}{4}$ of a gallon too little, as found by the last example, which may be depended on as very near. *Note*, If you use the dry inches you'll have the empty part's content.

Ex. 30. " Let *HddH* represent a lying cask full to *LOQ*, whose bung diameter *BD* is 32, head diameter *Hd* 26, length *EL* 40, and dry inches *BO* 12; and let *MD* a diameter in the middle between the head and bung be 30,5, all given in inches: To find the ullage or measure of the empty part *BLOQ*? " RULE,

To the sum of the areas of the segments at the bung and head, add 4 times the area of the middle diameters segments and multiply that sum by the length *LE*, and divide by 6 times 282 (viz.) 1692, or multiply by 000591 for ale &c. the same with the two forgoing rules. *Note*, You must take these areas from the liquors surface to the upperside of the cask, if it be more

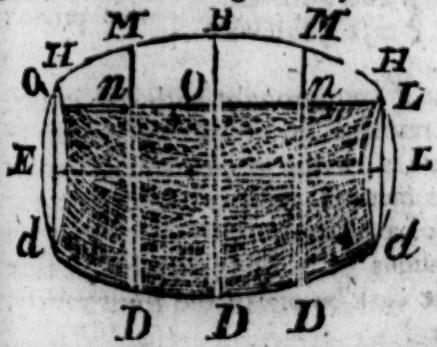
more than half full so yon'll have the content of the dry part, but if the cask be not half full, the wet inches must be the height for the greatest segment &c. the height of the middle segment is had by subtracting half the difference of the bung and middle diameters from the dry inches, or from the wet inches if the cask is not half full: In like manner you may get the height of the segment at the head; which 3 heights being had, annex 3 cyphers to each and divide it by its respective diameter, so will you have three versed-sines, with which enter the table under VS, and write out the 3 numbers against them under segment; and multiply each of those 3 numbers by the square of its respective diameter, so have you the three areas abovemention'd.----- So here half the difference between the bung and middle diameter is 0,75 which taken from the dry inches 12, leaves 11,25 = *Mn*. also, 3 taken from 12, leaves 9 the height of the segment at the head *HE*,

so { 12,000 / 11,150 / 9,000 } divided by { 32 / 30,5 / 26 } its diameter gives { 375 / 368 1/8 / 364 1/8 }

against which in the table 1/8 1/8 considered, stands { 269018 / 263052 / 241327 }

multiply'd by { 1024 / 3721 / 676 } square of { 32 / twice 30,5 / 26 } give { 275,469 / 978,816 / 163,137 }

the area of each segment by Ex. 22, sect. 5.



sum = 1417,422
LE = 40

56696,88
multiplier 000591

33,50795608

the answer for the ullage or measure of *LBQO*.

Which taken from the content of the whole cask 100,624, (found by Ex. 26.) leaves 67,116 ale gallons remaining in the cask. But if the same things were given and the wet inches 12, the work would be exactly the same, and you would have 33,5079 &c. ale gallons in the cask. This method of ullaging a standing or lying cask by the middle diameter, may always be

be practis'd where great exactneſs is required in common practice, as the line S S on the ſliding rule, ſerv'd for the laſt; ſo the line S L may ſerve to ullage a lying cask, thus, ſet the bung diameter 32 on the ſlider N, to 100 on the line S L then againſt the { wet inches 20 / dry inches 12 } on the ſlider N ſtands the reſerv'd number { 67,2 / 32,8 } Then ſet 100 on B, to 100,6 the content of the cask on A, aud againſt the reſerv'd numbet on S L { 67,2 / 32,8 } on B ſtands the ullage on A, viz. { 67,6 / 33,0 } ale gallons in the caſk,——and drawn out.

How to Ullage a Lying Caſk, *by the mean Diameter.*

RULE.

As directed in Ex. 27. find a meaſi ſuitable to the cask's form; then to twice the wet or dry inches, viz. the leaſt of the two, or to either of them when equal, (whlch is when the cask is juſt half full) add the mean diameter, and from the ſum take the mean diameter, half the remainder is the height of a ſegment, which divide by the mean diameter: and look for the quotient under V.S. in the table of ſegmens, and write out the number againſt it under ſegment, which multiply by the ſquare of the mean diameter; and that product by the length of the cask, and this laſt product multiply by { ,003546 / ,004329 } or divide by { 282 for ale / 231 for wine } this prodnct will give the ullage required.

That is what is drawn out if you wrought with dry inches, (they being leaſt) or what remains in, if the wet inches be leaſt. The reaſon of this rule is, if the mean diameter reduce the cask to a cylinder; this method reduces any ſegment or ſlice of the cask to the ſlice of a cylinder, *&c.*

Ex. 31. Let the dimenſions of a lying cask be the ſame as in the laſt example, and the cask be one of the third variety, alſo let the dry inches be 12?

The mean diameter of this cask was fonnd = 30,05, which added to 24 (twice the dry inches) gives 54,05, which leſſen'd by 32 the bung diameter leaves 22,05, half of which is 11,025 this divided by the mean diameter 30,35 gives 366,9 *fere*: againſt which in the table of ſegments under V S (after counted for the 0,9, ſee Ex. 22. ſect. 5.) ſtands 261204, which multiply'd

multiply'd by the ſquare of 30,05 the mean diameter thus,

261204
903

gives 235,939452
multiply by 40 cask's length.

divide by 282) 9437,578080 (33,46 *fere*.

So the anſwer is 33,46 but nearly 33,47 ale gallons for the content of the empty part; but if the wet inches were 12, then there would be 33,47 ale gallons nearly remaining in the cask. - - - - So that by the true method, *viz.* by the middle diameter the ullage of this cask is - - - - - - - 33,5079 *&c.*
by the line S L on the ſliding rule - - - - - - - 33,00
and by the mean diameter - - - - - - - - - - 33,47 *fere*.

Hence in this caſe it appears, that the line S L on the rule gives the ullage above half a gallon too little; but the method by the mean diameter and mean wet or dry inches, does not give ,04 of a gallon too little, ſo that this method is both eaſier and truer than the common one, and doth not require the content of the whole cask; and may alſo be perform'd by Mr. *Shirtcliffe*'s new line mark'd A L on the ſliding rule thus,

Set the mean diameter 30,05 on N to 11,94 on A L, then againſt the mean dry inches 11,025 on N ſtands 33,9 on A L; ſecondly, ſet 33,9 the reſerv'd number (found on A L) on D, to 40 the length of the cask on C, then againſt 30,05 the mean diameter on D, ſtands 33,47 *fere* on C, the ullage required.

Ex. 32. *If the caſk be of the* firſt variety, *of the ſame length and ſame bung and head diameters, with that in the laſt example, only let the wet inches be* 12: *To find its ullage or liquor undrawn out?*

In Ex. 27, the mean diameter of this cask is found 29,06, to which add 24 twice the wet inches, and the ſum is 53,06 from which take 32 the bung diameter and there leaves 21,06, half of which is 10,53 the mean wet inches. Then,

By the *Sliding-Rule*,

Sett 29,06 on N to 18,94 on A L, then againſt 10,53 on N ſtands 33,14 the reſerv'd number on A L. Again, ſet 33,14 on D to 40 on C, then againſt 29,06 on D ſtands 30,7 on C; ſo the liquor in this caſk is 30,7 ale gallons, which is both readier, (becauſe it doth not requiſe the content of the caſk) and truer in practice.

I might have added many more examples in this ſection, but theſe being well underſtood, I hope the practitioner will be at no loſs, for in every variety (beſides the general and new method) I have given a taſte; and if the following method of gauging without inches, be put in practice, I may ſay that you have the art of gauging in this book twice over.

To Gauge without Inches.

IN order to make this plain, let us ſuppoſe each ſide of a cube to be 12 inches, or 1 foot: Then $12 \times 12 \times 12$ is $= 1728$ the ſolid inches contain'd in that cube, or $1 \times 1 \times 1 = 1$ foot, the ſolidity of the ſame cube; ſo that if the ſolidity of any ſolid in inches be divided by 1728, the quotient will give its ſolidity in feet. ---- Now ſince the ſolidity of any cube containing 282 ſolid inches is a gallon of ale, it is evident a ſide of this cube will ſhew the length of 1 ale gallon; that is, the cube root of 282 is equal to the ſide of a cube which holds an ale gallon, which by Ex. 10, ſect. 4, is found equal to 6,558. So that if you take diamenſions by a line or ſcale of inches, and call every 6,558 inches 1; it is evident, the ſolidity of any ſolid given in theſe parts, will be ale gallons.— Now to make a rule or ſcale for this purpoſe; take a ſtreight four ſided rule of any convenient length, ſuppoſe a yard long, on which from the larger diagonal ſcale on the plain ſcale, or any other true ſcale of inches, take 6,558 and lay it upon the rule, making 1, 2, 3, &c. at the end of every 6,558 inches, denoting gallons in length; divide each of theſe parts into 10 equal parts, for tenths of gallons in length; and each of theſe parts into 10 more equal parts: So by this rule ſo divided you may take dimenſions in gallons, and hundred parts of a gallon, mark this face of the rule with the letters A M denoting ale meaſure. In like manner, with the cube roots of 231, 2150,42, and 268,8, divide the other three ſides of the rule, ſo you have a rule for any of the four principle meaſures, viz. ale or beer, wine, corn, and malt buſhels; and if it ſhut up in four pieces (9 inches in every piece) it will be very convenient for carriage, &c. Now ſince there is no mention made of the denumination of the dimenſions taken in ſect. 7, it is evident if thoſe be conſidered as taken by this

rule,

rule, you have the content of every ſolid there mentioned in gallons, of ſuch kind as you took the dimenſions.

Ex. 33. If each ſide of a cube be 60,2 ale meaſure? This cube contains 218167,208 ale gallons. Ex. 1. ſect. 7.

Ex. 34. If the length of a cheſt be 15, depth and width each 7,2 wine meaſure? --- This ſolid contains 777,6 wine gallons. Ex. 2. ſect. 7.

Ex. 35. If the diameter of an equal baſed round veſſel be 2,1 corn meaſure, and its length 8? --- This veſſel will hold 27,7 corn gallons. Ex. 5. ſect. 7.

Ex. 36. If the diameter of a cones baſe be 10 ale meaſure, and its axis 68,1? --- This cone will hold 1782,858 ale gallons. Ex. 8. ſect. 7.

Throughout the whole ſection, I preſume any gauger knows he muſt always take dimenſions anſwerable to thoſe in the inſide of the veſſel; but in meaſuring a ſolid, the dimenſions muſt be taken on the outſide.

(*See the* Table *of* Ullaging *at the end of the book.*)

SECT. X.

SECTION X.

PRACTICAL QUESTIONS.

Queſt. 1. *HOW many hewn ſtones each 3 feet long, and 2½ feet broad, will pave a walk, whoſe length is 40 yards, and breadth 3 yards?*

RULE. Divide 1080 feet the area of the walk, by 7,5 feet the area of one ſtone, and the quote 144 is the number of ſtones ſought.

Queſt. 2. *How many panes of glaſs each 7 inches ſquare, will ſuffice 4 windows, each 5 feet high, and 3 feet 7 inches broad?*

The area of all the windows is 10320 inches, divided by 49 inches the area of 1 pane, gives 210 $\frac{10}{49}$ for the number of panes required.

Queſt. 3. *How many rafts each 2½ inches broad, and 1½ inches thick, can be ſawn out of a piece of equal ſquar'd timber; the lengh of each end being 17 ½ inches, and breadth 10 inches?*

The area of each baſe or end of the timber is 175 inches, divided by 3,75 inches the area of one end of a raft, gives 46,66 for the anſwer.

Queſt. 4. *There is a room whoſe circuit is 20 yards and height 4 yards, to be hung about with tapeſtry 2 foot wide, all except a door caſe whoſe height is 8 feet and breadth 4 feet: How many yards of tapeſtry will ſerve?*

From 720 feet the area of the room, take 32 feet the area of the door caſe; the remainder 688 feet, divided by 2 feet the breadth of the tapeſtry, gives 344 feet, or 114 ⅔ yards for the required length of the tapeſtry.

Queſt. 5. *If a board be 9 inches broad, how much in length will make 3 feet?*

If the length multiply'd by the breadth gives the area, its evident that the area divided by either of theſe dimenſions, the quote will be the other dimenſion; ſo 432 inches, (viz. 3

3 times

times 144) divided by 9 inches, gives 48 inches or 4 feet, for the required length.

Queſt. 6. *If a plank be* 3 *inches thick and* 8 *broad, how much in length will make a ſolid foot?*

Since the length, breadth, and thickneſs multiply'd together gives the ſolidity; its plain, if the ſolidity be divided by the product of two of theſe, the quote will be the third of them. So by the *Sliding-Rule*, ſet 0,125 (viz 3 inches turn'd to a diviſor) on A to 3 inches on B, then againſt 1728 (the inches in a ſolid foot) on B, ſtands 72 inches on A, the required length 6 feet.

Queſt. 7. *If a brick wall be* 100 *feet long, and* 25,21 *feet high, and* 3 *bricks thick, how many roods are contain'd therein?*

Here 100 times 25,21 is 2521, and 6 times 2521 is 15129 which divided by 3 is 5042, and this divided by 272 gives 18 roods nearly. But thoſe two diviſions and one multiplication, may be reduc'd to one factor or diviſor which may be conſtant gauge point thus, always divide 3 times 272,25 by the number of half bricks thick, and the quote is gauge point for that thickneſs, ſo here 816,75 divided by 6 gives 136,12.

By the *Sliding-Rule*, ſet 136,12 on B to 100 on A, then againſt 25,21 on B, ſtands 18 roods *fere* on A.

Brick thick.	Multipliers.	Diviſors.
1	,00245	408,37
1½	,00367	272,25
2	,00489	204,19
2½	,00662	163,35
3	,00734	136,12
3½	,00857	116,68
4	,00881	102,1
4½	,01102	90,75

By the above method, this table is made for any thickneſs between ½ and 5 bricks thick; by which a brick wall of any of the thickneſſes, may be readily meaſured either by the pen or rule; thus, multiply or divide the area of the wall in feet by the multiplier or diviſor in the table, anſwer the thickneſs and the product, or quote, gives the roods in that wall.

Queſt. 8, *How many bricks each* 9 *inches long,* 4½ *inches broad, and* 3 *inches thick, muſt be taken to build a wall* 100 *feet long,* 20 *feet high, and* 1 *foot thick?*

Divide 3456000 inches the ſolidity of the wall, by 121,5 inches, the ſolidity of 1 brick, and the quote 2836 is the number of bricks.

Queſt. 9.

Quest. 9. *If a plank* 14 *feet long,* 1 ½ *foot broad, and half a foot thick, can be sold for* 8*d. a foot running measure,* 7*d. a foot superficial measure, and* 10*d. a foot solid measure; which of these ways must it be sold to make the most money?*

Its area is 21 feet, its solidity 10 ½ feet, and its length 14 feet, so 10 times 10,5 is 105 pence its price by the solid foot, and 7 times 21 is 147 d. its price by the flat foot, also, 8 times 14 is 112 d. its price by the running foot; so it sells best by the flat foot.

Quest. 10. *If a cellar be* 18,75 *feet long,* 10 *wide, and* 5,2 *feet deep, how many floors of earth are therein at* 324 *solid feet to one floor?*

The solidity of the cellar is 975 solid feet, which divided by 324 solid feet the solidity of one floor, gives 3 floors and 3 feet for the answer.

Quest. 11. *If the content of a cylinder be* 2150,42, *and its height* 10, *what is the diameter of its base?*

2150,42 the solidity divided by the height 10, and that quote by ,0,7854 the last quote is 273,67, the square of the diameter, whose square root is 16,5 the diameter requir'd.

Quest. 12. *If the length of a ship's keel be* 44 *feet, the depth of her hold* 9 *feet, and mid-ship beam* 20 *feet; what must these dimensions be in another ship of the same mould, that carries a double burthen?*

By Theo. 6, Prob. 162, If you cube any of these dimensions the cube root of the double thereof, will be the like dimensions of the ship of a double burthen; so 44 cubed is 85184, and doubled is 170368, whose cube root is 55,44 for the keel sought: and thus having found one of the dimensions the rest may be had by the rule of three, Thus,

As 44 : 55,44 :: 20 : 25,22 the mid-ship beam.
As 44 : 55,44 :: 9 : 11,34 depth of the hold. } requir'd

By the *sliding-rule.*] Because the burthens are as 1 to 2, therefore set 1 on E,

to $\left\{\begin{matrix}44\\20\\6\end{matrix}\right\}$ on D, then against $\left\{\begin{matrix}2\\2\\2\end{matrix}\right\}$ on E stands $\left\{\begin{matrix}55,44\\25,22\\11,34\end{matrix}\right\}$ on D.

Quest. 13. *If the keel of a ship be* 80 *feet long, mid-ship beam* 32 *feet, and depth of the hold* 14,1 *feet; what is her tunnage, or burthen in tuns?*

The

The uſual way to gauge a ſhip is to multiply theſe 3 dimenſions together, and divide the laſt product by 95. But if there be allowance made for guns, &c. you muſt divide by 100 and the quote is the tunnage. So by the ſliding-rule, ſet 9,78 (the ſquare root of 95) on D to 14,1 on C, then againſt 50,05 (a mean proportional between 80 and 32) on D, ſtands 380 *fere* the tuns requir'd; but if the diviſor be 100 the burthen will be 360,96 tuns.

Queſt. 14. *If the axis of a globe be 4 inches, and its weight 4 pounds, what will be the weight of another globe of the ſame metal, whoſe axis is 8 inches?*

As (64 cube axis) the cube of any part of a ſolid, is to its weight, (4 lb.) ſo is (512 cube of 8) the cube of the like part of any other like ſolid, to (32 lb.) its weight being both of the ſame metal

Queſt. 15. *Whether is half a foot ſquare, or half a ſquare foot greater?*----- Half a ſquare foot being 72 the half of 144, and half a foot ſquare is but 36 the ſquare of 6.

Queſt. 16. *If one fathom of a cable rope whoſe compaſs is 10 inches, weigh 17 pounds; what will be the weight of a fathom 18 inches about?*

As (100 I.) the ſquare of the periphery of any cylinder's baſe, is to (17 lb.) its weight or ſolidity, ſo is (324 I.) the ſquare of the periphery of any other cylinder's baſe of the ſame height and matter, to (55,08 lb.) its weight or ſolidity.

Queſt. 17. *If 26 lb. of powder be ſufficient to charge a cannon, whoſe diameter at the baſe is 8 inches, how much powder will charge one of 5 inches bore?*

Since bullets are as the cubes of their diameters, the weights of powder to drive thoſe bullets forth muſt be as the cubes of the diameters at the baſe: So as 512 (cube of 8) is to 26 lb. ſo is 125 (cube of 5) to $5\frac{178}{512}$ anſwer.

Queſt. 18. *If an oak cheſt be 8 ſolid feet when meaſured on the out-ſide, and but 7 ſolid feet when meaſured in the in-ſide, what is its weight; a cubic inch of oak being 0,537 parts of an ounce?*

Since ſolidity is as weight, therefore, As 1 I. is to its weight 0,537 Oz. ſo is 1728 I. (the difference between 8 F. and 7 F.) to 928 Oz. *fere*, anſwer.

Queſt. 19. *If a board be 8 inches broad at the leſſer end, and 10 feet long, and 10 inches broad at the greater end: how much in length at the leſſer will make a flat foot?*

Becauſe

Because the ends are given in inches, and the length in feet, we shall call the foot to be cut off 12 inches; then multiply (1) half the difference of the two ends by (48) 4 times the area of the part to be cut off, and that product by the length (10). To (480) this product add the product made by the multiplication of (100) the square of the length (10) into (64) the square of the lesser breadth, (8) and from (83 nearly) the square root of this sum take (80) the product of the length, into the lesser breadth, the remainder (3) divided by (2) the difference of the ends gives 1,5 F. for the answer.-------- *Demonstration*, Let b = the breadth of the lesser end, d = ½ difference of the two ends, l = length, s = area of the part to be cut off, and a = its length: Then *per* similar △'s, As $l : d :: a : \frac{da}{l}$ = ½ the difference between the breadths of the part to be cut, then $\frac{daa}{l} + ba = s$ which solv'd gives $a = \frac{\sqrt{4dls + llbb} : -bl}{2d}$ but if the part to be cut off be towards the greater end, you may take the area of the said part from the area of the whole board, and suppose the remainder to be cut off from the lesser end, and so work as before.

Quest. 20. *Let ABDE represent a pair of scales, whose center of grovity C of the beam DE, is supposed to be out of the middle of the beam DE: And a body weighing* 90 *pound in B, only weighs* 40 *pound in the scale A, what is the true weight, and how far is C from the center of gravity of the beam DE?*



Multiply the two weights 90 and 40 together, and the square root of the product 3600 is 60 pounds the true weight, by which scales may be prov'd, &c. Also, as 60 : 90 :: $CD : CE$ for let A and B = the weight of the body put into the scales A and B, then by mechanicks $DC : CE :: e : B$, hence, $A : c :: e : B$ (e representing the true weight) $\therefore e = \sqrt{AxB}$.

Quest. 21.

Queſt. 21. *What length of a rope will fit to tye to a cow's tail, the other end being faſt in the earth, that ſhe may graſe juſt an acre, or* 4840 *ſquare yards, allowing the cow's length* 4 *yards?* ------- Divide 4840 the area by 0,7854 the quote 6161,09 is the ſquare of the diameter of that circle whoſe area is 4840, whoſe ſquare root is 78,49, half whereof is 49,245 the radius, from which take 4 the cow's length, and there leaves 35,245 yards the anſwer.

Queſt. 22 *If the tranſverſe diameter of an ellipſis be* 40, *and the conjugate* 30, *what is its periphery?*

To (100) twice the ſquare root of the ſum of the ſquares of the tranſverſe and conjugate diameters, add $\frac{1}{4}$ part of the conjugate diameter, (10) the ſum (110) is the periphery of the ellipſis nearly.

Queſt. 23. *If a piece of ſquare tapering timber be* 10 *inches long, a ſide of the leſſer baſe* 8 *inches, and a ſide of the greater baſe* 10 *inches: How far muſt one meaſure from the leſs baſe to cut off a ſolid inch?*

Multiply (3) three times the part to be cut off, by (2) the difference between a ſide of each baſe, and the product (6) multiply by (100) the ſquare of (10) the height, to this product (600) add (512000) the product of the cube of the height, (1000) and (512) the cube of a ſide of the leſſer baſe; From (80,03) the cube root of this ſum (512600) take (80) the product of the height and a ſide of the leſſer baſe, and the remainder (,04) divided by (2) the difference between a ſide of each baſe, gives ,015 inches *Anſwer.*——For if $h =$ the frustum's height, $b =$ a ſide of the leſſer baſe, $d =$ the difference between a ſide of each baſe, $c =$ a factor by which if you multiply the ſquare of a ſide of the pyramid's baſe &c. and that product by its axis, may give the pyramid's ſolidity, and $a =$ the requir'd length, Then *per* ſimilar $\triangle$s, As $d : h :: b : \frac{bh}{d}$,

ſo $\frac{cbbbh}{d} =$ the ſolidity of the top pyramid, then by Prob. 162,

As $\frac{cbbbh}{d} : \frac{b^3 h^3}{d^3}$ or in lower terms, As $c : \frac{hh}{dd} ::$

$\frac{cbbbh}{d} + z : \frac{ch^3 b^3 + zdh^3}{cddd}$ the cube of the axis of the top pyramid and the part to be cut off, [z repreſenting the

part to be cut off,] whose cube root is $\sqrt[3]{:\frac{h^3b^3}{d}+\frac{zdh^2}{cd}:}$ from which take $\frac{hb}{d}$ and there leaves $\sqrt[3]{:\frac{h^3b^3}{d}+\frac{zdh^2}{cd}:}-\frac{hb}{d}=a.$

NOTE, If it be the frustum of a cone, you may multiply the part to be cut off by $\frac{1}{4}$ of 0,7854, and work with the diameters instead of the sides &c. by the rule; and so find the length of any proposed part from the lesser base; and consequently from the greater base too, by observing what is said in Quest. 19.

Quest. 24. *If the perimeter of a circle, of a trigon, and of a square be each = unity, which is the biggest.*

$1^2 \times ,07958 = 0,7958$
$\frac{1}{3} \times \frac{1}{3} \times 0,43303 = ,04811$ } area of the { circle. trigon.
$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16} = ,0625$ } { square.

By which it appears that of all the superficial figures contain'd the same perimeter, the circle is the greatest.

Quest. 25.—— *A globe, a cube, a cylinder,*
All three in surface equal are,
What do they differ in gravity
In weight or in solidity?

Suppose the surface, or superficies of each to be 3,1416, $0,5236 =$ the solidity of the globe, And $\left\{\frac{3,1416}{4,7124}\right\}^{\frac{1}{2}} = ,821$ the diameter and height of the cylinder, which cubed and multiply'd by 0,7854 gives 0,42985 the solidity of the cylinder; Also $\frac{3,1416}{6} = 0,5236$ the square of a side of the cube, whose square root 0,723 being cubed gives 0,3786 the solidity of the cube, so the globe is greatest, and the cube is the least: Hence, of all solids a globe is the greatest that can be under the same surface.

Quest. 26.— *A whin, a thorn, a sheaf of corn,*
Each to be measur'd are;
Come let us mind, if we can find,
A rule such things to clear.

The usual way to measure irregular solids, such as craggy stone, &c. is to fill a vessel with water, and then put in the solid, and by measuring the water that runs over the top of the vessel, or by taking out the solid and measuring the empty part of the vessel, you may know the solidity of such solids.

Quest. 27.

Queſt. 27.— *A cylinder its ſaid, there is to be made,*
To take the leaſt wood that can be:
Pray now let us ſee, what dimenſions be,
To hold a gallon of brandy.

Divide [462] twice the given ſolidity by ,7853 and [8,38 inches] the cube root of the quote is the diameter, by which the height [4,21.] may be found: [per queſtion 11.] For Let e = the diameter of each baſe, s = the ſolidity, and a = ,7854, then $\frac{4s}{e}$ = the curve ſurface, and aee = the area of the baſe, But $\frac{4s}{e} + aee$ = a maximum, which in fluxions will be $2aee + \frac{4se}{ee} = 0$, reduc'd gives $e = \sqrt[3]{\frac{2s}{a}}$

Queſt. 28.— *What length of wire will come out,*
One fourth of an inch about,
From braſs in meaſure a foot;
Pray ſir, try if you can do't?

That is, *Out of a ſolid foot of braſs, what length of wire may be drawn that's ,25 of an inch in circumference.*

Now if the periphery of a circle be 0,25, its area will be ,004973, ſo 1728 divided by ,004973 gives 34746 inches the length; which divided by 53360 the inches in a mile gives 6 miles, 758 yards, and 28 inches.

Queſt. 29.— *If in water, a cube of fir,*
Sink three inches down-right,
And if each ſide be twelve inch wide;
What ounces is its weight?

Any liquor will bear its own weight; That is, if a ſolid inch of any matter be of the ſame bulk with a ſolid inch of any liquid, this matter will have no weight in the ſaid liquid: So if one ſolid inch of water weigh 0,578697 ounces, it will be As 1 is to 0,578697 ſo is 432 inches the ſolidity of the wet part of the cube, to 249,997104 ounces, *Anſwer.*----- In like manner you may find the weight of any floating veſſel, *&c.* as a ſhip or the like, by finding the ſolidity of the wet part in inches and multiplying it by the weight of a cubic inch of ſuch water as it ſwims in; where Note, If you find the weight of a ſhip when ſhe is empty, and alſo when ſhe is loaden; it is evident the difference of theſe two weights will ſhew the weight of the ſhip's burthen,

burthen, which may be reduc'd into tuns, Thus, If a ſolid inch of ſea-water weigh 0,594894 *oz. Averdupoiſe*, then 1728 the inches in a ſolid foot will weigh 64,258552 pounds; now 2240 lb. being 1 tun, ſay, As 64,25 &c. is to 1 ſolid foot, ſo is 2240 lb. to 34,84 ſolid feet, whoſe weight is 1 tun of ſalt-water. So if you find the ſolidity of the whole ſpace in the inſide of the ſhip in feet, between the light mark and laden mark, and divide that ſolidity by 34,84 you'll have the true tunnage of the veſſel, which is a much truer method than the common one in Queſt. 13.

Queſt. 30.— *If a tub ten inches deep and no more,*
Hold in ale gallons juſt half a ſcore;
What muſt the two diameters be,
To be in the ratio five to three?

This veſſel being ſuppoſed a conical fruſtum, Find [128,284 inches] the content of a conical fruſtum, whoſe depth [10 I.] and proportion of diameters [3 and 5] are the ſame with thoſe given in the queſtion; Then ſay, As this ſolidity [128,284] is to the given ſolidity [2820 I] ſo is [9] the ſquare of [3] the leſſer ratio [3] to [190,05] the ſquare of the greater diameter: or ſo as [25] the ſquare of [5] the greater ratio [5] to the ſquare of the greater diameter, whoſe ſquare root is the diameter ſought: So the ſquare root of 190,05 is 13,6 inches the leſſer diameter, And as 3 : 5 : : 13,9 : 23 inches, the greater diameter.

☞. In the following Queſtions, I have omitted the Operations, in order to Exerciſe the Genius of the Learner.

Queſtion 1. A ſurveyor having meaſured three fields of encloſure, hath loſt the contents: But remembers, that if 847 acres be added to the 1ſt, the ſum will be equal to that of the other two fields; if 847 acres be added to the 2d encloſure, the ſum will be double to the contents of the 1ſt, and 3d; and 847 acres added to the 3d field, it will make treble the 1ſt, and 2d: How many acres are in each field?

Anſwer. The 1ſt field 77 A. 2d 385, and 3d 539.

Queſt. 2. I have a right-angled triangular meadow, whoſe ſum of the three ſides is 234 poles, and the product of the three ſides if multiply'd one into another, is 454960 poles. Requir'd the ſides and area in acres and poles?

Anſ. 97, 72 and 65.

Queſt. 3.

Queſt. 3. A maſon hath a piece of ſtone in form of a cone, whoſe baſe is 2 feet diameter, and its altitude is 7 feet; out of which he is deſirous to cut the greateſt cylinder or rolling-ſtone, ſo that its length, and the quantity of ſtone that is loſt by the cutting, may both be the leaſt poſſible. Requir'd the dimenſions of the the rolling-ſtone, and how much ſtone is waſted in cutting?

Anſ. 1 foot 4 inches, the diameter of the cylinder, ſolidity 3 feet 445,7445 inches.

Queſtion 4.

A malſter met a gauger on the way
And in their conference he thus did ſay:
I have a ciſtern that's exactly ſquare:
The length breadth and depth all three equal are.
Another too I have o'th' ſame quantity
Leſs in each other part inches 43 } [119,998 buſh.]
What both will hold, you in the margin ſee.
What hold they each, and their dimenſions,
Tell by your ſkill, and your invenſions?

The ſide of the { biggeſt / leſſer } ciſtern { 62,9999314825 / 19,9999314825 } inches.

The biggeſt 116,2793 buſhels, leſſer 3,7187 buſhels.

Queſt. 5. A gentleman has an horizontal triangular garden, at the angles of which are erected 3 towers A, B and C, their diſtances from each other, and heights as below. Now a ladder is to be erected at ſome point within the garden, ſo as from thence exactly to reach the top of each tower. The length of this ladder is requir'd?

Heights of the towers.		Diſtances.	
A 38 / B 42 / C 45 } feet.	from	A to B 50 / B to C 40 / C to A 47 } feet.	

Anſ. 49,5522 feet, the length of the ladder as requir'd.

Queſt. 6. There is a piece of timber 12 feet long, ſquaring 8 inches, and it is to be converted into a round piece 8 inches diameter. I demand how many ſolid inches muſt be taken off to convert it into a round priſm, and how many ſolid feet this cylinder will contain?

Anſ. The content of the cylinder is 4,1888 ſolid feet, and the content to be taken off is 1977,7536.

An

An UNIVERSAL SERIES.

IN PROB. 169, PART I, is laid down an universal method of measuring independant of the equation of the curve &c. It may not here be amiss to conclude with a short and easy *demonstration* of a series converging very swiftly, for squaring a *curve*, &c. of a given equation; and more especially because this series is laid down by a late author without *demonstration*; nor have I ever seen it done by this method I here offer.

First then, Let y = the Semiordinate, e = the Abscissa, and $de^{pn-1} \times \overline{z+fe^n}^m = y$, the Equation of Curve, First put $\overline{z+fe^n}^m$ into a series, and it will be $\overline{z+fe^n}^m = z^m + mfz^{m-1}e^n + \frac{m}{1} \cdot \frac{m-1}{2} \cdot f^2 z^{m-2} e^{2n} +$ &c. which multiplied by $de^{pn-1} \cdot e$ gives $dzme \cdot e^{pn-1} + dmfz^{m-1} \cdot ee^{pn-1+n} + \frac{m}{1} \cdot \frac{m-1}{2} \cdot df^2 z^{m-2} e \cdot e^{pn+2n-1} +$ &c. $= ye = A$ whose Fluent is $\frac{de^{pn}}{n} \times : \frac{z^m}{p} + \frac{mfz^{m-1}e^n}{p+1} + \frac{m}{1} \cdot \frac{m-1}{2} \cdot \frac{f^2 z^{m-2} e^{2n}}{p+2}$ &c. $=$ A, the Area of the required Curve. But to make the Series converge swifter, Let us put $v = z + fe^n$ then $z = v - fe^n$. which being put in the last Series instead of z, and then each Term reduced to a Series, it will stand Thus,

1st.

1st. $\frac{z^m}{p} = \frac{v-fe^n\ ^m}{p} = \frac{v^m}{p} - \frac{mv^{m-1}fe^n}{p} + \frac{m}{1}\cdot\frac{m-2}{2}\cdot\frac{v^{m-2}f^2e^{2n}}{p}$ — &c.

2dly. $\frac{mfz^{m-1}e^n}{p+1} = m\frac{v-fe^n\ ^{m-1}fe^n}{p+1} = \frac{mv^{m-1}fe^n}{p+1} - \frac{m}{1}\cdot\frac{m-1}{2}\cdot\frac{v^{m-2}f^2e^{2n}}{p+1}$ + &c.

3dly.

$\frac{m}{1}\cdot\frac{m-2}{2}\cdot\frac{f^2z^{m-2}e^{2n}}{p+2} = \frac{m}{1}\cdot\frac{m-1}{2}\cdot\frac{v-fe^n\ ^{m-2}f^2e^{2n}}{p+2} - \frac{m}{1}\cdot\frac{m-2}{2}\cdot\frac{v^{m-2}f^2e^{2n}}{p+2}$ — &c. &c. &c. which Series collected becomes

$\frac{v^m}{p} - \frac{mv^{m-1}fe^n}{p\cdot p+1} + \frac{m.m-1.v^{m-2}f^2e^{2n}}{p\cdot p+1\cdot p+2}$ — &c.

where the Law of Continuation is manifeſt; now this laſt Series multiply'd by $\frac{de^{pn}}{n}$, will become $\frac{de^{pn}v^m}{pn} \times : 1 - \frac{mfe^n}{v\cdot p+1} + \frac{m\cdot m-1\cdot f^2e^{2n}}{v^2\cdot p+1\cdot p+2}$ — &c. $= A$, by which the Area of a known Curve may be had in a few Terms, as for EXAMPLE, Let t = the Tranſverſe, c = the Conjugate, y = a Semi-ordinate, and e = the Abſciſſa of an Ellipſis or Hyperbola,; then by the Property of the Curves (Page 272, PART I.) $y = \frac{c}{t}\sqrt{:te \mp ee}$: or $\frac{2c}{t}\sqrt{:te \mp ee} :: 2y$, $\frac{2c}{t} \times e^{\frac{1}{2}} \times \sqrt{:t \mp e}$:, which compared with the general Ordinate $de^{pn-1} \times : z \pm fe^n\ ^m$: it will appear that $\frac{2c}{t} = d$, $e = e$, $n = 1$, $pn - 1 = \frac{1}{2}$ ∴ $p = \frac{1}{2}$, $t = z$, $f = \mp 1$, $m = \frac{1}{2}$, $v = t \mp e$, which ſubſtituted in the laſt Series, we have

$A =$

$A = 4y \times : \frac{e}{1.3} \pm \frac{ee}{1.3.5.v} - \frac{eee}{3.5.7.v^2} \pm \frac{e^4}{5.7.9.v^3} -$ &c. : That is

$4y : \times \frac{e}{1.3} - \frac{ee}{1.3.5.v} - \frac{eee}{3.5.7.v^2} - \frac{e^4}{5.7.9.v^3} -$ &c. : for the Area of the Hyperbola.

$4y : \times \frac{e}{1.3} + \frac{ee}{1.3.5.v} - \frac{eee}{3.5.7.v^2} + \frac{e_4}{5.7.9.v^3} -$ &c. : for the Area of the Elliptical Segment, v being $= t - e$, but in the Hyperbola, $v = t + e$. ---- ALSO, If $t = e$, the laſt Series gives the Area of a Circular Segment, whoſe Verſed-ſine is e, Semi-chord y, and Diameter of the Circle $= t$.

By the ſame general Series may be had the Solidity of a Solid, if the Equation of the generating Curve be known: Thus, Let t, c, e, and y, repreſent the ſame things as before, and put $q = 3,1416 \frac{cc}{tt}$, then by the Equation of the Curve $\frac{cc}{tt} \times : te \mp ee := yy$, ſo $3,1416yy = qte \mp qee = qe \times : t \mp e$: which compared with the general Ordinate $dc^{pn-1} \times \overline{z + fe^n}^m$ we have $q = d$, $e = e$, $m = 1$, $pn - 1 = 1$ ∴ $p = 2$, $z = t$, $f = \mp 1$, $m = 1$, by which the general Series becomes $\frac{qe^2}{2} \times : t \mp e \pm \frac{ee}{3} - 0 : = \frac{qtee \mp qeee}{2} \mp \frac{qe^3}{6} - 0$, that is $\frac{qtee}{2} + \frac{2qe^3}{6} =$ the Solidity of the Hyperbolic Conoid. ---- And $\frac{qtee}{2} - \frac{2qe^3}{6} =$ the Solidity of the Spheroidical Segment, which is the ſame in effect with *Theorem* 37, Page 233, PART I.

The Application of this Series may be carried a great length, and may alſo be applied to Surfaces, &c. But this Book ſwelling ſo far beyond its intended bounds, I muſt conclude humbly hoping my worthy Subſcribers will excuſe their Names being not inſerted, ſince here is much more Matter in the Book than at firſt intended, or mention'd in the propoſals.

FINIS.

MVSEVM BRITANNICVM